Computational Geometry Lecture 17

Lecturer Prof Pankaj Agarwal

Topic : Voronoi Diagrams

Nearest Neighbor searching

$S = \{p_1, \ldots, p_n\} \subseteq \mathbb{R}^2$

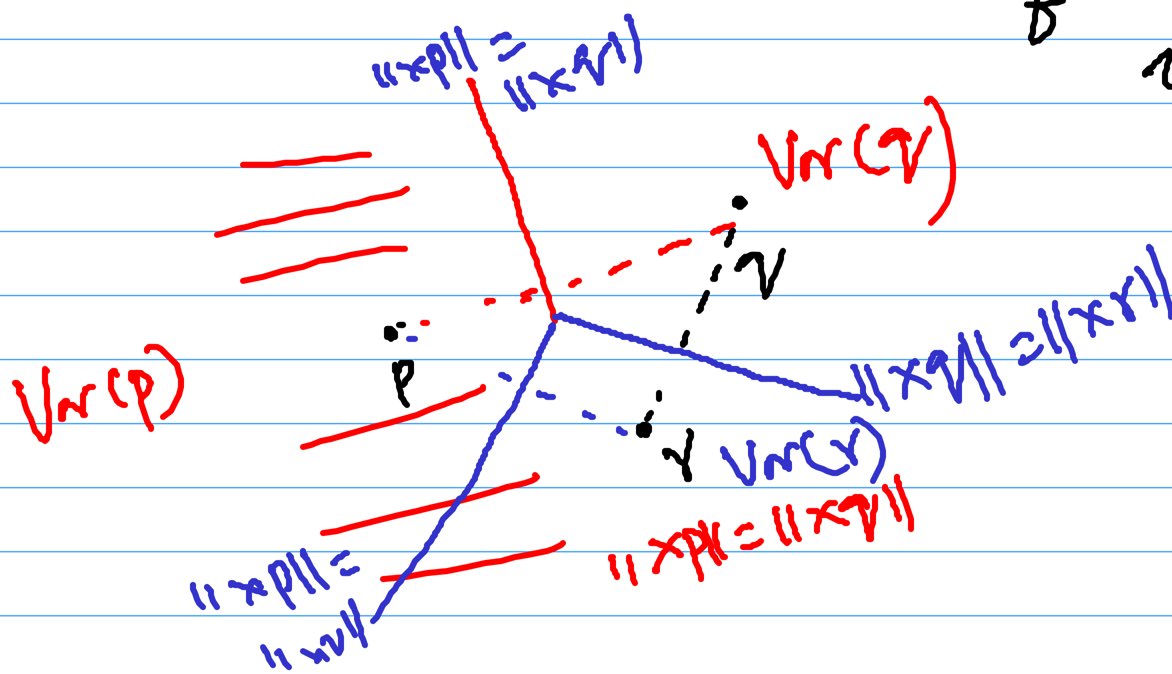
Preprocess S in a data structure s.t. for any $x \in \mathbb{R}^2$, report its nearest neighbor in S.

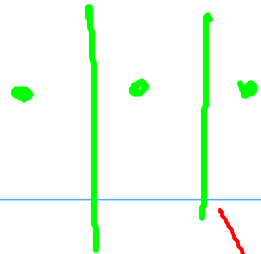
$$\arg\min_{p \in S} \|xp\|$$

Voronoi cell of p_i:

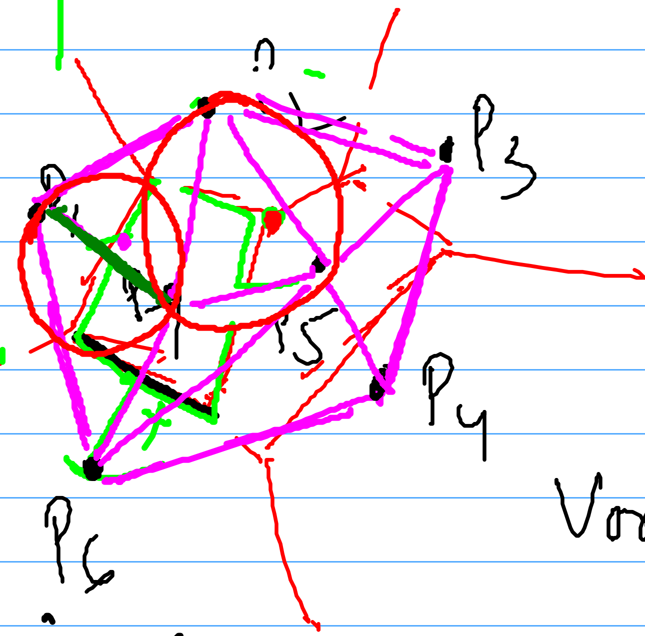
$$Vor(p_i) = \{ x \in \mathbb{R}^{\cancel{2}\, d} \mid \|xp_i\| \le \|xp_j\| \ \forall p_j \in S \}$$

$Vor(S)$: Voronoi Diagram of S is the collection of $Vor(p_i)$ $1 \le i \le n$.



