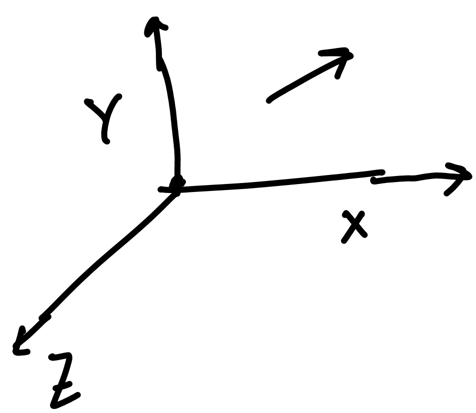
COL 702 Lecture 3 July 30

How about maximal points in three dimensions?

Brute force in d dimensions will work correctly using

$O(d \cdot n^2)$



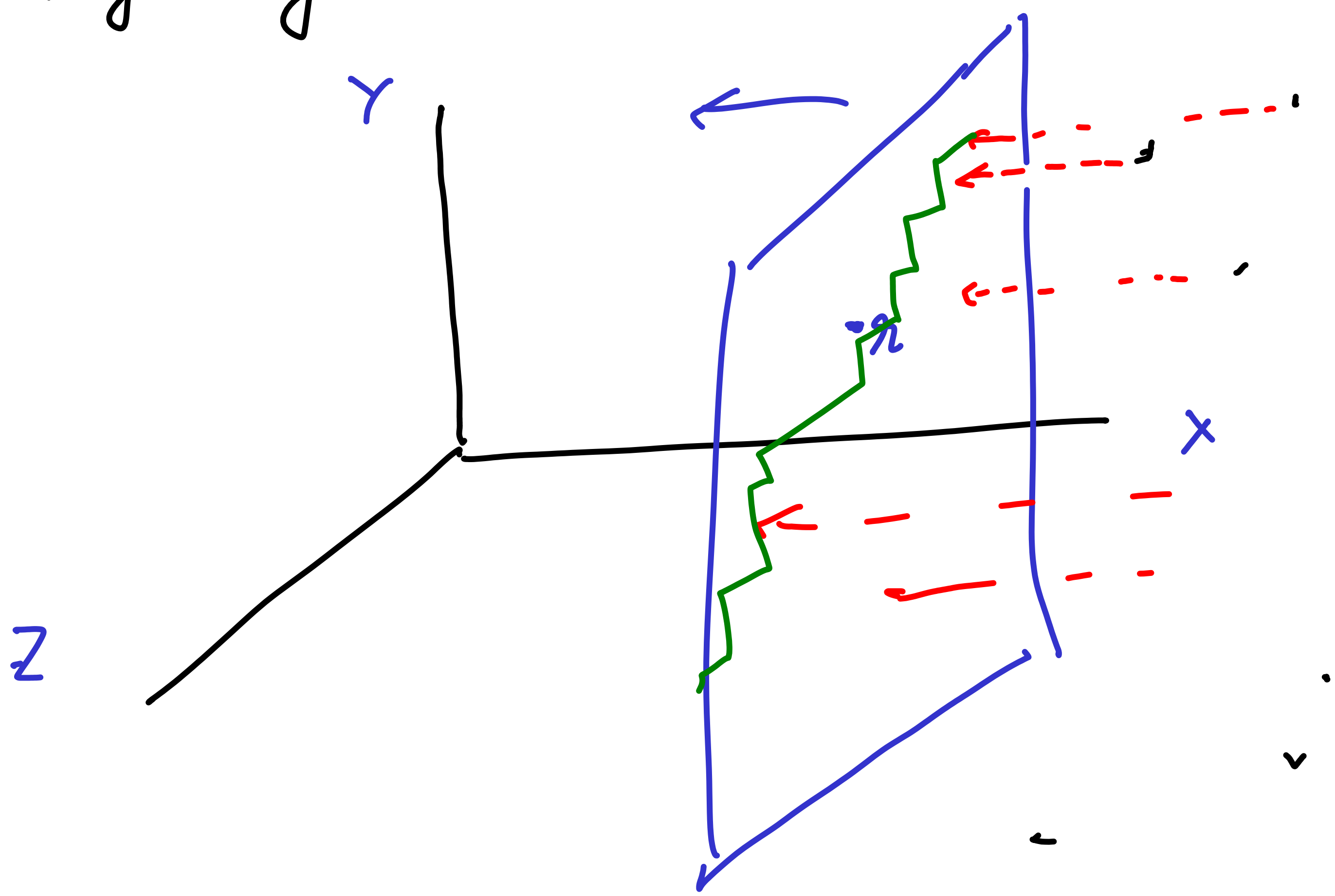
Sweeping by a plane ∥ to Y-Z in decreasing x coordinates

