# A Randomized Scheduler with Probabilistic Guarantees of Finding Bugs 

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#### Abstract

This paper presents a randomized scheduler for finding concurrency bugs. The scheduler improves upon current stresstesting methods by finding bugs more effectively, and by permitting us to quantify the probability of missing bugs. Key to its design is the characterization of the depth of a bug as the minimum number of scheduling constraints required to find it. In a single run of a program with $n$ threads and $k$ steps, our scheduler detects a bug of depth $d$ with probability at least $1 / n k^{d-1}$. We hypothesize that in practice, many bugs (including well-known types such as ordering errors, atomicity violations, and deadlocks) have small bug-depths, and we confirm the efficiency of our schedule randomization by detecting previously unknown and known concurrency bugs in several production-scale concurrent programs.


## 1. INTRODUCTION

Concurrent programming is known to be error prone. Concurrency bugs can be hard to find and are notorious for hiding in rare thread schedules. The goal of concurrency testing is to swiftly identify and exercise these buggy schedules from the astronomically large number of possible schedules. Popular testing methods involve various forms of stress testing where the program is run for days or even weeks under heavy loads with the hope of hitting buggy schedules. This is a slow and expensive process. Moreover, any bugs found are hard to reproduce and debug.

In this paper, we present PCT (Probabilistic Concurrency Testing), a randomized algorithm for concurrency testing. Given a concurrent program and an input test harness, PCT randomly schedules the threads of the program during each test run. In contrast to prior randomized testing techniques, PCT uses randomization sparingly and in a disciplined manner. As a result, PCT provides efficient mathematical probability of finding a concurrency bug in each run. Repeated independent runs can increase the probability of finding bugs to any user-desired level of certainty. In this paper, we demonstrate the ability of PCT to find bugs both theoretically, by stating and proving the probabilistic guarantees, and empirically, by applying PCT to several productionscale concurrent programs.

At the outset, it may seem impossible to provide effective probabilistic guarantees without exercising an astronomical number of schedules. Let us take a program with $n$ threads that together execute at most $k$ instructions. This program, to the first-order of approximation, has $n^{k}$ possible thread
schedules. If an adversary picks any one of these schedules to be the only buggy schedule, then no randomized scheduler can find the bug with a probability greater than $1 / n^{k}$. Given that realistic programs create tens of threads $(n)$ and can execute millions, if not billions, of instructions $(k)$, such a bound is not useful.

PCT relies on the crucial observation that bugs in practice are not adversarial. Concurrency bugs typically involve unexpected interactions among few instructions executed by a small number of threads [20, 22]. If PCT is able to schedule these few instructions correctly, it succeeds in finding the bug irrespective of the numerous ways it can schedule instructions irrelevant to the bug. The following characterization of concurrency bugs captures this intuition precisely.

We define the depth of a concurrency bug as the minimum number of scheduling constraints that are sufficient to find the bug. Intuitively, bugs with a higher depth exhibit in fewer schedules and are thus inherently harder to find. Fig. 1 explains this through a series of examples. The bug in Fig. 1(a) manifests whenever Thread 2 accesses $t$ before the initialization by Thread 1. We graphically represent this ordering constraint as an arrow. Any schedule that satisfies this ordering constraint finds the bug irrespective of the ordering of other instructions in the program. By our definition, this bug is of depth 1 . Fig. 1 shows two more examples of common concurrency errors, an atomicity violation in (b) and a deadlock in (c). Both these errors require two ordering constraints and are thus of depth 2 .

On each test run, PCT focuses on probabilistically finding bugs of a particular ${ }^{1}$ depth $d$. PCT is a priority-based scheduler and schedules the runnable thread with the highest priority at each scheduling step. Priorities are determined as follows. On thread creation, PCT assigns a random priority to the created thread. Additionally, PCT changes the thread priorities at $d-1$ randomly chosen steps during the execution.

These few but carefully designed random choices are provably effective for finding concurrency bugs of low depth. Specifically, when run on a program that creates at most $n$ threads and executes at most $k$ instructions, PCT finds

[^0]| Thread 1 | Thread 2 |
| :--- | :--- |
| $\ldots$ | $\ldots$ |
| $\mathrm{t}=$ new T() | $\ldots$ |
| $\ldots$ | if $(\mathrm{t}->$ state $==1)$ |
| $\ldots$ | $\ldots$ |
| $\ldots$ | $\ldots$ |

(a)

| Thread 1 | Thread 2 |
| :--- | :---: |
| $\ldots$ | $\ldots$ |
| $\ldots$ | $\ldots$ |
| $\mathrm{x}=$ null; | $\longrightarrow$ |
| $\ldots$ | if $(x!=$ null $)$ |
| $\ldots$ | $\ldots$ |

(b)

| Thread 1 | Thread 2 |
| :--- | :--- |
| $\ldots$ | $\ldots$ |
| $\operatorname{lock}(\mathrm{a}) ;$ | $\ldots$ |
| $\ldots$ | $\operatorname{lock}(\mathrm{b}) ;$ |
| $\ldots \operatorname{lock}(\mathrm{b}) ;$ | $\ldots$ |
| $\ldots$ | $\ldots$ |

