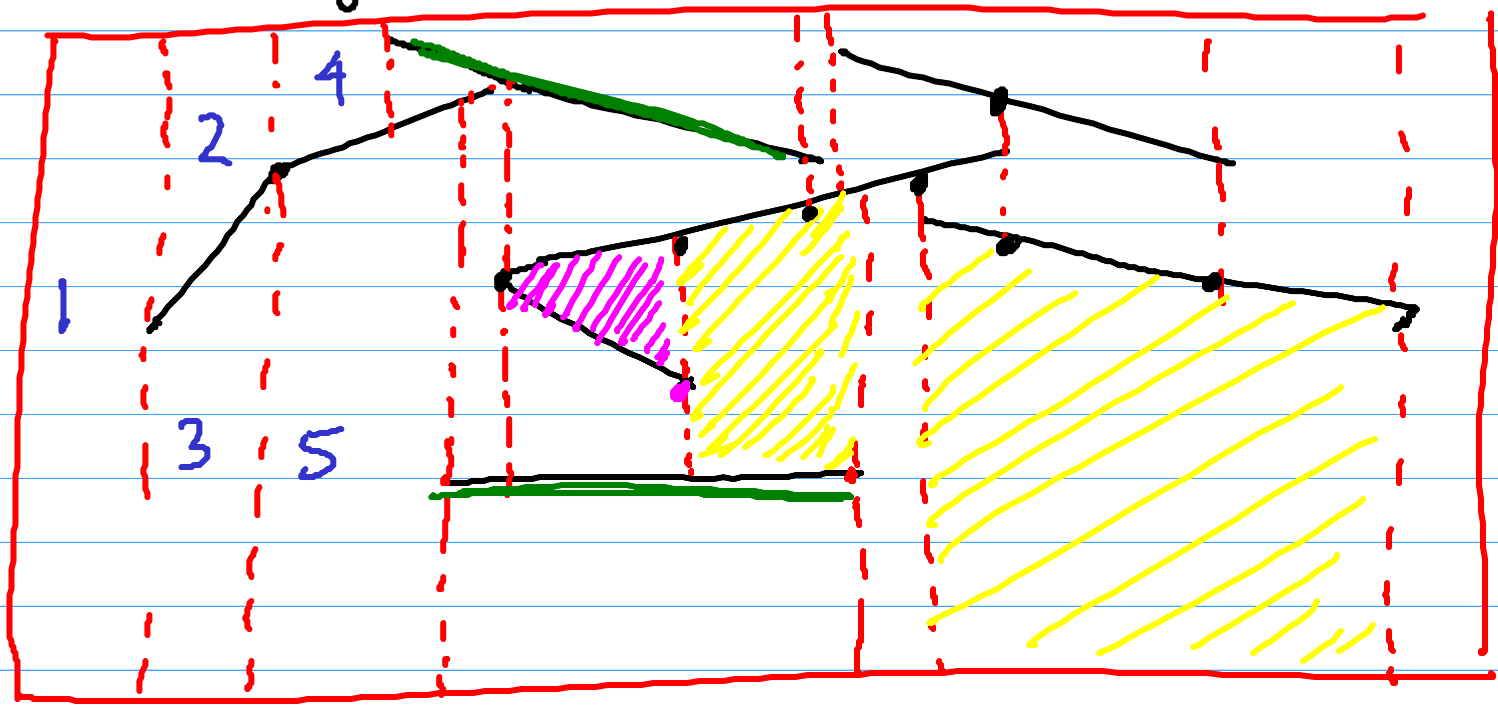
Computational Geometry Lecture 16

Triangulation of arbitrary polygon

n line segments

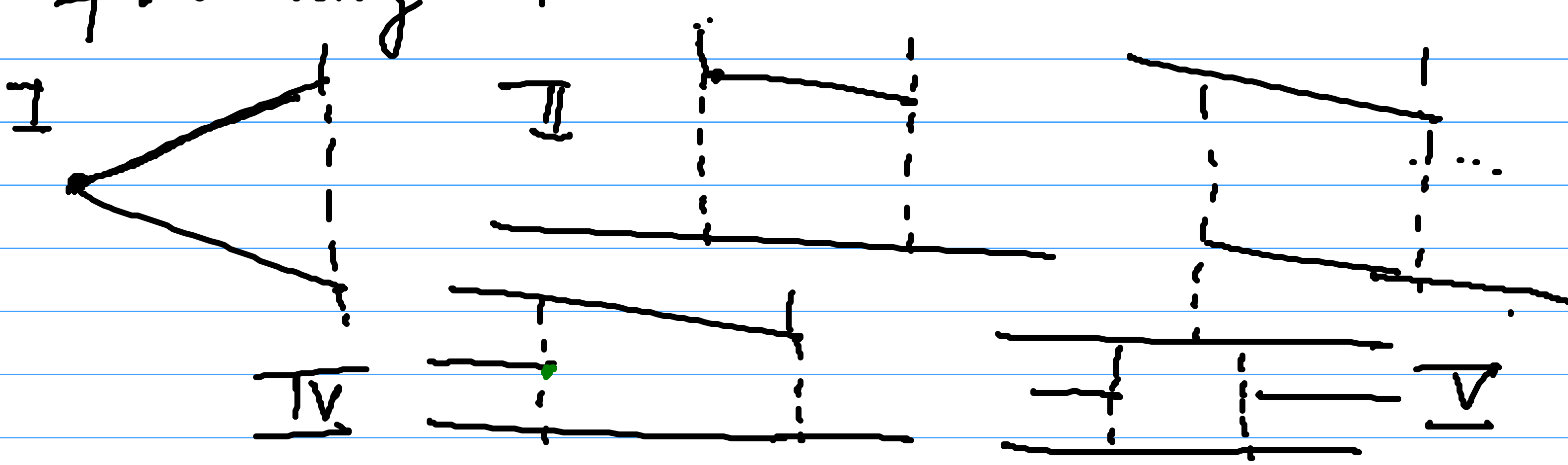


Initially $2n$ vertices $+$ $4n$ vertices

planar graph on $6n$ vertices

How many trapezoids?

A trapezoid $\triangle$ can be of the following kinds



