Axiomatic Semantic Models
We define store order $so \subseteq O \times O$ where $O = W \cup U \cup F$ such that $so$ is total.

**Axioms**

- hb is irreflexive. (ConsHB)
- so; hb is irreflexive. (ConsHBSO)
- fr; hb is irreflexive. (ConsFRHB)
- fr; so is irreflexive. (ConsFRSO)
- fr; so; rfe; po is irreflexive where rfe $= rf \setminus po$ (ConsFSRP)
- fr; so; [U \cup F]; po is irreflexive. (ConsFSUFP)

**Observation.**

Allows reordering of $W_x; R_y$ only where $x, y \in \text{Loc}$ and $x \neq y$. 
Transition on Execution Graph

We can construct an execution along with the execution.

At each step of execution we append an event to the execution graph.

\[
\text{Append}(G, a) = \langle G.E \cup \{a\}, \text{AddPO}(G, a), \text{AddRF}(G, a), \text{AddMO}(G, a) \rangle
\]

where

\[
\text{AddPO}(G, a) = G.po \cup (G.E^0 \cup G.E^a.Tid \times \{a\})
\]

\[
\text{AddRF}(G, a) = \begin{cases} 
\text{if } a \in R \text{ then } \exists w \in G.WU_x. \\
G'.rf = G.rf \cup \{(w, a)\} \land w.wVal = a.rVal \text{ else } G'.rf = G.rf
\end{cases}
\]

\[
\text{AddMO}(G, a) = \begin{cases} 
\text{if } a \in WU \text{ then } \exists O \subseteq G.WU_x. \\
G'.mo = G.mo \cup (O \times \{a\}) \cup (\{a\} \times G.WU_x \setminus O) \text{ else } G'.mo = G.mo
\end{cases}
\]
Consistent Execution Graph construction

Steps

- Graph $G$ is consistent.
- Append an event $a$ to $G$ and create a graph $G'$
- $G'$ is consistent.

\[
\begin{align*}
i &\in \text{Tid} \\
\ell &= \langle _, X, _\rangle \\
a &= \langle _, i, \ell \rangle \\
P(i), s &\xrightarrow{\ell} P'(i), s' \\
\text{Cons}_M(G) &\quad \text{Append}(G, a) \quad \text{Cons}_M(G') \\
P, G &\Rightarrow P', G'
\end{align*}
\]

\[
P, G_{\text{init}} \Rightarrow \cdots \Rightarrow \text{skip} \parallel \text{skip} \parallel \cdots \parallel \text{skip}, G_{\text{final}}
\]
Consistent Execution Graph construction

\[ \text{COH} \land \text{Atomicity} \iff \text{acy}(G.pox \cup G.rf \cup G.mo \cup G.fr) \]

**Proof Sketch**

(\(\iff\)) Follows immediately. (How?)

(\(\Rightarrow\)) Not so obvious.

\(\text{COH} \) and \(\text{Atomicity} \) has fixed number of relation compositions. But, a \(G.pox \cup G.rf \cup G.mo \cup G.fr\) path can be arbitrarily long.

- Follow the graph construction step.
- The condition holds at initial state.
- Assume the condition holds on consistent graph \(G\).
- Append an event \(a\) to get consistent \(G' = \text{Append}(G, a)\).
- Show that the condition holds on \(G'\)
  - all cases of \(a\) (i.e. \(W/R/U/F\))
  - possible relations among the events