(c)

Figure 1: Three typical concurrency bugs, and ordering edges sufficient to find each. (a) This ordering bug manifests whenever the test by thread 2 is executed before the initialization by thread 1. (b) This atomicity violation manifests whenever the test by thread 2 executed before the assignment by thread 1 , and the latter is executed before the method call by thread 2. (c) This deadlock manifests whenever thread 1 locks a before thread 2, and thread 2 locks b before thread 1.
a bug of depth $d$ with a probability of at least $1 / n k^{d-1}$. For small $d$, this bound is much better than the adversarial bound $1 / n^{k}$. In particular, for the cases $d=1$ and $d=2$ (which cover all examples in Fig. 1), the probability for finding the bug in each run is at least $1 / n$ and $1 / n k$, respectively. ${ }^{2}$

We describe the randomized algorithm informally in Section 2 and the formal treatment with the proof of the bound in Section 3. As described above, the scheduler is simple and can readily be implemented on large systems without knowledge of the proof mechanics. Note that the proof was instrumental to the design of PCT because it provided the insight on how to use randomization sparingly, yet effectively.

The probabilistic bound implies that on average, one can expect to find a bug of depth $d$ within $n k^{d-1}$ independent runs of PCT. As our experiments show (Section 5), PCT finds depth 1 bugs in the first few runs of the program. These bugs are certainly not trivial and were discovered by prior state-of-art research tools [26, 22] in well-tested realworld programs.

Our implementation of PCT, described in Section 4, works on executables compiled from $\mathrm{C} / \mathrm{C}++$ programs. In addition to the base algorithm described in Section 2, our implementation employs various optimizations including one that reduces $k$ to the maximum number of synchronization operations (rather than the number of instructions) performed in any run for a given test input.

We evaluate PCT on six benchmarks from prior work [26, 23] that contain known concurrency bugs. This includes the open source program PBZIP2 [26], three SPLASH2 benchmarks [26], an implementation of a work stealing queue [23], and Dryad Channels [23]. PCT finds all of the known bugs much faster than respectively reported in prior work. We also find two new bugs that were missed by prior work in these benchmarks. To test our scalability, we ran PCT on unmodified recently-shipped versions of two popular web

[^1]browsers Mozilla and Internet Explorer. We find one previously unknown bug in each of them. Finally, we empirically demonstrate that PCT often detects bugs with probabilities greater than the theoretical worst-case bound.

## 2. PCT OVERVIEW

In this section we provide necessary background and an informal description of our algorithm.

### 2.1 Concurrency Testing

The general problem of testing a program involves many steps. In this paper, we focus on concurrency testing. We define a concurrency bug as one that manifests on a strict subset of possible schedules. Bugs that manifest in all schedules are not concurrency bugs. The problem of concurrency testing is to find schedules that can trigger these bugs among the vast number of potential schedules.

We assume that inputs to our program are already provided, and the only challenge is to find buggy schedules for that input. Determining bug-triggering inputs for concurrent programs is a challenging open problem beyond the scope of this paper. Our assumption is validated by the fact that there already exists large suites of stress tests carefully constructed by programmers over the years.

### 2.2 State of the Art

We identify the following basic strategies for flushing out concurrency bugs. We describe them in detail in Section 6.

Stress Testing relies on repetition and heavy load to find bug-triggering schedules by chance. The schedules explored are not uniformly distributed and are determined by the chaotic noise in the system.

Heuristic-Directed Testing improves upon stress testing by using runtime monitors and heuristics to (1) detect suspicious activity in a program (such as variable access patterns that indicate potential atomicity violations [26], or lock acquisition orderings that indicate potential deadlocks [17]), and (2) direct the schedule towards suspected bugs.

Systematic Scheduling controls the scheduler to systematically enumerates possible schedules either exhaustively or within some bound (such as a bound on the number of preemptions) [23].


Figure 2: An example of a bug of depth 2 we found in pbzip. The bug surfaces if (1) the mutex is unlocked after it is freed, and (2) the mutex is unlocked before the main thread terminates the process by calling exit.

Randomized Scheduling is similar to stress testing, but attempts to amplify the 'randomness' of the OS scheduler[2]. It can do so by inserting random delays, context switches, or thread priority changes.

PCT falls in the last category. But unlike all the methods above, PCT provides a guaranteed probability of finding bugs in every run of the program. Our experiments validate this guarantee. Note that PCT is orthogonal to heuristic-directed testing methods above, in the sense that the analysis used in these methods can be used to further improve PCT.

### 2.3 Bug Depth

We classify concurrency bugs according to a depth metric. Intuitively, deeper bugs are inherently harder to find. PCT is designed to provide better guarantees for bugs with smaller depth.

Concurrency bugs happen when instructions are scheduled in an order not envisioned by the programmer. We identify a set of these ordering constraints between instructions from different threads that are sufficient to trigger the bug. It is possible for different sets of ordering constraints to trigger the same bug. In such a case, we focus on the set with lesser number of constraints. We define the depth of a concurrency bug as the minimum number of ordering constraints sufficient to find the bug.

