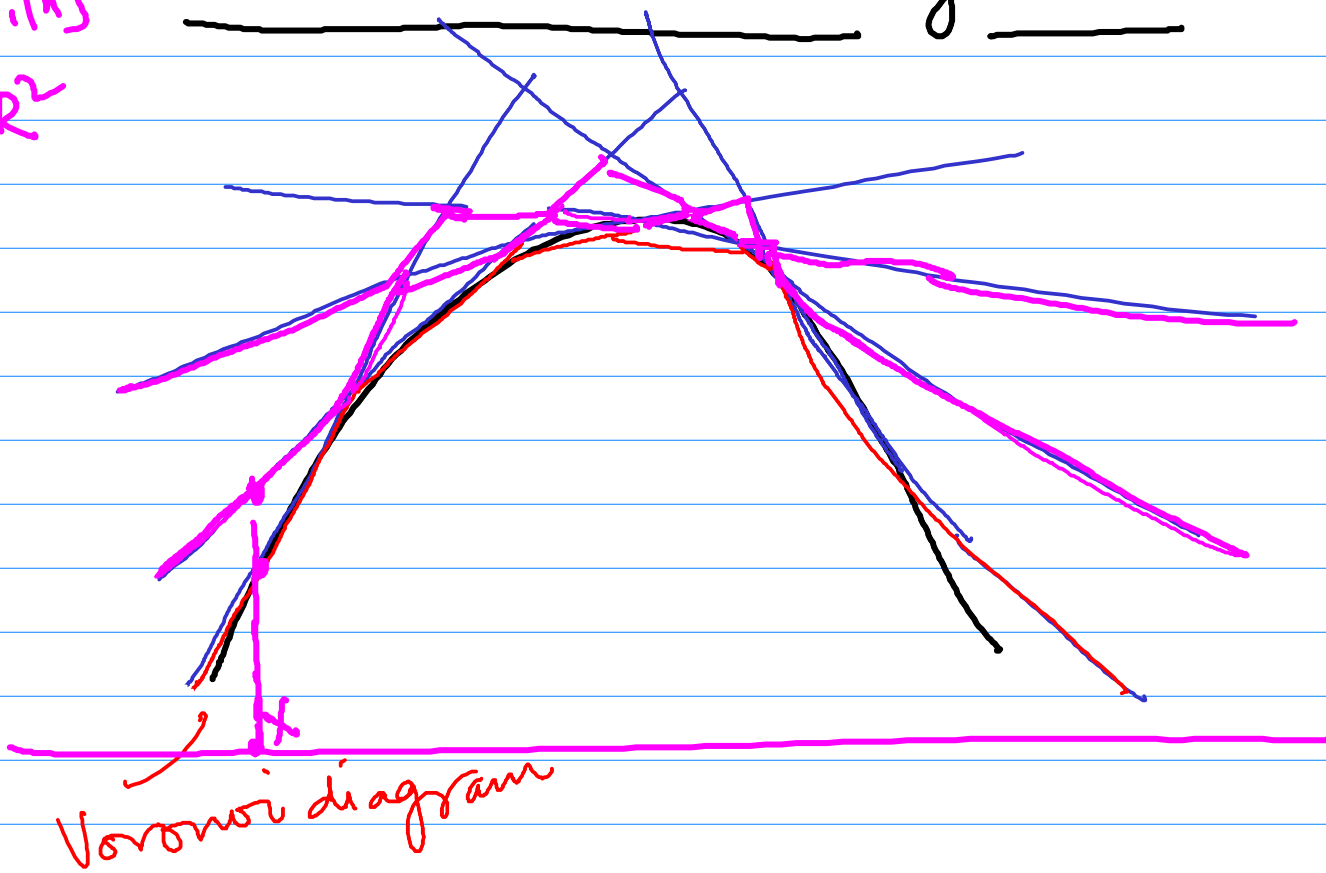
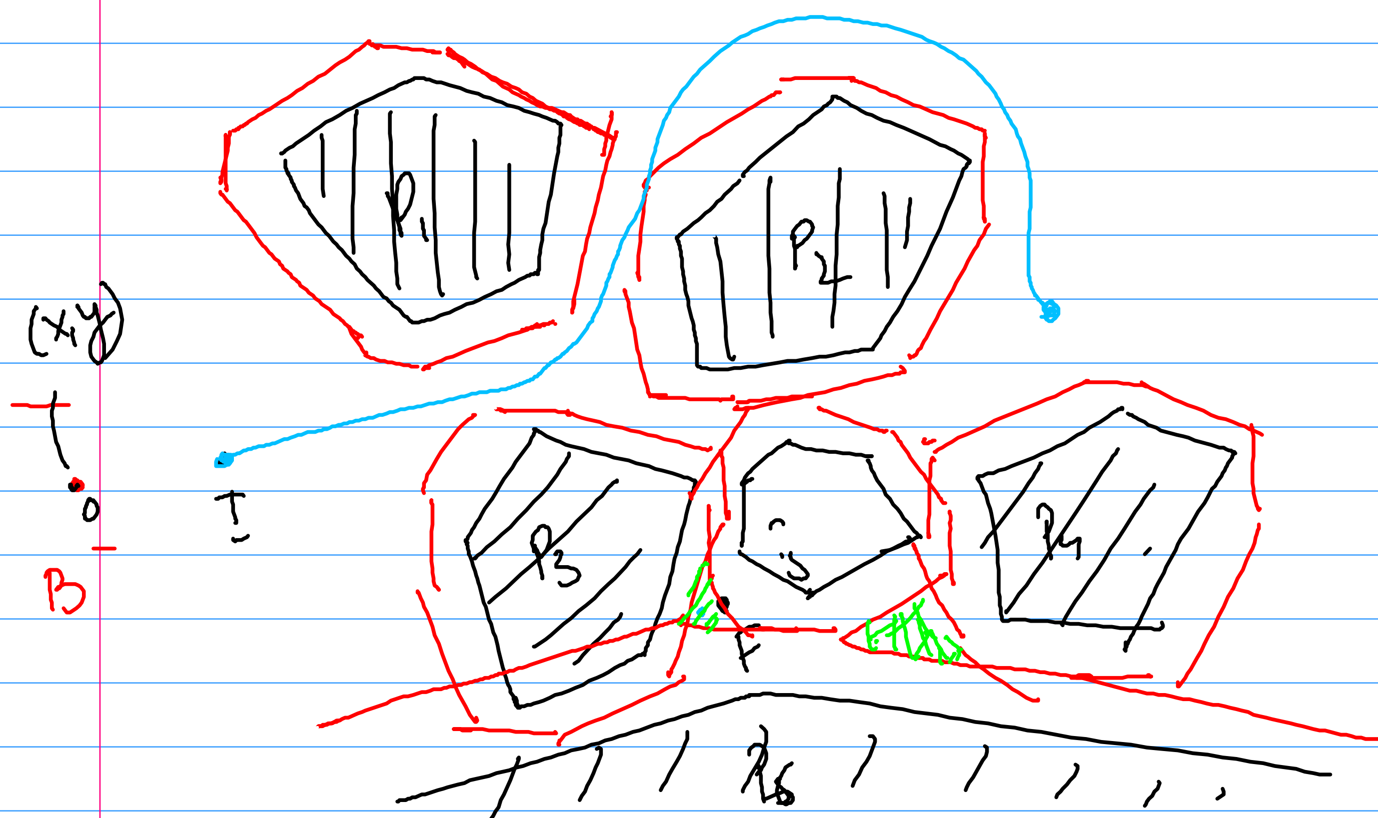
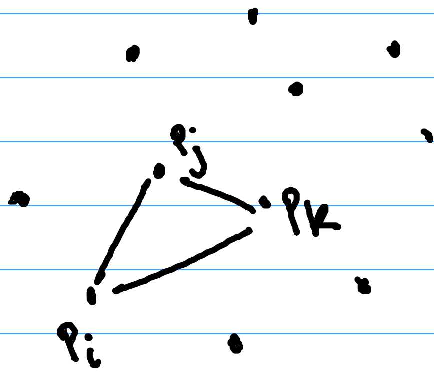
Lecture 23: Arrangements