The "potential" number of trapezoids is $\binom{n}{4}$ $n =$ #segments

assuming no two end-points lie on the same vertical line.

Exercise: Show that #trapezoids $\leq 3n+1$

This diagram is called "trapezoidal map" or equi. "vertical visibility map"

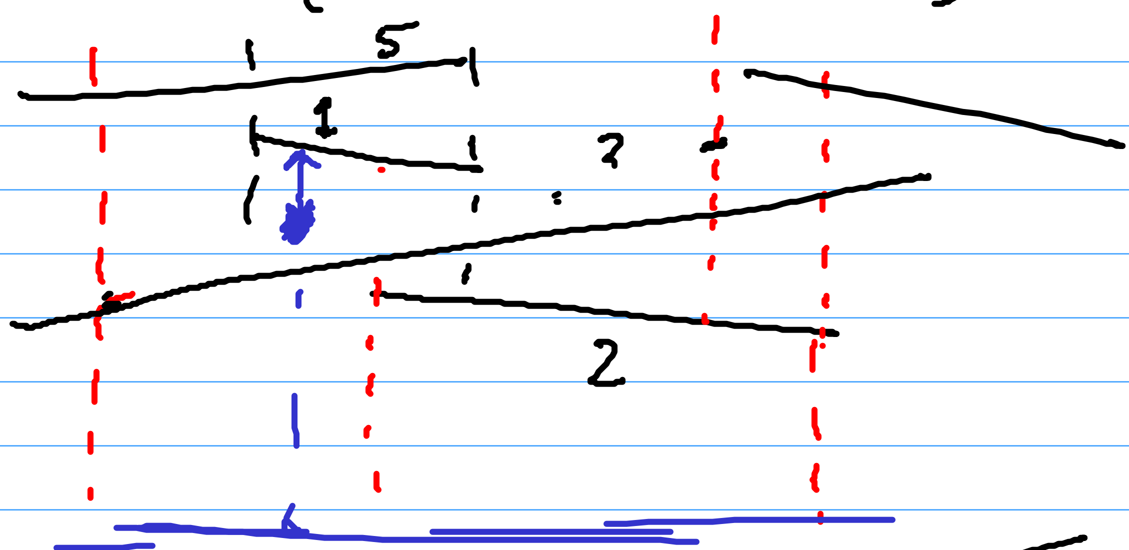
1. How can compute the trap map efficiently?