For example, Fig. 1 shows examples of common concurrency errors with ordering constraints, represented by arrows, that are sufficient to find the bug. Any schedule that satisfies these ordering constraints is guaranteed to find the bug irrespective of how it schedules instructions not relevant to the bug. For the examples in Fig. 1 the depth respectively is 1,2 , and 2 . We expect many concurrency bugs to have small depths. This is further validated by our experimental results.

### 2.3.1 Relationship with Prior Classification

Fig. 1 also demonstrates how previous classifications of concurrency bugs correspond to bugs of low depth. For example, ordering bugs [20] have depth 1 , atomicity violations and non-serializable interleavings [26], in general, have depth 2 , and deadlocks caused by circular lock acquisition [17] have depth 2 , or more generally $n$ if $n$ threads are involved. However, this classification is not strict. For instance, not all atomicity violations have a depth 2, and in fact, three of the bugs reported by prior work as atomicity violations [26] have a depth 1 .

| Thread 1 | Thread 2 |
| :--- | :--- |
| $\ldots$ | $\ldots$ |
| Set(e); | $\ldots$ |
| $\mathrm{t}=$ new $\mathrm{T}(\mathrm{O}$ | Wait(e); |
| $\ldots$ | t->state $=1$ |
| $\ldots$ | $\ldots$ |

Figure 3: A variation of the example in Fig. 1(a). This bug requires Thread 1 to be preempted right after the instruction that sets the event e. Hence this bug has a preemption bound of 1 while the bug in Fig. 1(a) has a preemption bound of 0. Both bugs are of depth 1 .

However, our notion of bug depth is more general and can capture concurrency bugs not classified before. Fig. 2 shows an example of a bug of depth 2 that does not directly fall into any of the mentioned categories. In particular, the ordering constraints do not have to be between instructions that access the same variable.

Another characterization of a concurrency bug is its preemption bound [22]. A preemption bound is the smallest number of preemptions sufficient to find a concurrency bug. To the best of our knowledge, there exists no relation between the preemption bound and the depth of a bug for arbitrary concurrent programs. For instance, Fig. 3 and Fig. 1(a) describe bugs that have the same bug depth but different preemption bounds.

### 2.3.2 Interaction with Control Flow

Fig. 4 shows a slight modification to Fig. 1(a). In this example, the program (incorrectly) maintains a Boolean variable init to indicate whether $t$ is initialized or not. Now, the single ordering constraint (black arrow) between the initialization and access of $t$ is not sufficient to find the bug. The scheduler should also ensure the right ordering constraint between init accesses (grey arrow). Thus, the presence of control flow increases the bug depth to 2 .

This example brings out two interesting points. First, the notion of bug depth is inherently tied to the difficulty of the concurrency bug. Fig. 4 is arguably a more subtle bug than Fig. 1(a). Second, in a program with complex control flow, the depth of a bug might not be readily apparent to the programmer. However, our technique does not require the programmer or a prior program analysis to identify these constraints explicitly. It relies on the mere existence of the right number of ordering constraints.

### 2.4 Naive Randomization

Using a randomized scheduler may appear like an obvious choice. However, it is not a priori clear how to design such a scheduler with a good detection probability for concurrency bugs. For illustration purposes, let us consider the simple case of a program shown in Fig. 5 with two threads containing a bug of depth 1 , shown by the black arrow. (Neglect the grey arrow for now.) Even this simple bug can frustrate a naive randomization technique.

| Thread 1 | Thread 2 |
| :--- | :--- |
| $\ldots$ | $\ldots$ |
| init $=$ true | $\ldots$ |
| $t=$ new $T()$ | if ( init ) |
| $\ldots$ | $\ldots$ |

Figure 4: Although it may seem like one constraint (black arrow) is sufficient to find this bug, an extra constraint (gray arrow) is needed to ensure that thread 2 really executes the access of $t$. Thus, the depth of this bug is 2 .


Figure 5: A program with two bugs of depth 1 that are hard to find with naive randomized schedulers that flip a coin in each step. PCT finds both these bugs with a probability $1 / 2$.

Consider a naive randomized scheduler that flips a coin in each step to decide which thread to schedule next. This scheduler is unlikely to detect the bug in Fig. 5 even though its depth is only 1 . To force the black constraint, this scheduler has to consistently schedule Thread 1 for $m+2$ steps, resulting in a probability that is inverse exponential in $m$ - a small quantity even for moderate $m$. One could then try to improve this scheduler by biasing the coin towards Thread 1 to increase the likelihood of hitting this bug. This still contains an exponential in $m$. But more importantly, any bias towards the black constraint, will be equally biased against the second bug represented by the grey constraint.

In contrast, our PCT scheduler will find both bugs (and all other bugs with depth 1 ) with probability $1 / 2$ for this program.

### 2.5 The PCT Randomized Scheduler

We now describe our key contribution, a randomized scheduler that detects bugs of depth $d$ with a guaranteed probability in every run of the program. Our scheduler is prioritybased. The scheduler maintains a priority for every thread, where lower numbers indicate lower priorities. During execution, the scheduler schedules a low priority thread only when all higher priority threads are blocked. Only one thread is scheduled to execute in each step. A thread can get blocked if it is waiting for a resource, such as a lock that is currently held by another thread, or more generally if it is performing some blocking synchronization of any kind.

