Context-free languages (CFG & PDA)

0 1 2 3 4 5 6 7 8 9 10...

0 0 0 0 1 1 1 1 1 1...

Finite state control

(overwrite)

\[ \delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R\} \]

\[ \Gamma : \text{tape alphabet} \]
\[ \text{contains } \Sigma : \text{input alphabet as well as some additional symbols including } b \]

The machine keeps executing until

1. the next move is undefined
2. it reaches a final state \( q_f \in F \)

Initially, the machine is in state \( q_0 \), and the tape head is on the leftmost cell.
$q \beta$ contain all the necessary information for the computation till a certain juncture.

$\alpha, \beta \in \Gamma^*$

$q \in Q$ $\alpha \beta \gamma$ represent the instantaneous description of a Turing Machine.

Initially, the I.D. $(\epsilon, q_0, I_n)$

In: input $I_n$

$\alpha_i q_i \beta_i \rightarrow \alpha_i q_i^* \beta_i \beta_{i+1} + \ldots$

Subterranean: $00 q_2 0101 \rightarrow 09 000001$

$\delta(q_2, 1)$ contains $(q_3, 0, L)$
$L = \{ a^n b^n c^n, \ n \geq 1 \}$

- $a$ $a$ $a$ $b$ $b$ $b$ $c$ $c$ $c$

$(0, a)$

Multi-track Turing Machine

Claim: Multi-track can be simulated by a Normal 1-track machine. (by blowing up the tape alphabet)
Multithread Turing Machine

$\delta : Q \times \Gamma^k \times \Gamma^k \rightarrow Q \times \Gamma^k \times \Gamma^{k+1} \times \Gamma^k \\ \leq \prod_{i=k}^{k+1} \Gamma^k \\
\times \xi \in L, R^2_k \leq$
Simulate a $K$-multitape TM by a
Multi-track $2K$-track TM

<table>
<thead>
<tr>
<th>Contents of tape 1</th>
<th>Contents of tape 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>The head position of tape 1</td>
<td>The head track on tape</td>
</tr>
<tr>
<td>Contents of Km tape</td>
<td>Head position in Km tape</td>
</tr>
</tbody>
</table>

The head scans from leftmost cell till it has counted $K$ markers
(must be done using some special states)

2nd scan: It replaces the symbols according the $S$ function of the multitape machine

How do we remember the head markers as we scan from left to right. The state space of the multi-track machine can be as follows:

$[a, 0, 1, 0, 0, 0, 0, 0]$

scanning for head position/replace the tape symbols
Suppose the multi-tape TM takes \( T \) steps.

How many steps (in \( O(n) \) notation) will the simulation take?

About \( O(T^2) \)