Post Correspondence Problem (PCP) is not recursive

\[ L_u \leq_f L_{\text{pcp}} \]

\[
\begin{array}{c|c}
A & B \\
\hline
B_1 & \omega_1 \\
\vdots & \vdots \\
B_k & \omega_k \\
\end{array}
\]

\[ f(\langle M, \omega \rangle) = (A, B) \]

s.t. \( A, B \in L_{\text{pcp}} \)

if \( M \) accepts \( \omega \)

\( \beta_1, \beta_2, \beta_3, \ldots, \beta_m = \omega_1, \omega_2, \omega_k \)

\( i_d \in \{1, \ldots, k\} \)
Time and Space Complexity

Time complexity $T(n)$ is the length of the input string $k$-tape $TM$ $K \geq 1$

The $TM$ always halt (recursion language)

For a given $TM$ $M$, the time complexity is $T(n)$ if the maximum no of steps taken by $M$ on any input of size $n$ is $T(n)$.

( for acceptance )

A language $L$ is in the time complexity class $T(n)$ if there is a $TM$ that accepts $L$ has time complexity $T_k(n)$ ($k$-tapes)

$T(n)$ is at least $n$ (by convention)

Unless the entire input is read, the language / problem is not considered verifiable
A similar definition also holds for non-deterministic TM.

\[ T(n) = \max_{\text{all inputs of length } n} \max_{\text{all choices for a fixed input}} T(n) \]

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**Space Complexity**

k-tape TM but we have a read-only input tape.

Max \{Rightmost head position in any of the k storage tapes\} not including input tape
Consider \( L_{\text{pal}} = \{ wc w^R \mid w \in \Sigma^* \} \)

Time complexity of \( L_{\text{pal}} \)

By copying \( w \) in tape 2 and then checking \( w \) and \( w^R \) by moving the heads in opposite directions \( \mathcal{T}(n) = n \) for \( L_{\text{pal}} \)

Space complexity of \( L_{\text{pal}} \)

Read only

Space complexity using the previous approach

\( s = \frac{n}{2} \) (length of \( w \))

By using counters we can compare the corresponding characters of \( w \) and \( w^R \)
A counter takes log₂ n space, so space complexity is $\frac{3\log n}{n}$.