Threads can change priorities during execution when they pass a priority change point. Each such point is a step in the dynamic execution and has a predetermined prior-
ity value associated with it. When the execution reaches a change point, the scheduler changes the priority of the current thread to the priority value associated with the change point.

Given inputs $n, k$, and $d$, PCT works as follows.

1. Assign the $n$ priority values $d, d+1, \ldots, d+n$ randomly to the $n$ threads (we reserve the lower priority values $1, \ldots,(d-1)$ for change points).
2. Pick $d-1$ random priority change points $k_{1}, \ldots, k_{d-1}$ in the range $[1, k]$. Each $k_{i}$ has an associated priority value of $i$.
3. Schedule the threads by honoring their priorities. When a thread reaches the $i$-th change point (that is, when it executes the $k_{i}$-th step of the execution), change the priority of that thread to $i$.

This randomized scheduler provides the following guarantee.
Given a program that creates at most $n$ threads and executes at most $k$ instructions, PCT finds a bug of depth $d$ with probability at least $1 / n k^{d-1}$.

### 2.6 Intuition Behind the Algorithm

See Fig. 6 for an illustration of how our algorithm finds the errors in Fig. 1. This figure shows the initial thread priorities in white circles and the priority change points in black circles. To understand the working of the scheduler, observe that a high priority thread runs faster than a low priority thread. So, barring priority inversion issues, an ordering constraint $a \rightarrow b$ is satisfied if $a$ is executed by a higher priority thread. In Fig. 6(a), the bug is found if PCT chooses a lower priority for Thread 1 than Thread 2. The probability of this is $1 / 2$ and thus PCT is expected to find this bug within the first two runs.

If there are more than two threads in the program in Fig. 6(a), then the algorithm has to work harder because of priority inversion issues. Even if Thread 1 has a lower priority than Thread 2, the latter can be blocked on a resource held by another thread, say Thread 3. If Thread 3 has a priority lower than Thread 1, then this priority inversion can allow Thread 1 to execute the initialization before Thread 2 reads $t$. However, such a priority inversion cannot happen if Thread 1 has the lowest priority of all threads in the program. The probability of this happening is $1 / n$ which is our guarantee.

For bugs with depth greater than 1 , we need to understand the effects of priority change points. (Our algorithm does not introduce priority change points when $d=1$.) In Fig. 6(b), the atomicity violation is induced if PCT inserts a priority change point after the null check but before executing the branch. The probability of this is $1 / k$ as PCT will pick the change point uniformly over all dynamic instructions. In addition, PCT needs to ensure the first constraint by running Thread 1 with lowest priority till Thread 2 does the null check. Together, the probability of finding this atomicity violation is at least $1 / n k$.

(a)

(b)

(c)

Figure 6: Illustration on how our randomized scheduler finds bugs of depth $d$, using the examples from Fig. 1. The scheduler assigns random initial thread priorities $\{d, \ldots, d+n-1\}$ (white circles) and randomly places $d-1$ priority change points of values $\{1, \ldots, d-1\}$ (black circles) into the execution. The bug is found if the scheduler happens to make the random choices shown above.

The same argument holds for the deadlock in Fig. 6(c). PCT has to insert a priority change point after Thread 1 acquires the first lock before acquiring the second. The probabilistic guarantee of our algorithm with multiple priority change points in the presence of arbitrary synchronizations and control flow in the program, and issues of priority inversion is not readily apparent from the discussion above. Section 3 provides a detailed formal proof that accounts for all these complications.

### 2.7 Probabilistic Worst-Case Bound

The probabilistic guarantee provided by PCT is a worst-case bound. In other words, for any program than an adversary might pick, and for any bug of depth $d$ in that program, PCT is guaranteed to find the bug with a probability not less than $1 / n k^{d-1}$. This bound is also tight. There exists programs, such as the one in Fig. 4, for which PCT can do no better than this bound.

In practice, it is possible for PCT to perform better than this worst-case bound. Our experiments in Section 5 validate this possiblity. Here are several reasons as to why PCT performs better than the worst-case bond. (1) Sometimes it is good enough for priority change points to fall in some range of instructions. For example, Thread 1 in Fig. 6(c) may perform lots of instructions between the two acquires. PCT will find the deadlock if it picks any one of them to be a priority change point. (2) Sometimes a bug can be found in different ways. For instance, in Fig. 6(c), there exists a symmetric case in which PCT inserts a priority change point in Thread 2. (3) Sometimes a buggy code fragment is repeated many times in a test, by the same thread or by different threads, and thus offers multiple opportunities to trigger the bug.

## 3. ALGORITHM

In this section, we build a formal foundation for describing our scheduler and prove its probabilistic guarantees. For conciseness, some technical proofs are available in the full version of the paper [1].

