\(|H| \geq \gamma_0\)

\(\forall z_i \in \mathcal{Z}\), \(\exists \Delta \subset \mathcal{Z} \cup \{\gamma_0\} - \text{cutting a } H\)

\(H_i : \text{ set of lines intersecting } z_i\)

Make each \(z_i\) a child of \(\Delta\)

Cutting Tree \((H_i, z_i)\) for \(1 \leq i \leq s\)

\(n_{z_i} : \text{ for each } z_i \text{, store the number of } H\)

\(\text{lines lying above } z_i\).

Query procedure

\(\text{Query}((\Delta, p))\)

If \(\Delta\) is a leaf, return the

\(#\) lines of \(HA\) lying above \(p\)

Else

\(z : \text{ triangle of } \Delta \text{ that contains } p\)

\(\text{return } n_{z} + \text{Query}(z, p)\)

\(\text{Time: } O\left(\log n\right)\)
$S(n)$: max # leaves in a cutting tree built on $n$ lines.

$$S(n) \leq \begin{cases} 
1 & n \leq r_0 \\

C \cdot r_0^2 \cdot S(n/r_0) & n > r_0
\end{cases}$$

$$S(n) \leq C r_0^2 S(n/r_0)$$

$$\leq (C r_0^2)^i S(n/r_0^i)$$

$$\leq (C r_0^2)^{\log_{r_0} n}$$

$$= c \log_{r_0} n \left(\frac{r_0^2}{r_0}\right)^{\log_{r_0} n}$$

$$\leq \frac{\log_{r_0} c}{2+\log_{r_0} n} \leq n^2$$
Linear size data structure

$S: n$ points in $\mathbb{R}^2$

Construct a spanning path $\pi$ s.t.
every line intersects as few
edges of $\Pi$ as possible.

$\varepsilon$: spanning path
with small
crossing number.

$\chi(\Pi, \varepsilon) = \#$ crossing points

$\Pi = \max \chi(\Pi, \varepsilon)$

$\chi(S) = \min_{\Pi} \chi(\Pi)$
What is the value of \( x(s) \)?

\[
\begin{align*}
x(n) &= \max_{|s|=n} x(s) \\
S &= \{ \mathbf{s} \subseteq \mathbb{R}^2 \}
\end{align*}
\]

\[\Omega(n) \cdot x(n) = O(\sqrt{n})\]

S: set of points

L: set of lines

w: \( L \rightarrow \mathbb{N} \)

\[w(l) = 1 \quad \forall \mathbf{l} \in L\]

i-th step:

\[S_i: \text{endpoint from each connected component}\]

\[n_i = |S_i| = n - (i+1)\]

Choose a constant \( \alpha \), \[\sum_{i} \]

Compute a \((\alpha/\sqrt{n_i})\)-cutting \( L \)
\[ r_i = \sqrt{m_i} \alpha \]

\[ |\Xi_i| = O(r_i^2) = O\left(\frac{m_i}{\alpha^2}\right) \leq n_i - 1 \]

There exists a \( \Delta \in \Xi_i \) that contains 2 points \( p, q \in S_i \).

Add \((p, q)\) to \( \Pi \).

For each line \( \ell \) in \( L \) that intersects \((p, q)\) set \( \text{w}(\ell) = 2 \cdot \text{w}(\ell) \)

\( n_i : \text{w}(L) \) after i-th iteration
\[ w_0 = m \]

\[ w_{i+1} \leq w_i + \frac{w_i}{r_i} \]

\[ = w_i \left(1 + \frac{\alpha}{\sqrt{n-i+1}}\right) \]

\[ w_{n-1} \leq w_0 \prod_{i=0}^{n-1} \left(1 + \frac{\alpha}{\sqrt{n-i+1}}\right) \]

\[ = w_0 \prod_{j=1}^{n} \left(1 + \frac{\alpha}{\sqrt{j}}\right) \]

\[ \leq w_0 \prod_{j=1}^{n} e^{\frac{\alpha}{\sqrt{j}}} \]

\[ \prod_{j=1}^{n} e^{\frac{\alpha}{\sqrt{j}}} \]

\[ w_{n-1} \leq m \cdot \exp \left(\alpha \cdot \sqrt{n}\right) \]

\[ 2^k \leq m \cdot \exp \left(\alpha \cdot \sqrt{n}\right) \]

\[ k \geq 0 \left(\sqrt{n}\right) \]