Point Location - continued

Algorithm for preprocessing a planar subdivision.

1. Find a "fractional independent" set. (An independent set of vertices that has size \( \geq \alpha n \) for some constant \( \alpha \)). Call it \( F \).

We had observed that there are at least \( \Omega n \) vertices in a planar graph that have degrees \( \leq 12 \), denoted by \( V' \). Select vertices from \( V' \) such that if we chose a vertex \( v \in V' \), we exclude all the neighbors of \( v \).

\[ \text{N}(v) \leq 12 \]

Build \( F \) in this greedy fashion, it guaranteed that \( |F| \geq \frac{|V'|}{25} \).
2. Exclude (or delete F) from the graph $G = G_0(V_0, E_0)$ resulting in $G_1(V_1, E_1)$

$$V_1 = V_0 - F$$

Repeat step 1. Notice that $G_1$ is also planar and similar arguments apply

$$G_0 \rightarrow G_2 \rightarrow G_2 \rightarrow \cdots \rightarrow G_k$$

$$k \sim \log_2 |V| \sim O(\log_2 n)$$

$G_k$ is a very small graph, say 10 vertices.

Initially we assume that $G_0$ is a Delta ted graph.

(Hand-drawn diagram of a graph with labeled vertices and edges.)
For point location, we can do it in \( O(1) \) time in \( G_k \) (using some brute force).

Having located in \( G_k \), we want to refine the point location w.r.t. \( G_{k-1} \).

What is the cost of refinement, i.e., locality in \( G_{k-1} \), given location in \( G_k \)?

Search levelled graph

Query line: \( \text{levels} \times O(1) = O(n) \)
Preprocessing Space and Time

Time at level $i$ : $O(v_i)$

$V_i$ : # vertices in level $i$.

Total preprocessing time : $O(i \leq V_i)$

$V_i \leq \alpha V_{i-1} = O(lv_i)$

Space is proportional to

$\leq \sum_{i} |F_i| \quad F_i$ : set of fences in level $i$

$= O(lv_i)$

Kirkpatrick's

A By product decomposition is the hierarchy of the planar subdivision.
Triangulation

(input: line segments)

(constrained Delaunay)

(no Steiner points)

general

mountain chain