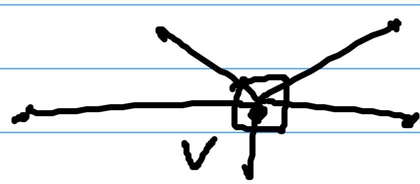
Computational Geometry Lecture 15

Point Location - continued

Algorithm for preprocessing a planar subdivision.

1. Find a "fractional independent" set. (An independ set of vertices ~~that~~ has size $\geq \alpha n$ for some constant α). - call it F

We had observed that there are at least $\frac{n}{2}$ vertices in a planar graph that have degrees ≤ 12, denote by V'. Select vertices from V' such that if we chose a vertex $v \in V'$, we exclude all the neighbours of v



$N(v) \leq 12$

Build F in this greedy fashion, it guaranteed that $|F| \geq \frac{|V|}{25}$

2. Exclude (or delete F) from the graph $G = G_0(V_0, E_0)$ resulting in $G_1(V_1, E_1)$

$$V_1 = V_0 - F$$

Repeat step 1. Notice that G_1 is also planar and similar arguments apply

$$G_0 \longrightarrow G_1 \longrightarrow G_2 \longrightarrow \cdots \underline{G_k}$$

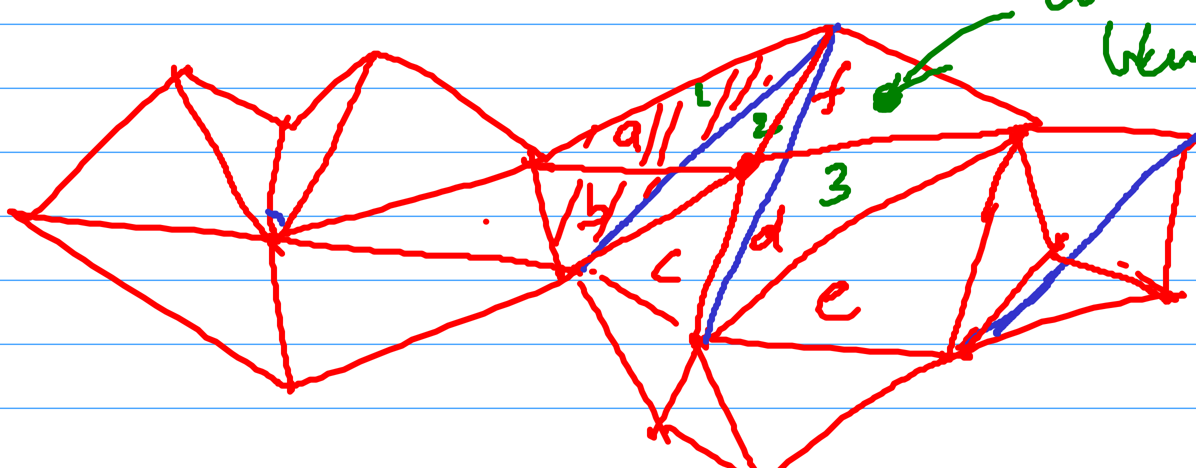
← point local

$$k \sim \log_{\frac{1}{1-\alpha}} |V| \sim O(\log_2 n)$$

G_k is a very small graph, say 10 vertices

Initially we assume that G_0 is a $\triangle$ted graph.

