1. (Practice problems for induction) Must state the induction assertion formally and what is the induction on.
   (i) For all \( n \geq 1 \), \( x^{2n-1} + y^{2n-1} \) is divisible by \( x + y \).
   (ii) For all \( n \geq 1 \),
   \[
   1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} \cdots \frac{1}{\sqrt{n}} > 2(\sqrt{n} + 1 - 1)
   \]
   (iii) Prove that every integer divisible by 9 satisfies the property that, the sum of the digits is also divisible by 9.
   (iv) The greatest-common-divisor (gcd) of two non-negative integers \( m, n \) is known to satisfy the identity
   \( \text{gcd}(m, n) = \text{gcd}(m, n + m) \). Prove it.

2. Consider the following two definitions of the strings of balanced parentheses:
   A. A string \( w \) is balanced iff
      (i) \( w \) has an equal number of "(" and ")"
      (ii) Any prefix of \( w \) has at least as many "(" as ")"
   B. (i) \( \epsilon \) is balanced.
      (ii) If \( w \) is balanced, so is \( (w) \).
      (iii) If \( w \) and \( x \) are balanced then so is \( wx \)
      (iv) Nothing else is balanced.
   Show that the above definitions A and B are equivalent.

3. Show that the Principle of Mathematical Induction and the Principle of Complete induction are equivalent.
   Hint: Express them rigorously as sentences in first order logic.

4. Show two distinct bijective mappings between integers and rationals.

5. What is fallacy in applying a diagonalization argument to the set of rationals?

6. A relation \( \leq_{\#} \) is defined as follows
   If \( A, B \) are sets, then \( A \leq_{\#} B \) iff there exists a 1-1 mapping \( f : A \rightarrow B \) and an onto mapping
   \( g : B \rightarrow A \).
   If \( f : A \rightarrow B \) is a bijection then, \( A =_{\#} B \).
   What can you say about the pairs of sets
   (i) integers and rationals (ii) integers and Reals
   The Bernstein-Schroeder theorem says that If \( A \leq_{\#} B \) and \( B \leq_{\#} A \) then \( A =_{\#} B \).