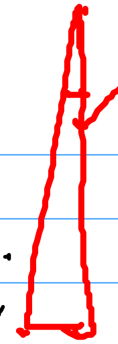


v_{ijk} :



Delaunay triangulation

. each Voronoi cell is convex



$Vor(p_i) = \bigcap_{j \neq i} \gamma_{ij}$

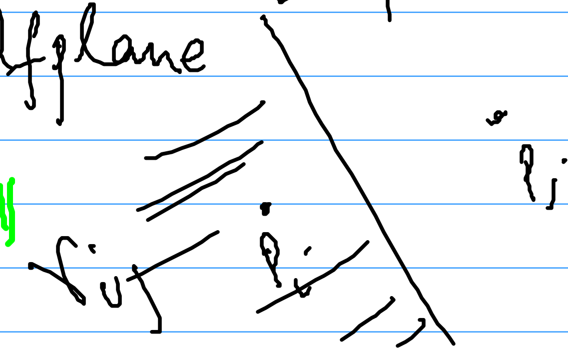
e_{ij}: Voronoi edge sharing $Vor(p_i)$ & $Vor(p_j)$

$\forall x \in e_{ij}$

$\|xp_i\| = \|xp_j\| \le \|xp_k\|$ $\forall k$

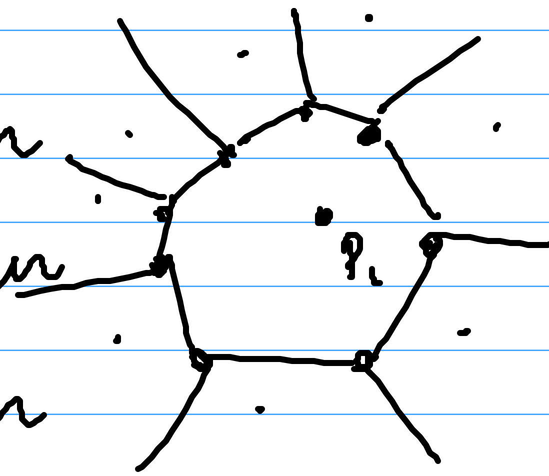
$\gamma_{ij} = \{x \mid \|xp_i\| \le \|xp_j\|\}$

halfplane



Lemma: $Vor(p_i)$ is unbounded $\iff$ p_i lies on $\partial\, conv(S)$

Size of $Vor(S)$



- # faces $= n$
- # vertices $\leq 2n$
- # edges $\leq 3n$

Lemma: Complexity of $Vor(S)$ is $O(n)$

$S = \{p_1, \ldots, p_n\} \subseteq \mathbb{R}^d$

$Vor(S)$ in $\mathbb{R}^d$ has

$\Theta\left(n^{\lceil \frac{d}{2} \rceil}\right)$ vertices!

$S = \{p_1, \ldots, p_n\} \subseteq \mathbb{R}^2$

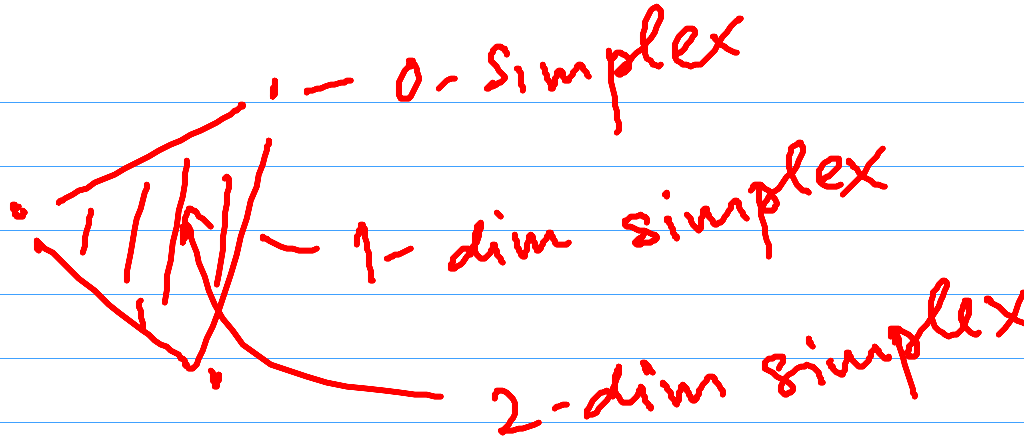
$G = (S, E) \longrightarrow$ Delaunay triangulation

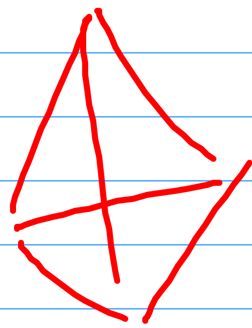
$E \Rightarrow (p_i, p_j)$ is an edge in G on

if $\exists$ circle passing thru p_i & p_j

& not containing any point of

P in its interior





k-dim face of Vor(S)

$\Longleftrightarrow$

(d-k) simplex in DT(S)

Delaunay triangulation of S.