Important observation about linear decision trees.

Given any input point \( x \in \mathbb{R}^n \), we follow some path from the root to some leaf node. \( \Rightarrow \) Any decision tree also implies a partitioning of \( \mathbb{R}^n \) in the following way:

"Every node \( t \) corresponds to some subset \( W(t) \subset \mathbb{R}^n \), namely all\( \) points that pass through \( t \).

Every node is associated with a \underline{convex region} (intersection of linear inequalities)."
The number of leaf nodes must exceed the number of connected components of the solution space.

\[ \text{Let a linear decision tree is } O \left( \log (\#W) \right) \]

where \( \#W \) = no. of connected components in the solution space

\[ \bigwedge_{i \neq j} (x_i - x_j) = 0 \text{ iff answer is no} \]

Convex hulls: relaxed version

(i) Enumerate (in any order) the corner points

(ii) Given n points, do all points appear on the convex hull (Decision problem)
Given a set of \( n \) points on the plane and a number \( h \leq n \), are there \( n \) points on the convex hull?

Special case: \( P_2 \)

Can we go from one shaded wedge to another using a path that completely lies in the shaded region?

\( n-h \) points have to be distributed to the \( h \) wedges

\[ h^{n-h} \log \left( \frac{n-h}{(n-h)h} \right) \]
Fix the $k$ points to be the regular $k$-gon and the remaining $n-k$ points are chosen arbitrarily in the annular region between $C_I$ and $C_o$.

Are there $k$ points on the convex hull?

Claim: $P_2 \prec P_1$

$\Rightarrow$ lower bound if $P_2$ is a lower bound for $P_1$