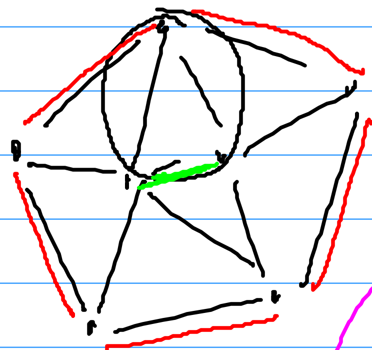
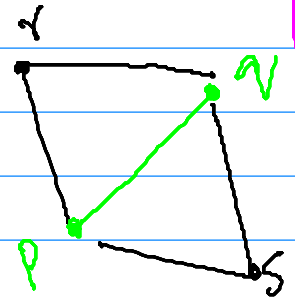
Lecture 19: Delaunay Triangulation



$S = \{p_1, \dots, p_n\} \subseteq \mathbb{R}^2$

T: Triangulation of conv(S)

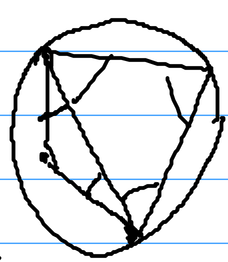
$\boxed{T \text{ is } DT(S) \iff \forall \Delta \in T,\ \Delta = \Delta pqr,\ \text{int circ}(pqr) \cap S = \phi}$



Definition: pq is legal if circle(p,q,r) does not contain s & circ(p,q,s) does not contain r.

Lemma: If all edges in T are legal then T is DT(S)!

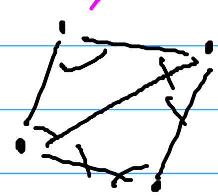
All edges in T are legal $\iff$ T is DT(S)



min angle

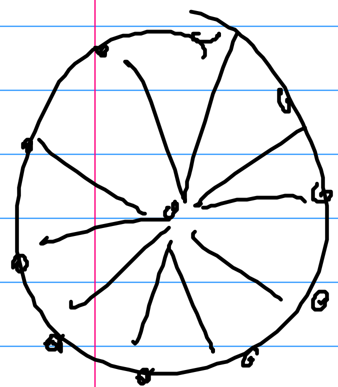
edge flip

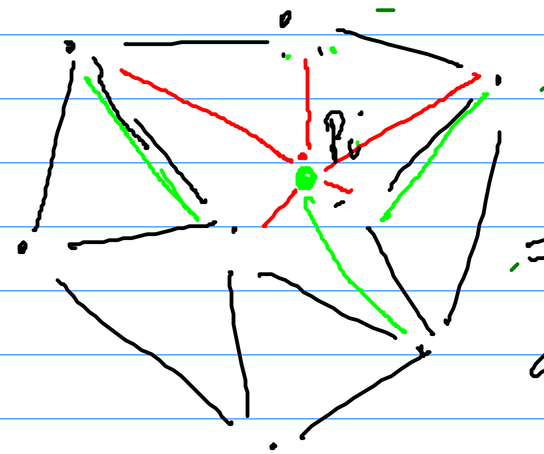
$<$



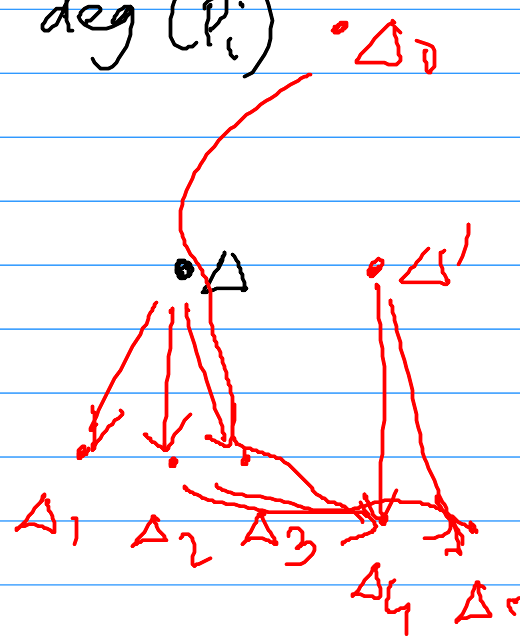
min angle

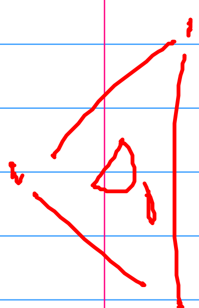
The flip sequence will terminate after $\Theta(n^2)$ steps. (in 2D)