### 3.1 Definitions

We briefly recount some standard notation for operations on sequences. Let $T$ be any set. Define $T^{*}$ to be the set of finite sequences of elements from $T$. For a sequence $S \in T^{*}$, define length $(S)$ to be the length of the sequence. We let $\epsilon$ denote the sequence of length 0 . For a sequence $S \in T^{*}$ and

```
Require: program \(P, d \geq 0\)
Require: \(n \geq\) maxthreads \((P), k \geq\) maxsteps \((P)\)
Require: random variables \(k_{1}, \ldots, k_{d-1} \in\{1, \ldots, k\}\)
Require: random variable \(\pi \in \operatorname{Permutations}(n)\)
procedure \(\operatorname{RandS}(n, k, d)\) begin
    \(\operatorname{var} S\) : schedule
    \(\operatorname{var} p: \operatorname{array}[\mathrm{n}]\) of \(\mathbb{N}\)
    \(S \leftarrow \epsilon\)
    // set initial priorities
    for all \(t \in\{1, \ldots, n\}\) do
        \(p[t] \leftarrow d+\pi(t)-1\)
    end for
    while \(e n_{P}(S) \neq \emptyset\) do
        /* schedule thread of maximal priority */
        \(t \leftarrow\) element of \(e n_{P}(S)\) such that \(p[t]\) maximal
        \(S \leftarrow S t\)
        /* are we at priority change point? */
        for all \(i \in\{1, \ldots, d-1\}\) do
            if length \((S)=k_{i}\) then
                \(p[t]=d-i\)
            end if
        end for
    end while
    return \(S\)
end
```

Figure 7: The randomized scheduler.
a number $n$ such that $0 \leq n<\operatorname{length}(S)$, let $S[n]$ be the $n$-th element of $S$ (where counting starts with 0 ). For $t \in T$ and $S \in T^{*}$, we write $t \in S$ as a shorthand for $\exists m: S[m]=$ $t$. For any $S \subset T^{*}$ and for any $n, m$ such that $0 \leq n \leq$ $m \leq$ length $(S)$, let $S[n, m]$ be the contiguous subsequence of $S$ starting at position $n$ and ending at (and including) position $m$. For two sequences $S_{1}, S_{2} \in T^{*}$, we let $S_{1} S_{2}$ denote the concatenation as usual. We do not distinguish between sequences of length one and the respective element. We call a sequence $S_{1} \in T^{*}$ a prefix of a sequence $S \in T^{*}$ if there exists a sequence $S_{2} \in T^{*}$ such that $S=S_{1} S_{2}$. A set of sequences $P \subseteq T^{*}$ is called prefix-closed if for any $S \in P$, all prefixes of $P$ are also in $P$.

Definition 1. Define $T=\mathbb{N}$ to be the set of thread identifiers. Define Sched $=T^{*}$ to be the set of all schedules. Define a program to be a prefix-closed subset of Sched. For
a given program $P \subseteq$ Sched, we say a schedule $S \in P$ is complete if it is not the prefix of any schedule in $P$ beside itself, and partial otherwise.

Thus, we represent a program abstractly by its schedules, and each schedule is simply a sequence of thread identifiers. For example, the sequence 1221 represents the schedule where thread 1 takes one step, followed by two steps by thread 2 , followed by another step of thread 1 . We think of schedules as an abstract representation of the program state. Not all threads can be scheduled from all states, as they may be blocked. We say a thread is enabled in a state if it can be scheduled from that state.

DEfinition 2. Let $P \subseteq$ Sched be a program. For a schedule $S \in P$, define en $n_{P}(S)$ to be the set $\{t \in T \mid S t \in$ $P\}$. Define maxsteps $(P)=\max \{\operatorname{length}(S) \mid S \in P\}$ and maxthreads $(P)=\max \{S[i] \mid S \in P\} \quad$ (or $\infty$ if unbounded).

Finally, we represent a concurrency bug abstractly as the set of schedules that find it:

DEFINITION 3. Let $P \subseteq$ Sched be a program. Define a bug $B$ of $P$ to be a subset $B \subset P$.

### 3.2 The Algorithm

We now introduce the randomized scheduler (Fig. 7). It operates as described informally in Section 2.5. We expect $\operatorname{RandS}(n, k, d)$ to be called with a conservative estimate for $n$ (number of threads) and $k$ (number of steps). During the progress of the algorithm, we store the current schedule in the variable $S$, and the current thread priorities in an array $p$ of size $n$. The thread priorities are initially assigned random values (chosen by the random permutation $\pi$ ). In each iteration, we pick an enabled thread of maximal priority $t$ and schedule it for one step. Then we check if we have reached a priority change point (determined by the random values $k_{i}$ ), and if so, we change the priority of $t$ accordingly. This process repeats until no more threads are enabled (that is, we have reached a deadlock or the program has terminated).

### 3.3 Probabilistic Coverage Guarantee

In this section, we precisely state and then prove the probabilistic coverage guarantees for our randomized scheduler, in three steps. First, we introduce a general mechanism for identifying dynamic events in threads, which is a necessary prerequisite for defining ordering constraints on such events. Next, we build on that basis to define the depth of a bug as the minimum number of ordering constraints on thread events that will reliably reveal the bug. Finally, we state and prove the core theorem.

