Lecture 19: Delaunay Triangulation

\[ S = \{ P_1, \ldots, P_n \} \subseteq \mathbb{R}^2 \]

T: Triangulation of Conv(S)

\[ T \text{ is } DT(S) \iff \forall \Delta \in T \]

\[ \text{circ}(Pqy) \cap S = \emptyset \]

Definition: \( Pq \) is legal if \( \text{circle}(Pqy) \) does not contain \( S \) & \( \text{circ}(Pqy) \) does not contain \( y \).

Lemma: If all edges in \( T \) are legal then \( T \) is \( DT(S) \).

All edges in \( T \) are legal \( \iff \) \( T \) is \( DT(S) \).

\[ \text{min angle} \rightarrow \text{edge flip} \leftarrow \text{min angle} \]
The flip sequence will terminate after $O(n^2)$ steps. (in 2D)

Expected depth $O(\log n)$
$S = \{ e_i, \ldots, e_n \}$

$f(x) = d(x, e_i)$

$F = \{ f_1, \ldots, f_n \}$

$\text{Var}(S) \cdot \text{Minimization diagram of } F.$
\[ d(x, C_i) = \| x - \Pi_{C_i} x \| \]
$d(x_i, c_i) = \sqrt{11 \times \| \mathbf{p} \|^2 - r_i^2}$

Power Diagram