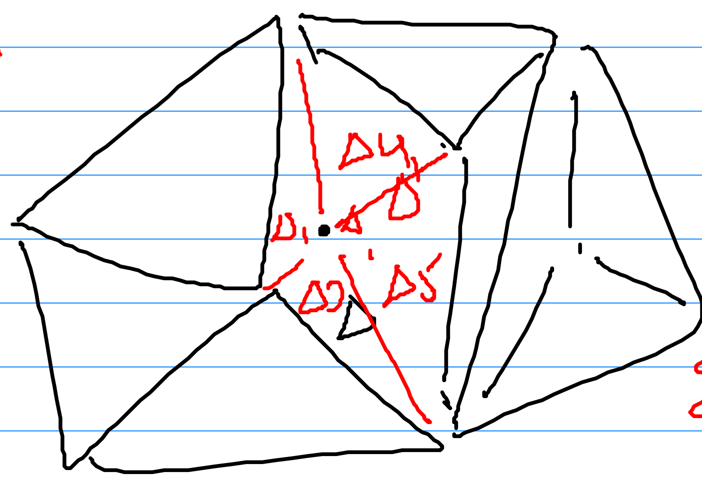




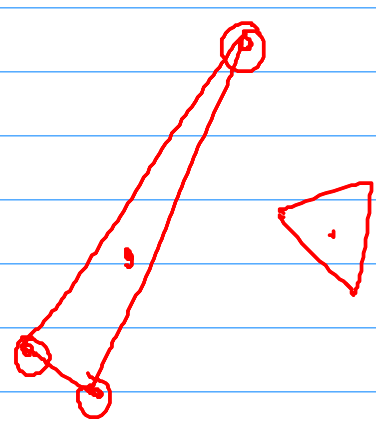
$\#$ edge flips $\propto \deg(p_i)$

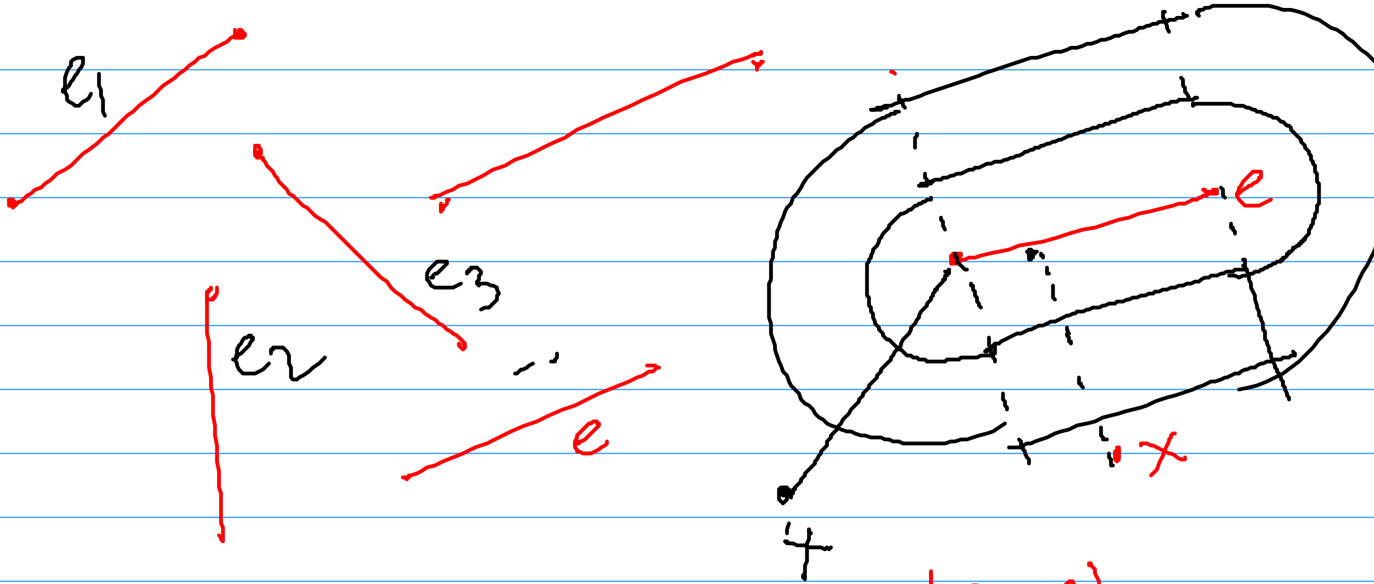


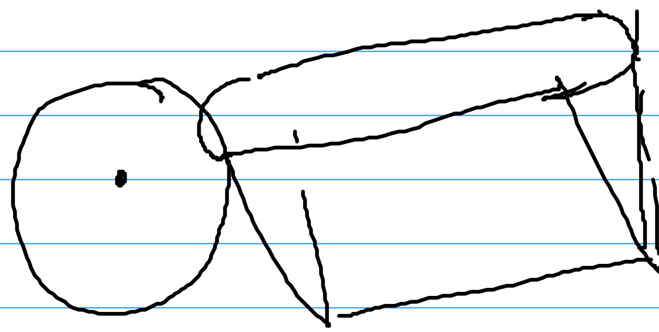