Obs · Point having maxm x-coord is maximal, say p

When we visit the next point, say q we know $x(q) < x(p)$

$$y(q) : y(p) \qquad z(q) : z(p)$$

In the generic step, the latest point visited, say r will not be maximal if and only if one of the points visited previously has a higher y coordinate and a higher z coordinate

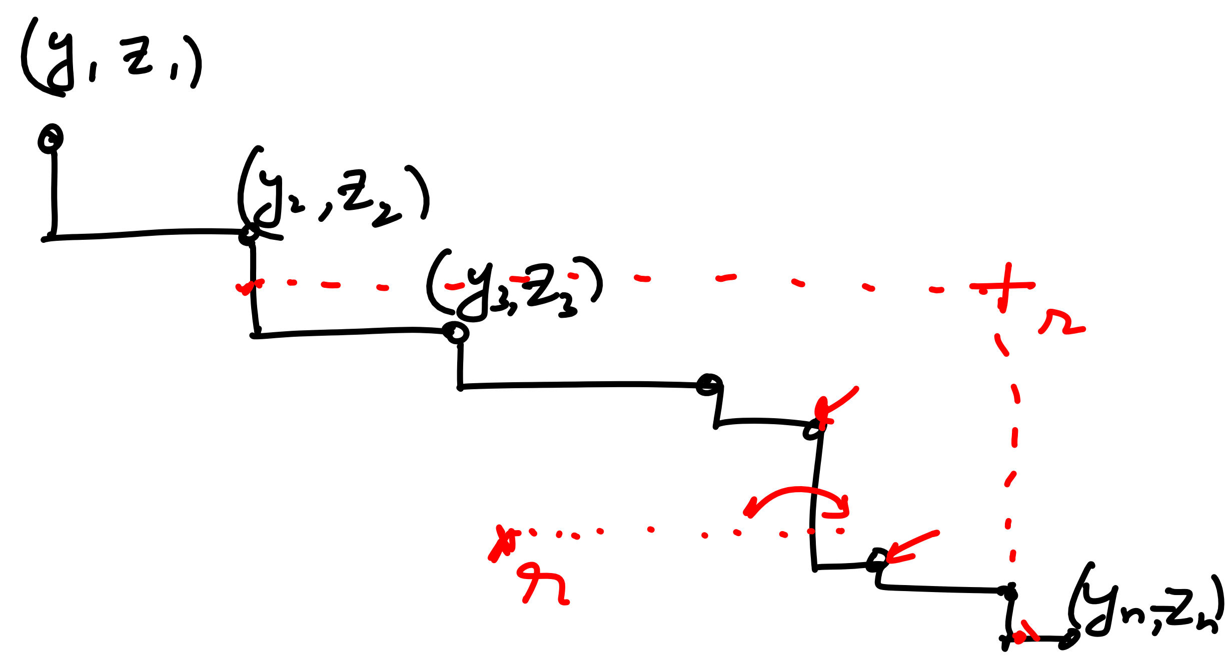


We want to test if r is inside or outside the staircase formed by the previously visited points

1. Can we do it quickly even if staircase is large?

2. Can we update the staircase quickly

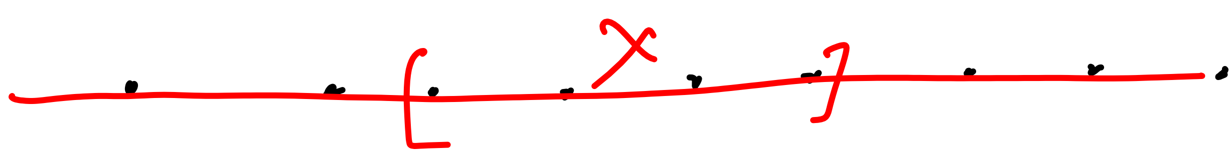
<u>Problem</u>: Design an efficient data structure for the staircase so that ① and ② can be supported



Given a Balanced BST, can we support insertions, search and "arbitrary # of deletions" in $O(\log n)$ time.

In general the following data structure operation on sorted sets are considered very useful

1. Splicing out an interval of points
2. Concatenating two sorted intervals into a single interval



$\boxed{\text{Concatenable queues}}$

In the staircase, we will delete the points one by one paying $O(\log n)$ cost per deletion

One single iteration may be expensive (lots of points deleted)

but overall each point can be inserted or deleted at most once

$\Rightarrow$ $O(n\log n)$ overall for 3D maxima

Amortized Analysis