Lecture 39: Shape Representation

**Sampling**

How densely one should sample a geometric shape to reconstruct it?

\[ M = \{ x | \text{B}_x \cap \partial P \neq \emptyset \} \]

If \( \partial P \) is connected, Medial axis of \( P \) is a tree.

**Retraction** \( \mathbb{Q} : \partial P \rightarrow M \)

\[ \mathbb{Q}(p) = x \quad \text{if} \quad p \in \text{B}_x \cap \partial P \]

\[ \text{LFS}(p) = ||p - \mathbb{Q}(p)|| \]
Sampling density $\propto \frac{1}{\text{LPCS}(p)}$

curve simplification

$P = \{P_i, \ldots, P_n\}$

$\varepsilon$: tolerance error

$Q \subseteq P: Q = \{q_1, \ldots, q_k\}$

each $q_i$ lies within distance $\varepsilon$ from an chain $q_1 q_2 \ldots q_k$.

$k$ should be small.

- Refinement
- Decimation
Douglas–Peucker algorithm

Decimation Method

Edge contraction method

Optimal algorithm

\[ G = (P, E) \]

\[ (p_i, p_j) \in E \text{ if } d(p_k, \overline{p_ip_j}) \leq \varepsilon \quad \forall i \leq k \leq j \]
Find the shortest path from \( p_1 \) to \( p_n \) in \( G \).

\[ O(n^3) : \text{straightforward.} \]

\[ O(n^2 \log n) \]

Surface Simplification

\[
\frac{4}{3} \quad n \quad \text{Chebyshev metric}
\]
Decimation method.

Garland-Heckbert algorithm.