### 3.3.1 Event Labeling

The first problem is to clarify how we define the events that participate in the ordering constraints. For this purpose, we introduce a general definition of event labeling. Event labels must be unique within each execution, but may vary across executions. Essentially, an event labeling $E$ defines a set of labels $L_{E}$ (where each label $a \in L_{E}$ belongs to a particular
thread thread $\left.{ }_{E}(a)\right)$ and a function $\operatorname{exxt}_{E}(S, t)$ that tells us what label (if any) the thread $t$ is going to emit if scheduled next after schedule $S$. More formally, we define:

Definition 4. Let $P$ be a program. An event labeling $E$ is a triple $\left(L_{E}\right.$, thread $_{E}$, next $\left._{E}\right)$ where $L_{E}$ is a set of labels, thread $_{E}$ is a function $L_{E} \rightarrow T$, and next $E_{E}$ is a function $P \times T \rightarrow\left(L_{E} \cup\{\perp\}\right)$, such that the following conditions are satisfied:

1. (Affinity) If $\operatorname{next}_{E}(S, t)=a$ for some $a \in L_{E}$, then thread $_{E}(a)=t$.
2. (Stability) If $\operatorname{next}_{E}(S, t)=a$ for some $a \in L_{E}$, and if $t \neq t^{\prime}$, then $\operatorname{next}_{E}\left(S t^{\prime}, t\right)=a$.
3. (Uniqueness) If next ${ }_{E}\left(S_{1}, t\right)=\operatorname{next}_{E}\left(S_{1} S_{2}, t\right)=$ a for some $a \in L_{E}$, then $t \notin S_{2}$.
4. (NotFirst) $\operatorname{next}_{E}(\epsilon, t)=\perp$ for all $t \in T$.

Sometimes, we would like to talk about labels that have already been emitted in a schedule. For this purpose we define the auxiliary functions label $_{E}$ and labels $_{E}$ as follows. For $S \in P$ and $0 \leq m<\operatorname{length}(S)$, we define $\operatorname{label}_{E}(S, m)=$ $a$ if the label $a$ is being emitted at position $m$, and we define labels $_{E}(S)$ to be the set of all labels emitted in $S$ (more formally, $\operatorname{label}_{E}(S, m)=a$ if there exists $k<m$ and an $a \in L_{E}$ such that $\operatorname{eext}_{E}(S[0, k], S[m])=a$ and $S[m] \notin S[k+$ $1, m-1]$, and $\operatorname{label}_{E}(S, m)=\perp$ otherwise; and labels ${ }_{E}(S)=$ $\left.\left\{\operatorname{label}_{E}(S, m) \mid 0 \leq m<l e n g t h(S)\right\}\right)$.

### 3.3.2 Bug Depth

We now formalize the notion of 'ordering constraints' and 'bug depth' that we motivated earlier. Compared to our informal introduction from Section 2.3 , there are two variations worth mentioning. First, we generalize each edge constraint $(a, b)$ (where $a$ and $b$ are event labels) to allow multiple sources $(A, b)$, where $A$ is a set of labels all of which have to be scheduled before $b$ to satisfy the constraint. Second, because we are using dynamically generated labels as our events, we require that the ordering constraints are sufficient to guide the scheduler to the bug without needing to know about additional constraints implied by the program structure (as motivated by the example in Fig. 4).

We formulate the notion of a directive $D$ of size $d$, which consists of a labeling and $d$ constraints. The idea is that a directive can guide a schedule towards a bug, and that the depth of a bug is defined as the minimal size of a directive that is guaranteed to find it.

Definition 5. A directive $D$ for a program $P$ is a tuple $\left(E, A_{1}, b_{1}, A_{2}, b_{2}, \ldots, A_{d}, b_{d}\right)$ for some $d \geq 1$ (called the size of $D$, and denoted by size $(D)$ ) where $E$ is an event labeling for $P$, where $A_{1}, \ldots, A_{d} \subseteq L_{E}$ are sets of labels, and where $b_{1}, \ldots b_{d} \in L_{E}$ are labels that are pairwise distinct $\left(b_{i} \neq b_{j}\right.$ for $i \neq j$ ).

Definition 6. Let $P$ be a program and let $D$ be a directive for $P$. We say a schedule $S \in P$ violates the directive $D$

```
Require: program \(P, d \geq 0\)
Require: \(n>=\) maxthreads \((P)\)
Require: \(k_{1}, \ldots, k_{d-1} \geq 1\)
Require: \(\pi \in\) Permutations \((n)\)
Require: random variables \(k_{1}, \ldots, k_{d-1} \in\{1, \ldots, k\}\)
Require: random variable \(\pi \in \operatorname{Permutations}(n)\)
Require: bug \(B\)
Require: directive \(D=\left(E, A_{1}, b_{1}, \ldots, A_{d}, b_{d}\right)\) for \(B\)
    procedure \(\operatorname{DirS}(n, k, d, D)\) begin
    \(\operatorname{var} S\) : schedule
        \(\operatorname{var} p: \operatorname{array}[\mathrm{n}]\) of \(\mathbb{N}\)
        \(S \leftarrow \epsilon\)
    // set initial priorities
    for all \(t \in\{1, \ldots, n\}\) do
        \(p[t] \leftarrow d+\pi(t)-1\)
    end for
    [ assert: \(p\left[\right.\) thread \(\left.\left._{E}\left(b_{1}\right)\right]=d\right]\)
    while \(e n_{P}(S) \neq \emptyset\) do
        /* schedule thread of maximal priority */
        \(t \leftarrow\) element of \(e n_{P}(S)\) such that \(p[t]\) maximal
        \(S \leftarrow S t\)
        /* change priority first time we peek a \(b\)-label */
        for all \(i \in\{1, \ldots, d-1\}\) do
            if \(\operatorname{next}_{E}(S, t)=b_{i+1}\) and \(p[t] \neq d-i\) then
                    \(p[t]=d-i\)
                    [ assert: length \(\left.(S)=k_{i}\right]\)
            end if
        end for
        end while
        return \(S\)
    end
```