2. What are the applications of this map?

Given a trapezoidal map, we can add the Δtion edges in linear time.

Incremental Construction

We add objects $O_1, O_2, \ldots O_n$ one after the other and at the i^{th} stage, we compute the map corresponding to $\{O_1, O_2, \ldots O_i\}$



Cost of the construction $= \sum_i$ Cost of adding the i^{th} object

T_i : corresponds to the trapezoidal map of $\{O_1, O_2 \ldots O_i\}$

All the modification required to update T_{i-1} to T_i

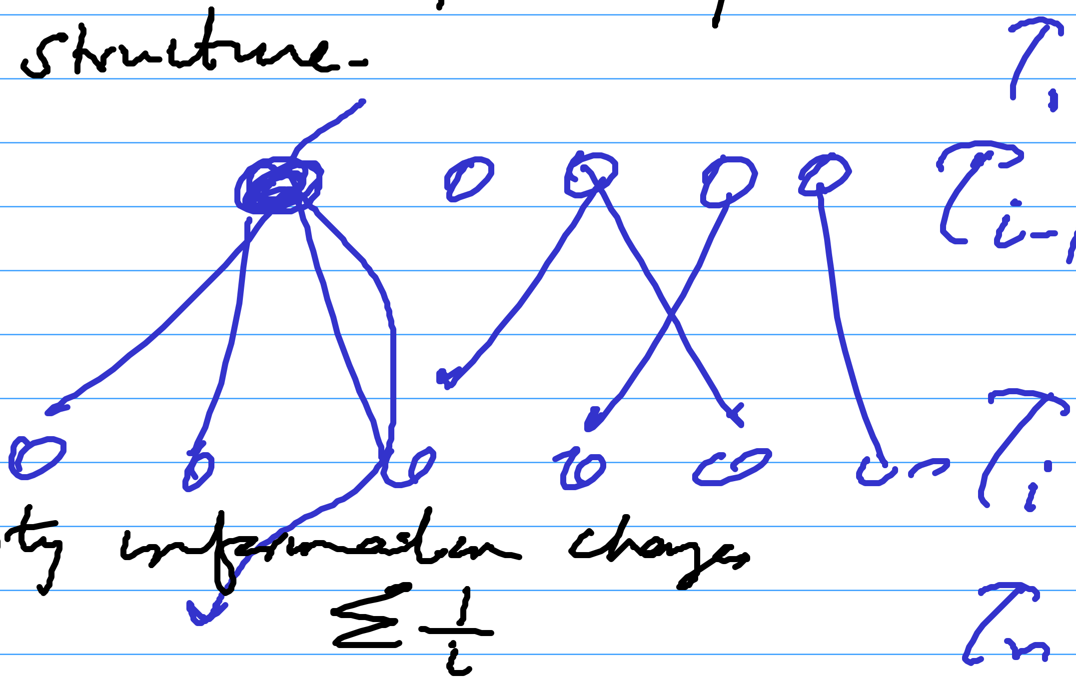
Cost of construction depends on the order of insertions.

Randomized Incremental Construction (RIC)

- Generate a random permutation π of the objects $\sigma_1, \sigma_2, \ldots \sigma_n$
- Insert according to π
- Expected running time is

$$\frac{1}{n!} \sum_{\pi} T_{\pi}$$

Constructing a trapezoidal map also has a byproduct in the form of a point-location data structure.



At the i^{th} stage stage the prob that the visibility information changes $\sim \frac{1}{i}$ $\quad \sum \frac{1}{i}$