1. $S \rightarrow ab$  
2. $S \rightarrow aSb$  
3. $S \rightarrow \varepsilon$  
   $S \rightarrow aSb \quad \rightarrow \quad aaaSbb \quad \rightarrow \quad aaaaSbbbb$  
   $\quad \rightarrow \quad aaaaabbbbb$  

4. $S \xrightarrow{*} w$  
   $w = a^i b^i$, $i \geq 0$  

apply repeatedly one of the rules

Any sequence of substitution must begin with the special variable $S$

$G: \{ V = S \quad S \rightarrow S \}$  
$T = \{ a, b, \varepsilon \}$  
$P = \{ 1, 2, 3 \}$  

Convention: Capital letters for variables, small case for terminals

$AB \rightarrow ABBB$

Context Free Grammar (CFG), the production rules have exactly one symbol on the LHS
Grammar $G = (V, T, P, S)$

- Set of variables $V$
- Start symbol $S$
- Set of terminals
- Set of rules $P$
- Production $\rightarrow E \in V$

Case: Equal number of $a$'s and $b$'s
$q^i b^i a b a b a b a b a b$, $a a b a b a b b$

Is $L(eq)$ regular?

Context-Free Language (CFL): all languages that can be generated using CFG

Is $L(eq)$ CFL?

$S, \{a, b, \epsilon\}, S, S \rightarrow \epsilon | b a | a b | b a S | a b S$
\[ V = \{ S, A, B \} \quad T = \{ a, b \} \]

\[ S \to aB \mid bA \mid \varepsilon \]

\[ A \to a \mid bAA \mid aS \]

\[ B \to b \mid aBB \mid bS \]

Claim 1: \( S \not\Rightarrow w \) iff \( w \) has equal \( a \)'s and \( b \)'s

for \( |w| \geq 1 \)

2. \( A \not\Rightarrow w \) iff \( w \) has one more \( a \) than \( b \)

3. \( B \not\Rightarrow w \) iff \( w \) has one more \( b \) than \( a \)

Proof by induction on \(|w|\):

\( |w| = 1 \):

Base case: \(|w| = 1\):

- \( S \): no strings of length 1, so true
- \( A \): \( A \to a \) only string of length 1
- \( B \): \( B \to b \)

Suppose true for all \( |w| \leq k-1 \)

Consider any string \(|w| = k\)

\[ S \]

\[ a_{\omega_1} \quad \xrightarrow{w} \quad b_{\omega_1} \quad |\omega_1| = k-1 \]

\[ S \to aB \not\Rightarrow a_{\omega_1} \]

Some \( B \) generates all strings \( |w| = k-1 \) with one extra \( b \).
So \( B \to^1 w \).

Conversely if \( S \to^1 w \) then \( w = aw_1b \).
\( aw = bw_2 \).
\( \uparrow \) extra \( a \)

Let \( S \to aB \) \( S \to B \to^1 w \).

From I.H. \( w \) has an extra \( b \).

Prime it for all the three assertions \( A, B, S \) and their converse.