Initially $2n$ vertices + $4n$ vertices
planar graph on $6n$ vertices
How many trapezoids?

A trapezoid $\triangle$ can be of the following kinds:

I

II

...
The “potential” number of trapezoids is \( \binom{n}{4} \) \( n = \# \text{segments} \), assuming no two end-points lie on the same vertical line.

**Exercise:** Show that \( \# \text{trapezoids} \leq 3n + 1 \)

This diagram is called "trapezoidal map" or equivalently "vertical visibility map".

1. How can compute the trap map efficiently?

2. What are the applications of this map?

Given a trapezoidal map, we can add the \( \Delta \) ton edges in linear time.
Incremental Construction

We add objects one after the other and at the $i^{th}$ stage, we compute the map corresponding to $\{0, 0_2, \ldots 0_i\}$.

Cost of the construction $\leq \text{Cost of adding } i$th object.

$T_i$ corresponds to the hapezoidal map of $0, 0_2, \ldots 0_i$.

All the modification required to update $T_{i-1}$ to $T_i$.

Cost of construction depends on the order of insertions.
Randomized Incremental Construction (RIC)

- Generate a random permutation $\pi$ of the objects $\sigma_1, \sigma_2, \ldots, \sigma_n$
- Insert according to $\pi$
- Expected running time is
  $$\frac{1}{n!} \sum_{\pi} T_\pi$$

Constructing a trapezoidal map also has a byproduct in the form of a point-location data structure.

At the

in stage the

prob that

the visibility information changes

$$\sim \frac{1}{n!} \leq \frac{1}{c^i}$$