Topic: Duality (continued)

Point $p: (a, b)$  \quad Line $L: (m, c)$

$D(p): (a, b) \rightarrow L: y = 2ax - b$

$m = \frac{2a}{2} = a$
$c = -b$

$D(L: y = 2ax - b) \rightarrow (a, b)$

Incidence property: Suppose $p$ lies on $L: (a, b), y = mx + c$

$\Rightarrow b = am + c \text{ } \text{(1)}$

$D(p): \text{line } y = 2ax - b \quad D(L): \left( \frac{m}{2}, -c \right)$

$-c = 2a \cdot \frac{m}{2} - b \Rightarrow -c \neq \frac{am - b}{a}$

$b = am + c$
what is the equation of the tangent at \((a, a^2)\)? 

\[ y = 2ax - a^2 \]

\[ D(a, a^2) : y = 2ax - a^2 \]

In higher dimensions \(x_1, x_2, \ldots, x_d\)

\[ x_d = x_1^2 + x_2^2 + \cdots + x_{d-1}^2 \]

\[ \frac{\partial x_d}{\partial x_i} = 2x_i \]

\[ D(a_1, a_2, a_3, a_d) : x_d = 2a_1x_1 + 2a_2x_2 + \cdots + a_d \]

distance along x axis
Problem: Element uniqueness

Given \( n \) elements \((x_1, x_2, \ldots, x_n)\) determine if all of them are unique (or there is a repetition)?

\((0.3, 1.15, 1.5, 6, 10, 2.8)\)

Decision problem: Yes/No

Suppose \( x_i \in \{1, 2, 3, \ldots, n\} \) with indrect addressing, \( O(n) \) time

\( x_i \in \mathbb{R} \)

E.U. \& sorting \( \Rightarrow \) L.B. \& E.U. applies to sorting

Lower bound of sorting assumes a certain model of computation

: Comparison tree model
\[ x_i : x_j \]

Only count comparisons

\[
\begin{align*}
\text{leaves} &: \text{each correspond to a specific ordering} \\
\Rightarrow \# \text{leaves} &\geq n! \Rightarrow \text{longest path} \\
&\geq \log_2(n!) \\
&\geq \Omega(n \log n)
\end{align*}
\]

A general inequality could be

\[ a_1 x_i + a_2 x_j + b \geq 0 \]

\[ + a_3 x_k \]
Linear decision Tree

- (Fixed degree algebraic decision tree)
- Arithmetic Computable Tree

\[ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \]

In \( \mathbb{R}^n \), some subset, say \( W \subseteq \mathbb{R}^n \), contains all the Yes answers and the complement \( \mathbb{R}^n - W \) are the "No" answers.

\[ w \]

\[ w \]

Hyperplanes

\[ x_i = x_j \]

\[ i \neq j \]