$S = \{p_1, \dots, p_n\}$ in $\mathbb{R}^2$





$S = \{p_1, \dots, p_n\} \subseteq \mathbb{R}^2$

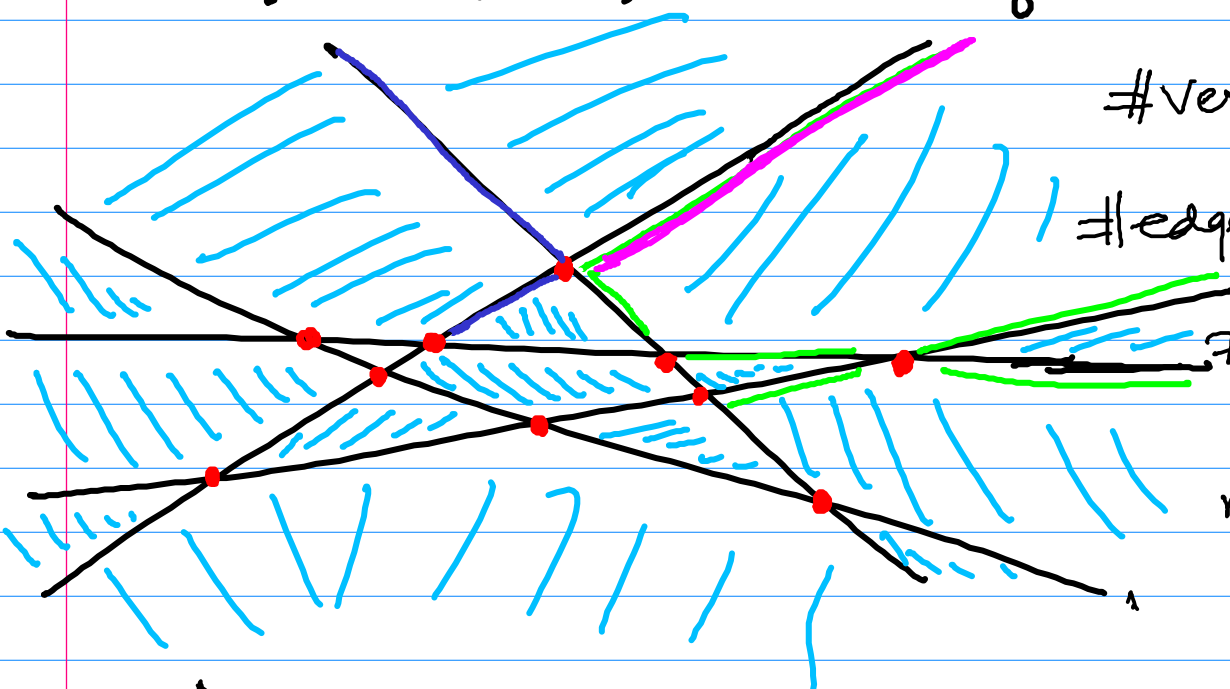


Find the minimum area triangle spanned by S, i.e.

$$\underset{p_i \neq p_j \neq p_k}{\arg\min} \text{ Area } \triangle p_i p_j p_k$$