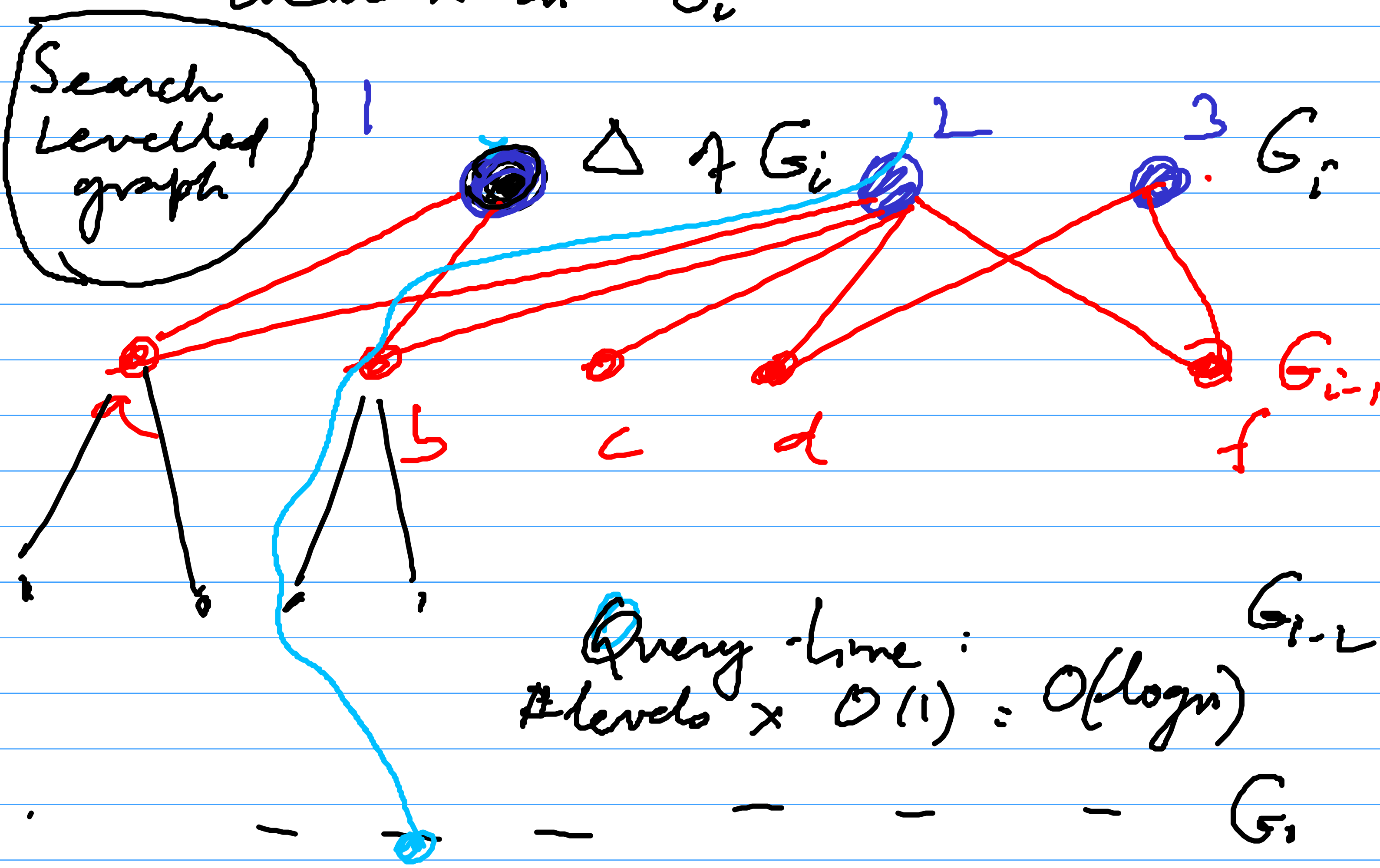
location w.r.t. blue subdiv



For point location, we can do it in $O(1)$ time in G_K (using some brute force)

Having located in G_K, we want refine the point location w.r.t G_{K-1}

What is the cost of refinement i.e. locating in G_{i-1}, given location in G_i



Query time:
$\#\text{levels} \times O(1) = O(\log n)$

Preprocessing Space and Time

Time at level i : $O(V_i)$

V_i = # vertices in level i.

Total preprocessing-time : $O\left(\sum_i V_i\right)$

$$V_i \leq \alpha V_{i-1} \quad = O(|V|)$$

Space is proportional to

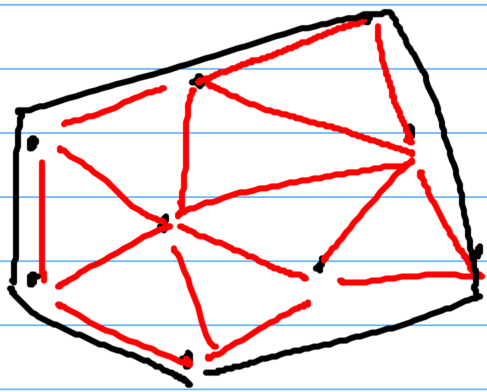
$$\sum_i |F_i| \qquad F_i : \text{set of faces in level } i$$

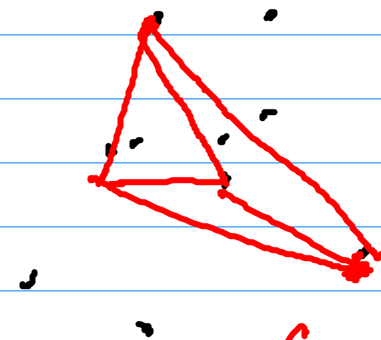
$$= O(|V|)$$

Kirkpatrick's decomposition

A By product is the hierarchy of the planar subdivision.

