Given DFAs $M_1$ and $M_2$, how can we determine if $L(M_1) = L(M_2)$? (can be re. $r_1$ and $r_2$ also)

$L_1 = L_2 \iff (L_1 - L_2) \cup (L_2 - L_1) = \emptyset$

$L_1 - L_2 = L_1 \cap \overline{L_2}$ If $L_1, L_2$ reg
- then $L_1 - L_2$ is reg.

So can we test if $L(M) = \emptyset$ for a given $M$

Try all possible strings $x \in \Sigma^*$ |$x$| < $n$
If some string is accepted, clearly $L(M) \neq \emptyset$
If no string is accepted declare $L(M) = \emptyset$

Proof: Suppose $x, \in L(M)$ and |$x$| > $n$ and among all such strings, this is the shortest.
From P.L. \( x_i = u v w \) and \( u w \in L \) 
\[ x_i \text{ is a shorter string, so contradiction.} \]

**Problem**

Let \( L \) be a regular language

Then the language

\[ L_1 = \left\{ a_1 a_3 a_5 \ldots a_{2n-1} \mid a_1, a_3, a_5, \ldots, a_{2n-1} \in L \right\} \]

\( a_i \in \Sigma \)

is regular / not regular

Suppose \( \Sigma = \{0, 1\} \)

and say \( L \) consists of the string

\[ L = \{10010011, 11010, 10010000, x_0, x_0 x_2, \ldots, x_k \} \]

\[ L_1 = \{1001, 100, x_1, x_2, x_3, x_4, \ldots, x_k \} \]

Can we use PL to prove that some language \( L \) is regular?

If \( L \) is regular \( \Rightarrow ( \) ?

??
We have DFA $M$ recognizing $L$.

If $w_1w_2w_3 \ldots w_k \in L$, then there exists $x_1, x_2, x_3, \ldots, x_k \in \Sigma$

such that $w, x, wx, wx^2, \ldots, wx^{k-1} \in L$

$\iff \delta(q_0, w, x, wx, \ldots, wx^k) \notin F$

For every state $q \in M$, say $q$

$\delta'(q, a) = \{ q \} \quad \text{if } q = \delta(q, ax) \quad x \in \Sigma$

new transition function

\[ 0 \]
Claim 1 \quad \text{If } w, w_2, w_3, \ldots, w_k \in L(M) \text{ then } \exists x_1, x_2, x_3, \ldots, x_k \mid w, x_1, w_2, x_2, \ldots, w_k x_k \in \Sigma^+

Claim 2 \quad \text{If } w, x, w_2 x_2, \ldots, w_k x_k \in L(M) \Rightarrow w, w_2, w_3, \ldots, w_k \in L(M')