Figure 8: The directed scheduler.
if either (1) there exists an $i \in\{1, \ldots, d\}$ and an $a \in A_{i}$ such that $b_{i} \in \operatorname{labels}_{E}(S)$, but $a \notin$ labels $_{E}(S)$, or (2) there exist $1 \leq i<j \leq d$ such that $b_{j} \in \operatorname{labels}_{E}(S)$, but $_{i} \notin \operatorname{labels}_{E}(S)$. We say a schedule $S \in P$ satisfies $D$ if it does not violate $D$, and if $b_{i} \in$ labels $_{E}(S)$ for all $1 \leq i \leq d$.

Definition 7. Let $P$ be a program, $B$ be a bug of $P$, and $D$ be a directive for $P$. We say $D$ guarantees $B$ if and only if the following conditions are satisfied:

1. For any partial schedule $S \in P$ that does not violate $D$, there exists a thread $t \in e n_{P}(S)$ such that $S t$ does not violate $D$.
2. Any complete schedule $S$ that does not violate $D$ does satisfy $D$ and is in $B$.

DEfinition 8. Let $P$ be a program, and let $B$ be a bug of $P$. Then we define the depth of $B$ to be

$$
\operatorname{depth}(B)=\min \{\text { size }(D) \mid D \text { guarantees } B\}
$$

### 3.3.3 Coverage Theorem

The following theorem states the key guarantee: the probability that one invocation $\operatorname{Rand} S(n, k, d)$ of our randomized scheduler (Fig. 7) detects a bug of depth $d$ is at least $\frac{1}{n k^{d-1}}$.

ThEOREM 9. Let $P$ be a program with a bug $B$ of depth $d$, let $n \geq$ maxthreads $(P)$, and let $k \geq$ maxsteps $(P)$. Then

$$
\operatorname{Pr}[\operatorname{Rand} S(n, k, d) \in B] \geq \frac{1}{n k^{d-1}}
$$

Proof. Because $B$ has depth $d$, we know there exists a directive $D$ for $B$ of size $d$. Of course, in any real situation, we do not know $D$, but by Def. 8 we know that it exists, so we can use it for the purposes of this proof. Essentially, we show that even without knowing $D$, here is a relatively high probability that $\operatorname{Rand} S(n, k, d)$ follows the directive $D$ by pure chance. To prove that, we first construct an auxiliary algorithm $\operatorname{DirS}(n, k, d, D)$ (Fig. 8) that uses the same random variables as RandS, but has knowledge of $D$ and constructs its schedule accordingly.

Comparing the two programs, we see two differences. First, Line 13 uses a condition based on $D$ to decide when to change priorities. In fact, this is where we make sure the call to $\operatorname{DirS}(n, k, d, D)$ is following the directive $D$ : whenever we catch a glimpse of thread $t$ executing one of the labels $b_{i}$ (for $i>1$ ), we change the priority of $t$ accordingly. Second, DirS has assertions which are not present in RandS. We use these assertions for this proof to reason about the probability that $\operatorname{DirS}$ guesses the right random choices. The intended behavior is that $\operatorname{DirS}$ fails (terminating immediately) if it executes a failing assertion.

The following three lemmas (the proofs are available in the full version [1]) are key for our proof construction:

Lemma 10. DirS succeeds with probability $\geq \frac{1}{n k^{d-1}}$.

Lemma 11. If DirS succeeds, it returns a schedule that finds the bug

Lemma 12. If $\operatorname{DirS}$ succeeds, it returns the same schedule as RandS.

We can formally assemble these lemmas into a proof as follows. Our sample space consists of all valuations of the random variables $\pi$ and $k_{1}, \ldots, k_{d-1}$. By construction, each variable is distributed uniformly and independently (thus, the probability of each valuation is equal to $n!k^{d-1}$ ). Define $\mathcal{S}$ to be the event (that is, set of all valuations) such that $\operatorname{DirS}(n, k, d, D)$ succeeds, and let $\overline{\mathcal{S}}$ be its complement.