Expected depth $O(\log n)$







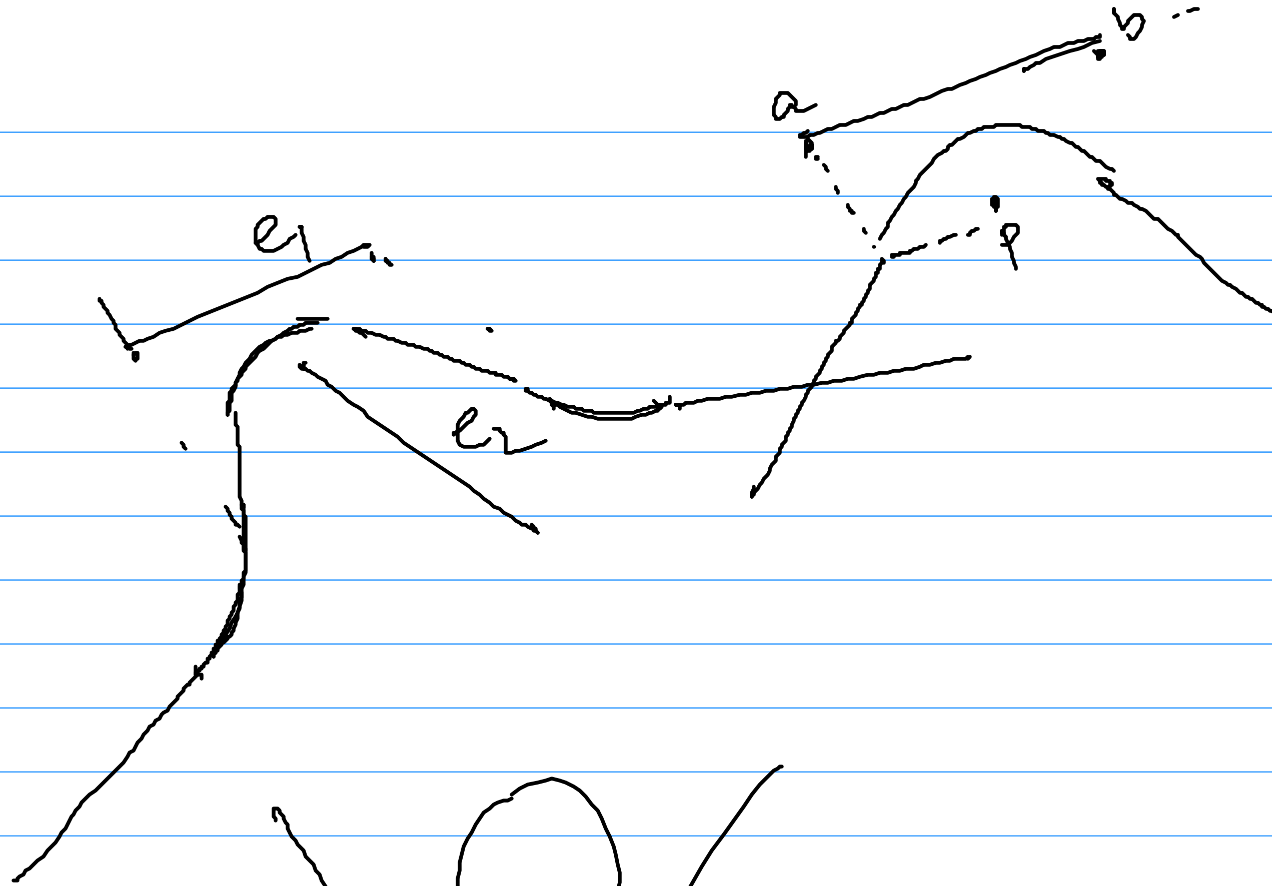
$d(x,e) = \min_{p \in e} \|xp\|$

$$S = \{e_1, \dots, e_n\}$$

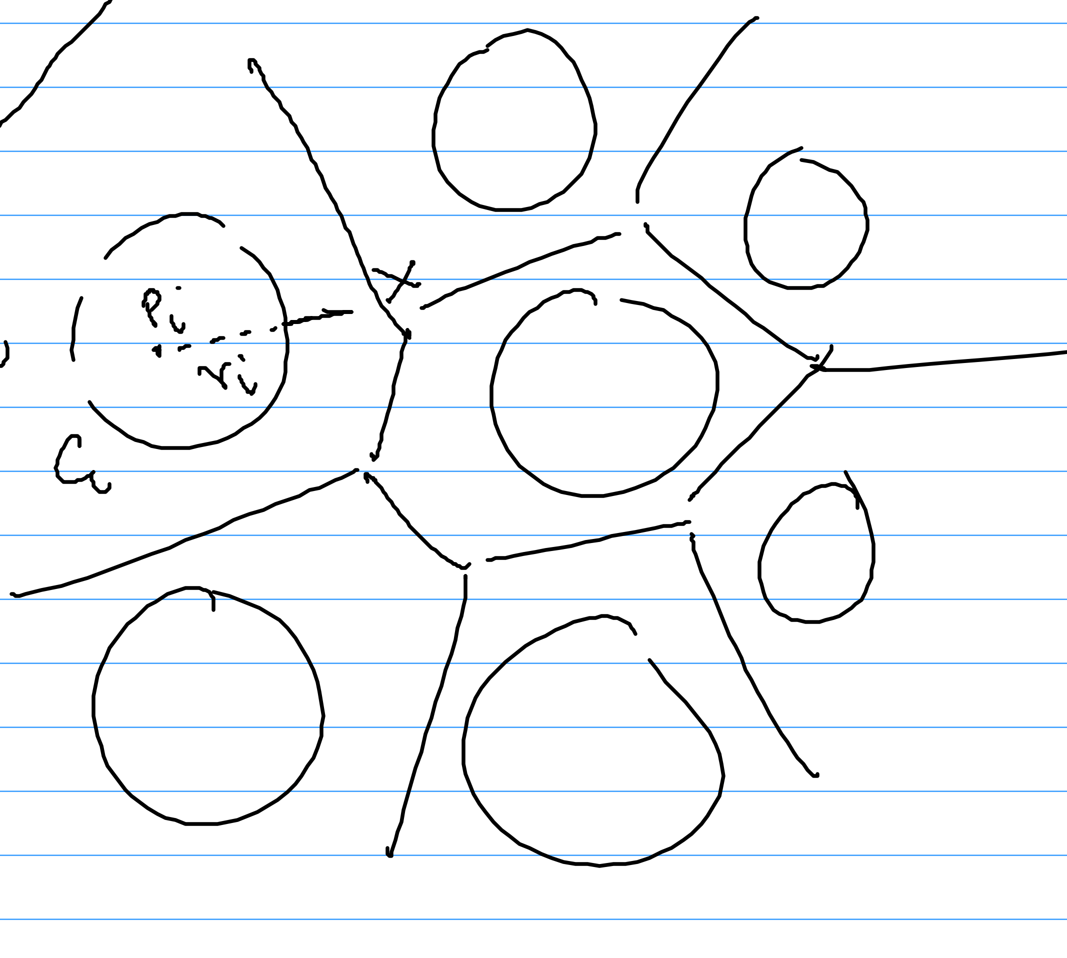
$$f_i(x) = d(x, e_i)$$

$$F = \{f_1, \dots, f_n\}$$

Var(S) : Minimization diagram of F.



$d(x, C_i) =$
$\|x p_i\| - r_i$



$$d(x, c_i) = \sqrt{\|x - p_i\|^2 - r_i^2}$$

Power Diagram

$$\|x - p_i\|^2 - r_i^2$$