Computational Geometry CSL 852

Lecture 30

Topic: Quadrees → EWS PD

Quadree: HT? | log | spread |
Canonical Quadtree

If gridpoints have some specific values, say powers of 2, then it is a canonical grid. We want to align our quadtree with such a family of grids.

\[ G_i : \quad 0, \ 2^i, \ 2 \cdot 2^i, \ldots \]

\[ G_1 \]

\[ G_2 \]

\[ G_i < G_j \quad j < i \]

\[ G_i : \quad \frac{1}{2^i} \]

**Advantage:** At depth \( i \) from root, which subquadrants can be present? Can be hashed.
Point location, which is the smallest sub-square, i.e., largest $j$, for which $G_j$ contains $p$.

Query: The set of quadrants.
Space: Unbounded.

Observation: Compress the nodes having only one child.

Observation: A compressed quadrant of $n$ points has at most $O(n)$ nodes.
By hashing, we mean a performance $O(n)$ space $O(1)$ search time (e.g., Universal hash functions).

Speed up point location using "binary search" on the search path.

Claim: In a compressed quadtree, we can do point location in $O(\log n)$ time.
Constructing a Quadtree efficiently

Can we find a "canonical sub-square" that contains a large fraction of the n points, i.e., \( \alpha n \) for some constant \( 0 < \alpha < 1 \)?

\[
T(n) = T(\alpha n) + T((1-\alpha)n) + O(n)
\]

Claim: Given a point set \( P \) of \( n \) points, we can compute a disk \( D \) that contains \( K \) points (\( K \leq n \)) such that \( \text{radius}(D) \leq 2 \text{Ropt}(K) \)
D can be found in $O(n \cdot \left(\frac{n}{k}\right)^2)$ steps.

If $k = \alpha n$, then $O(n)$.

Claim: Compressed quadtree can be constructed in $O(n \log n)$ time.