Triangulation





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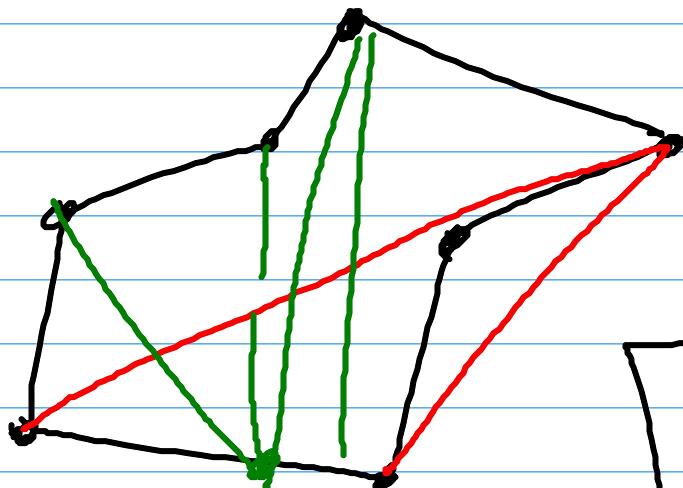
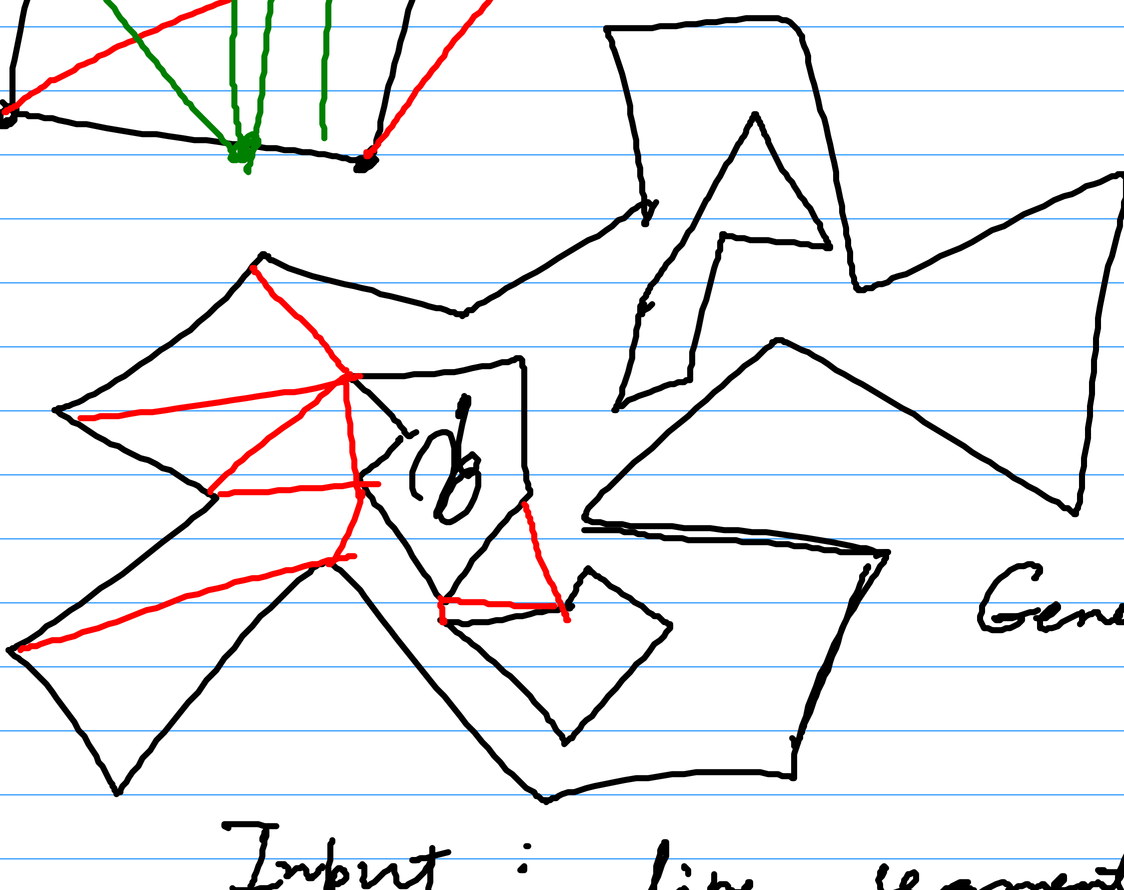


Constrained
Δ-tion



General

Input : line segments

monotone chain