1. (Practice problems for induction) Must state the induction assertion formally and what is the induction on.
   (i) For all $n \geq 1$, $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$.
   (ii) For all $n \geq 1$,
   \[ 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} \cdots \frac{1}{\sqrt{n}} > 2(\sqrt{n + 1} - 1) \]
   (iii) Prove that every integer divisible by 9 satisfies the property that, the sum of the digits is also divisible by 9.
   (iv) The greatest-common-divisor (gcd) of two non-negative integers $m, n$ is known to satisfy the identity $\text{gcd}(m, n) = \text{gcd}(m, n + m)$. Prove it.

2. Consider the following two definitions of the strings of balanced parentheses:
   A. A string $w$ is balanced iff
      (i) $w$ has an equal number of "(" and ")"
      (ii) Any prefix of $w$ has at least as many "(" as ")"
   B. (i) $\epsilon$ is balanced.
      (ii) If $w$ is balanced, so is $(w)$.
      (iii) If $w$ and $x$ are balanced then so is $w \cdot x$
      (iv) Nothing else is balanced.
   Show that the above definitions A and B are equivalent.

3. Show that the Principle of Mathematical Induction and the Principle of Complete induction are equivalent.
   Hint: Express them rigorously as sentences in first order logic.

4. Show two distinct bijective mappings between integers and rationals.

5. What is the fallacy in applying a diagonalization argument to the set of rationals?

6. A relation $\leq_\#$ is defined as follows
   
   If $A, B$ are sets, then $A \leq_\# B$ iff there exists a 1-1 mapping $f : A \to B$ and an onto mapping $g : B \to A$.
   If $f : A \to B$ is a bijection then, $A = \# B$.
   
   What can you say about the pairs of sets
   (i) integers and rationals (ii) Reals in $[0, 1]$ and reals in $(10, 100)$ (open interval)
   The Bernstein-Schroeder theorem says that If $A \leq_\# B$ and $B \leq_\# A$ then $A = \# B$. 
