Conversions between Binary to Octal and Hexadecimal (and vice-versa)

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Let us assume a \( n \)-bit number, \( N \), in binary format. Let it be represented as follows:

\[ N = x_n x_{n-1} \ldots x_1 \]

Here, \( x_1 \) is the LSB (least significant bit) and \( x_n \) is the MSB (most significant bit). For simplicity let us assume that \( n \) is a multiple of 3 and 4. We have:

\[ N = x_n 2^{n-1} + x_{n-1} 2^{n-2} + \ldots + x_2 2^1 + x_1 2^0 \tag{1} \]

Here, \( x_1 \ldots x_n \) are binary digits. They can either be 0 or 1. We can subsequently write:

\[
N = x_n 2^{n-1} + x_{n-1} 2^{n-2} + \ldots + x_2 2^1 + x_1 2^0 \\
= \underbrace{(x_n \times 2^2 + x_{n-1} \times 2^1 + x_{n-2} \times 2^0)}_{y_{n/3}} 2^{2-3} + \ldots + \underbrace{(x_6 \times 2^2 + x_5 \times 2^1 + x_4 \times 2^0)}_{y_2} 2^0 + \ldots + y_2 \times 8^1 + y_1 \times 8^0 \\
= y_{n/3} \ldots y_2 y_1 \quad \text{(in octal)} \tag{2}
\]

We thus have a method of converting a binary number into the octal (base 8) format by grouping bits in blocks of 3. We start from the LSB,
move leftward, group bits in a block of 3, and replace them by an octal
digit.
Example:
Convert (110 001) in binary to base 8. Answer: 0 61

Example:
Convert (0 74) in base 8 to binary. Answer: 111 100

We can use the reverse technique to convert a number in base 8 to binary.

To convert a binary number to the hexadecimal format and vice-versa,
we can follow the same logic and design a proof that says that we need to
group bits starting from the LSB in groups of 4.
Example:
Convert (1100 0011) in binary to base 16. Answer: 0x C3

Example:
Convert (0x FE) in hex to binary. Answer: 1111 1110