Computational Geometry Lecture 12

Topic : Duality (continued)

point p : (a, b) $L: (m, c)$

$D(p): (a, b) \rightarrow L: y = \underbrace{2a}_{m}x - \underbrace{b}_{c=-b}$

$D(L: y = 2ax - b) \rightarrow (a, b)$

<u>Incidence property</u> : Suppose p lies on L:

(a, b) $y = mx + c$

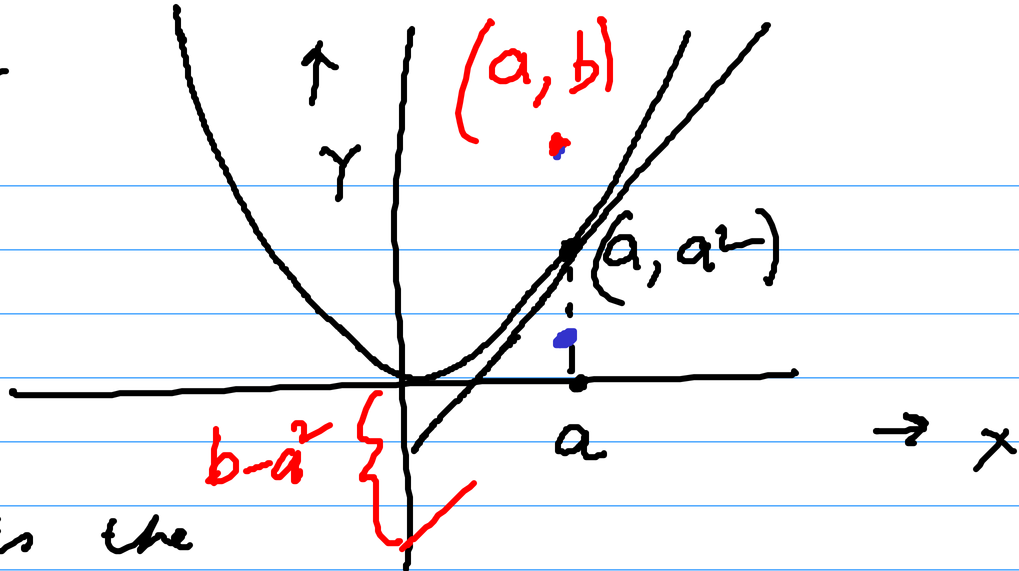
$\Rightarrow$ $b = am + c$ ①

$D(p)$: line $y = 2ax - b$ $D(L): \left(\frac{m}{2}, -c\right)$

$-c \stackrel{?}{=} 2a \cdot \frac{m}{\cancel{2}} - b$ or $-c \stackrel{?}{=} am - b$

or $b \stackrel{?}{=} am + c$ ✓

$y = x^2$



What is the equation of the tangent at (a, a^2)? $y = 2ax - a^2$

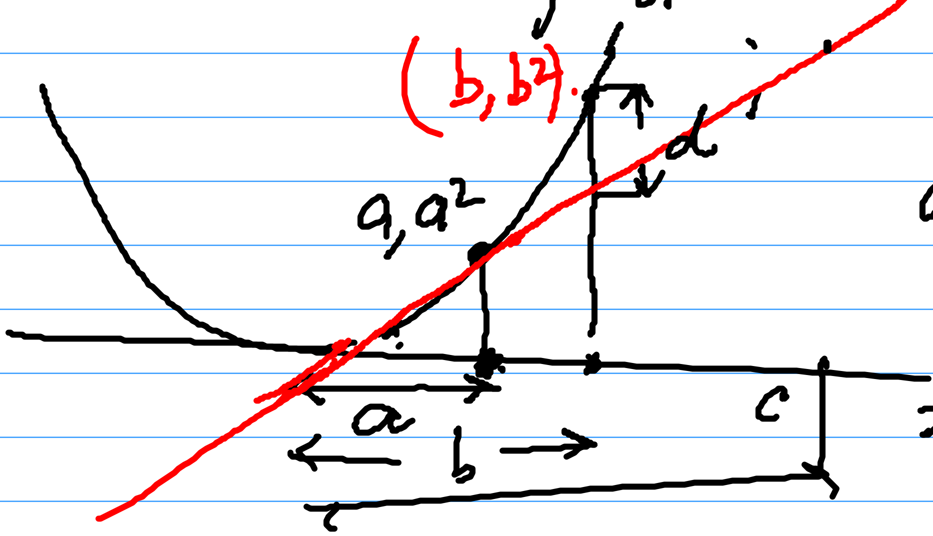
$D(a, a^2) : y = 2ax - a^2$

In higher dimensions $x_1, x_2, \ldots x_d$

$x_d^2 = x_1^2 + x_2^2 + \cdots x_{d-1}^2.$

$$\frac{\partial x_d}{\partial x_i} = 2x_i$$

$D(a_1, a_2, a_3\ a_d) : x_d = 2a_1x_1 + 2a_2x_2 + \cdots - a_d$



$d = (b-a)^2.$

$=$ distance along x axis

Lower bounds for geometric problems

Problem : Element uniqueness

Given n elements $(x_1, x_2, \ldots x_n)$ determine if all of them are unique (or there is a repetition)?

