Claim: Reporting range query for orthogonal rectangles take \( O(\sqrt{n} + \frac{r}{\epsilon}) \) steps using a K-d tree (2-d tree in two dimensions).

Suppose \( Q(n) \) is the worst case query time (# nodes visited) for \( n \) points.

\[
Q'(n) = 2 \cdot Q'(\frac{n}{4}) + 2^k \quad \text{nodes visited}
\]

\( k \) of rectangular regions are edge intersect, where the rectangular region contains \( n \) points.

\( Q(n) \leq 4 \cdot Q'(n) \)
\( K-d \) tree is for \( K \)-dim range search

\[ O_k(n) = \Omega(n^{1-\frac{1}{k}}) \]

\[ S_2(n) = O(n) \quad \text{a point is stored exactly once} \]

\[ S_k(n) = \Omega(kn) \quad \text{(is)} \]

Can we improve the \( \sqrt{n} \) bound?

Perhaps space must increase

Inherent space-time tradeoffs

must be lower bounds

Build a 1-dimensional \( K-d \) tree.

Observation: Any arbitrary interval \([x_1, x_2]\) can be written as a union of \( \lceil 2 \log n \rceil \) disjoint "canonical" intervals.
A two dimensional range tree is a nested data structure where the primary tree is built on x coordinates. And within each node we have a 1 dimensional range data structure on y coordinates.

Query time: \( O(\log^2 n + k) \)

Space: \( O(n \log n) \)