COL 352 Intro to Automata and Theory of Computation

Tutorial Sheet 3

1. Instead of defining the terminating condition of a Turing machine using final states, it is possible to define it using the condition of the head falling off the leftmost cell. Show that the two classes of machines are equivalent, i.e., we can simulate one by the other.

2. For any arbitrary TM M, show that it can be simulated by another with tape alpahbet restricted to $\{0, 1, B\}$. Discuss if it is possible to restrict to only two tape symbols.

3. Formally prove that a TM can recognize any Context Free Language. Moreover prove that it is strictly more powerful by showing that the language $L = \{ww | w \in (0+1)^*\}$ is recursive. Recall that L is not CFL.

4. Show that a $T(n)$ time bounded k tape Turing machine can be simulated by a 1 tape machine in $O(T^2(n))$ steps.
   Use this result to show how a Random Access Machine can be simulated by a 1 tape TM incurring at most a polynomial overhead.

5. Show that every TM can be simulated by an off-line TM having one storage tape with two symbols 0 (blank) and 1 (non-blank). Further this machine can overwrite a 0 by 1 but cannot overwrite a 1 by a 0.

6. Show formally that the *halting problem* is undecidable - Given a string $< M, w >$, is there an algorithm to decide if M halts on w (does not necesarily accept w) ?

7. Given input $< M_1, M_2 >$, is the problem $L(M_1) \cap L(M_2) \neq \phi$ decidable ? Justfy. (M_1, M_2 are codes of TM).

8. The proof of Rice's theorem exploits the result that L_u (universal language) is non-recursive. Given Rice's theorem, can you prove that L_u is not recursive ?

9. Are the following problems decidable ?

   (a) Given a TM M, whether M ever writes a specific non-blank symbol when started on an empty tape.

   (b) Given a TM M, whether there is a w such that M enters each of its states during the computation on w.

   (c) Given a TM M and an input w, does the head ever visit the B-th square for a given integer B.

10. Show that for every r.e. language L, there is an infinite recursive language L', where $L' \subset L$.
    Conversely, is it true that for every infinite recursive language, there is a subset that is not recursive ?