1. For $M = O(B)$, what is the I-O complexity (number of block transfers) to transpose an $n \times n$ matrix?

2. Design an efficient version of partition sort (quicksort with multiple pivots) for the external memory model with parameters $M$ and $B$. Show that it is comparable to mergesort.
   
   Hint: You may want to use the sampling lemma used for PRAM based partition sort.

3. Show that the average case lower bound for permutation is asymptotically similar to the worst-case bound.

4. A $k$-transposition permutes $n = k \cdot \ell$ elements as follows:
   
   $x_1 x_2 x_3 x_\ell x_{\ell+1} x_{\ell+2} \ldots x_{\ell k}, x_{\ell k+1} \ldots x_{\ell k+\ell}$ are mapped to $x_1, x_{\ell+1}, x_{2 \ell+1} \ldots x_{\ell k}, x_2, x_{\ell+2} \ldots x_{2 \ell k} \ldots$

   Show how to do this in an external memory model using $O\left(\frac{n}{B} \log_{M/B} k\right)$ I-Os.

5. Describe a cache-efficient algorithm for computing the matrix product
   
   $C_{n \times n} = A_{n \times n} \cdot A_{n \times n}$

   for parameters $M, B$.

6. Describe an cache efficient implementation of shear sort in the external memory model with parameters $M, B$.

7. Describe a cache efficient algorithm for constructing planar convex hull of $n$ points in the external memory model.

8. Describe a cache efficient algorithm for finding the maximal elements of $n$ points on the plane in the external memory model.

9. Describe a cache efficient algorithm for computing All nearest smaller value problem in the I-O model.

10. Design a cache-oblivious algorithm for computing matrix transpose for the case $M \geq B^{3/2}$.