

CSL COL Intro to Automata and Theory of Computation, Tutorial Sheet 1

1. Design DFA for the following languages over $\{0, 1\}$
 - (a) The set of all strings such that every block of five consecutive symbols have at least two 0's.
 - (b) The set of all strings beginning with a 1 which interpreted as an integer is congruent to zero modulo 5.
 - (c) The set of strings with an equal number of 0's and 1's such that each prefix has at most one more 0 than 1's and at most one more 1 than 0's.
 - (d) The set of strings not containing the substring 110.

2. Design NFA for the following languages
 - (a) The set of strings over $\{0, 1\}$ such that *some* pair of 0's is separated by a string of length $4i$ $i \geq 0$.
 - (b) The set of strings over $\{a, b\}$ that have the same value when multiplied from left to right as from right to left. The rules of multiplication are $a \times a = b, b \times b = a, a \times b = b, b \times a = a$. Note that $((a \times b) \times b) = a$ and $(a \times (b \times b)) = b$, i.e. they are not the same, i.e. it is not associative.
 - (c) The set of strings of the form $\{x \cdot w \cdot x^R \mid x, w \text{ are strings over } 0,1 \text{ of non-zero length}\}$

3. Prove or disprove the following about regular expressions r, s, t where $r = s$ implies $L(r) = L(s)$
 - (a) $r(s + t) = rs + rt$
 - (b) $(r^*)^* = r^*$
 - (c) $(r^*s^*)^* = (r + s)^*$
 - (d) $(r + s)^* = r^* + s^*$

4. Which of the following are regular sets - Prove them
 - (a) $\{0^{2^n} \mid n \geq 1\}$
 - (b) $\{0^m 0^n 0^{m+n} \mid m, n \geq 1\}$
 - (c) $\{0^n \mid n \text{ is prime}\}$
 - (d) The set of all strings with equal number of 0s and 1's.
 - (e) Set of all palindromes over 0,1.
 - (f) $\{xx^R w \mid x, w \in (0 + 1)^+\}$

5. Let L be a regular set. Which of the following are regular
 - (a) $\{a_1 a_3 \dots a_{2n-1} \mid a_1 a_2 a_3 \dots a_{2n} \text{ is in } L\}$
 - (b) $MAX(L) = \{x \text{ is in } L \mid \text{no extension of } x \text{ is in } L\}$
 - (c) $L^R = \{x \mid x^R \text{ is in } L\}$
 - (d) $\frac{1}{2}(L) = \{x \mid \text{for some } y \text{ such that } |x| = |y|, xy \in L\}$.

6. A set of integers is *linear* if it is of the form $\{c + p \cdot i \mid i \geq 0\}$. A set is *semilinear* if it is a finite union of linear sets. Let $R \subset 0^*$ be regular. Prove that R is semilinear.

7. What is the relationship between class of regular sets and the least class of languages closed under union, intersection and complement containing all finite sets ?

8. What will be the statement of the converse of Pumping Lemma for regular language ?
 What does it imply for the language $\{0^i \cdot 1^j \cdot 2^k\}$ where $i = 0$ or $j = k$?