$\ell_i : y = a_i x + b_i$

$L = \{\ell_1, \ell_2, \dots, \ell_n\}$: set of n lines in $\mathbb{R}^2$

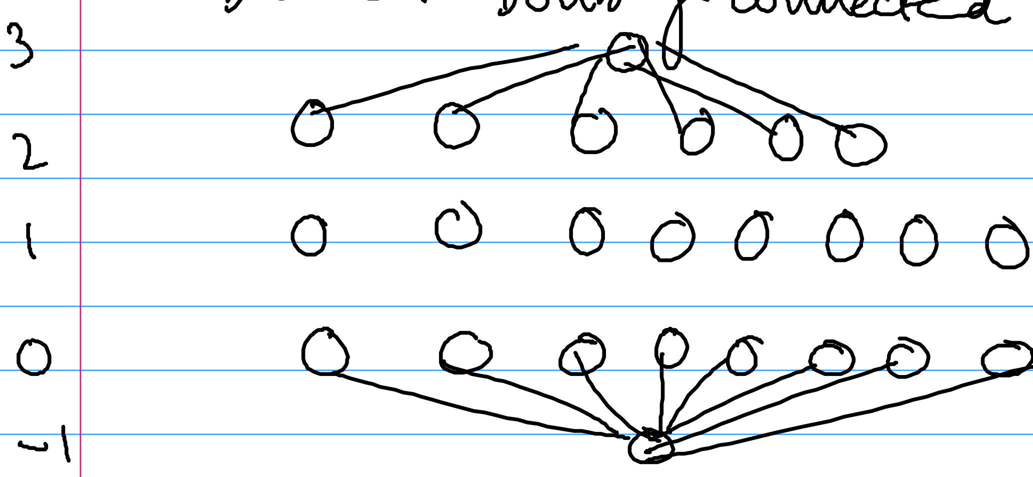


#vertices $= \binom{n}{2}$

#edges $= n^2$

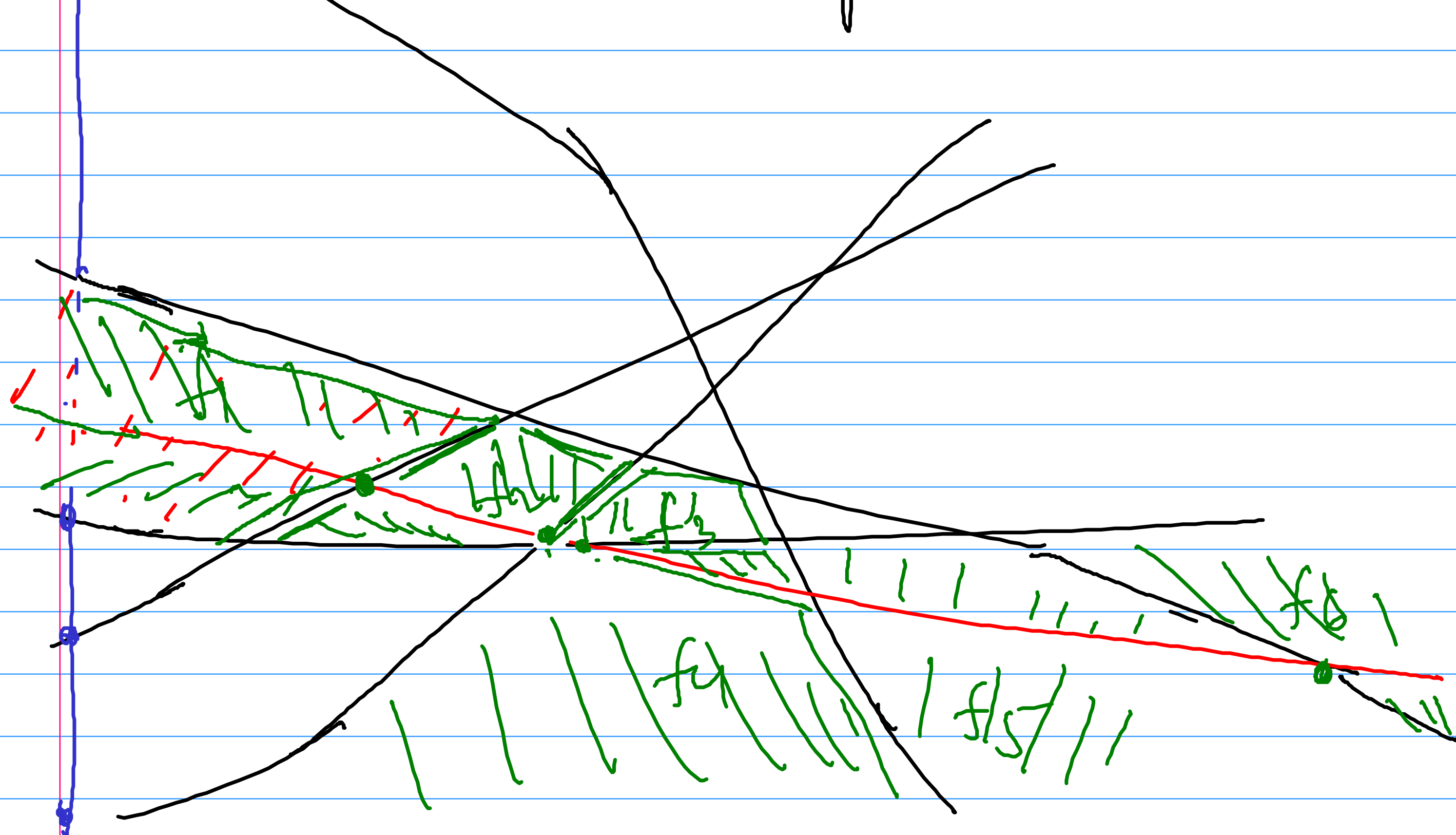
#faces $= n^2 - \binom{n}{2} + 1$

DCEL: Doubly connected edge list



A(L): Arrangement of L

$O(n^2 \log n)$ time to compute A(L) | sweepline.



n_i : # edges in f_i

$\sum n_i$

Zone(ℓ, L) zone of a line in A(L)

set of 2-faces of A(L) that ℓ intersects.

$\mu(\ell, L)$ = # edges in the faces of zone(ℓ, L)

Lemma (zone theorem)

$$\mu(n) = O(n)$$

$L = \{l_1, \dots, l_n\}$ n lines

n_f: # edges in a face f of $A(L)$

$$\sum_{f \in A(L)} n_f = O(n^2) \checkmark$$

Thm:

$$\sum_{f \in A(L)} n_f^2 = O(n^2)$$

Proof:

$$\sum n_f^2 = \sum_f 2\binom{n_f}{2} + n_f$$

$$= \sum_f 2\binom{n_f}{2} + O(n^2)$$

$$\sum 2\binom{n_f}{2} = \sum_{\ell \in L} M(\ell, L \setminus \{\ell\})$$

$$= \sum_{\ell \in L} O(n)$$

$$= O(n^2).$$