$$(0.3,\ 1.15,\ 1.56,\ 1.1,\ 2.8)$$

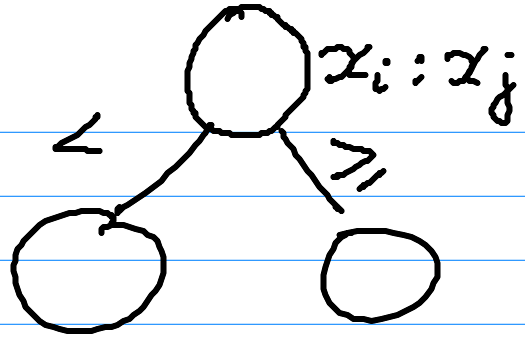
Decision problem Yes / No

Suppose $x_i \in \{1, 2, 3, \ldots n^c\}$
with indirect addressing, $O(n)$ time

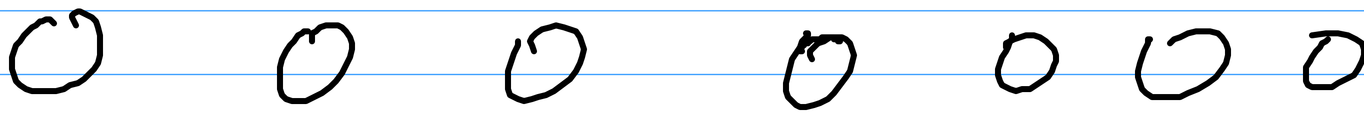
$$x_i \in \mathbb{R}$$

E.U. $\propto$ sorting $\Rightarrow$ L.B. of E.U applies to sorting

Lower bound of sorting assumes a certain model of computation : Comparison tree model



Only counts comparisons



leaves : each corresponds to a specific ordering

$\Rightarrow$ # leaves $\geq n!$ $\Rightarrow$ longest path $\geq \log_2(n!)$ $\sim \Omega(n \log n)$

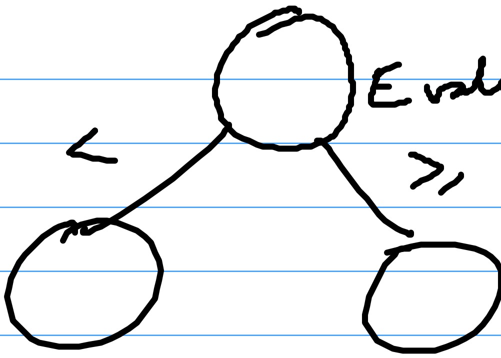
$$x_i - x_j \overset{?}{\geq} 0$$

A general inequality could be

$$a_1 x_i + a_2 x_j + b \geq 0 \; ?$$

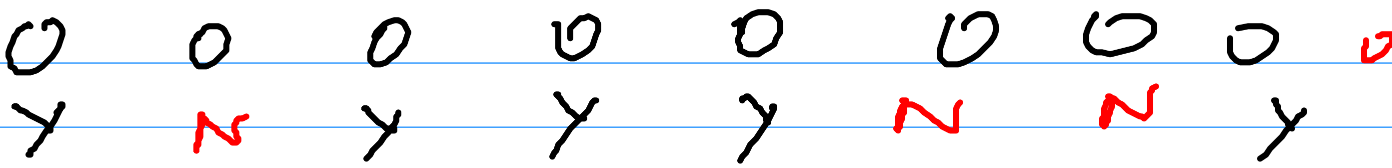
$$+ a_3 x_l$$

Linear decision Tree



Evaluate some linear inequality

- (Fixed degree algebraic decision tree)
- [Arithmetic computation Tree



In $\mathbb{R}^n$, some subset, say $W \subset \mathbb{R}^n$ contains all the Yes answers and the complement $\mathbb{R}^n - W$ are the "No" answers

Hyperplanes

$x_i = x_j$

$i \neq j$