$$
\begin{array}{ll}
\operatorname{Pr}[\operatorname{RandS}(n, k, d) \in B] & \\
=\operatorname{Pr}[\operatorname{RandS}(n, k, d) \in B \mid \mathcal{S}] \cdot \operatorname{Pr}[\mathcal{S}] & \\
\quad+\operatorname{Pr}[\operatorname{Rand} S(n, k, d) \in B \mid \overline{\mathcal{S}}] \cdot \operatorname{Pr}[\overline{\mathcal{S}}] & \\
\geq \operatorname{Pr}[\operatorname{Rand} S(n, k, d) \in B \mid \mathcal{S}] \cdot \operatorname{Pr}[\mathcal{S}] & \\
=\operatorname{Pr}[\operatorname{Dir} S(n, k, d, D) \in B \mid \mathcal{S}] \cdot \operatorname{Pr}[\mathcal{S}] & \text { (by Lemma 12) } \\
=1 \cdot \operatorname{Pr}[\mathcal{S}] & \text { (by Lemma 11) } \\
\geq \frac{1}{n k^{d-1}} & \text { (by Lemma 10) } \tag{byLemma10}
\end{array}
$$

(by Lemma 11)

## 4. IMPLEMENTATION

This section describes our implementation of the PCT scheduler.

### 4.1 Design Choices

The PCT scheduler, as informally described in Section 2, is based on thread priorities. The obvious way to implement PCT would be to reuse the priority mechanisms already supported by modern operating systems. We chose not to for the following reason. The guarantees provided by PCT crucially rely on a low priority thread proceeding strictly slower than a high priority thread. OS priorities do not provide this guarantee. In particular, priority boosting [15] techniques can arbitrarily change user-intended priorities. Similarly, our scheduler would not be able to control the relative speeds of two threads with different priorities running concurrently on different processors.

For fine-grained priority control, we implemented PCT as a user-mode scheduler. PCT works on unmodified x86 binaries. It employs binary instrumentation to insert calls to the scheduler after every instruction that accesses shared memory or makes a system call. The scheduler gains control of a thread the first time the thread calls into the scheduler. From there on, the scheduler ensures that the thread makes progress only when all threads with higher priorities are disabled. Thread priorities are determined by the algorithm as described in Section 2.

Our scheduler is able to reliably scale to large programs. We are successfully able to run unmodified versions of Mozilla Firefox and Internet Explorer, two popular web browsers, and find bugs in them. Our initial prototype slows down the execution of the program by 2 to 3 times. This is well within the expected slowdowns for any binary instrumentation tool.

One challenge we identified during our implementation is the need for our scheduler to be starvation free. It is common for concurrent programs to use spin loops. If, under PCT, a high priority thread spins waiting for a low priority thread, the program will livelock - to guarantee strict priorities, PCT would not schedule the low priority thread required for the high priority thread to make progress. To avoid such starvation, PCT uses heuristics to identify threads that are not making progress and lowers their priorities with a small probability.

### 4.2 Optimizations

The base algorithm described in Section 2 requires that the scheduler have the capability of inserting a priority change point at randomly selected instructions. This has two disadvantages. First, the need to insert a change point at an arbitrary instruction requires PCT to insert a callback after every instruction, slowing down the performance. Second, by counting the number of instructions executed the large value for the parameter $k$ can reduce the effectiveness especially for bugs with depth $\geq 2$. We introduced the optimizations below to address this problem.

Identifying Synchronization Operations: The first optimization relies on identifying synchronization operations and inserting prority change points only at these operations.

We first classify thread operations into two classes: synchronization operations and local operations. A synchronization operation can be used to communicate between threads, while a local operation is used for local computation within a thread. Synchronization operations include system calls, calls to synchronization libraries (such as pthreads), and hardware synchronization instructions (such as interlocked instructions). In addition, we also treat accesses to flag variables, volatile accesses, and data races (both programmer intended and unintended) as "shared-memory synchronization." Our classification reflects the fact that these memory accesses result in communication between the threads. Local operations include instructions that do not access memory and memory accesses that do not participate in a data race (such as accessing the stack or accessing consistently protected shared memory). Any of the existing data-race detection tools [10, 12] or hardware mechanisms [24] can be used to classify memory accesses into local or synchronization operations. Other forms of synchronization are straightforward to identify from the program binary.

This optimization relies on the following observation. For every execution in which a priority change point occurs before a local operation, there exists a behaviorally-equivalent execution in which the priority change point occurs before the synchronization operation following the local operation. This is because the two executions differ only in the order of local operations. This means that we only need to insert priority change points before synchronization operations. This effectively reduces $k$ in the probabilistic bound by several orders of magnitude, from the maximum number of instructions executed by the program to the maximum number of syncrhonization operations. In the rest of the paper, we only report the number of synchronization operations as $k$.

Identifying Sequential Execution: We observed for some benchmarks that a significant portion of a concurrent execution is actually sequential where there is only one enabled thread. Inserting priority change points during this sequential execution is not necessary. The same effect can be achieved by reducing the priority at the point the sequential thread enables/creates a second thread.

Identifying Join Points: Programs written with a forkjoin paradigm typically have multiple phases where a single thread waits for a flurry of concurrent activity belonging to one phase to finish before starting the next phase. This is also a typical behavior of long running stress tests that perform multiple iterations of concurrency scenarios. Our implementation of PCT identifies these phases whenever the program enters a state with one thread enabled. The effective $k$ is the maximum number of synchronization operations performed per phase.

Final Wait: Some concurrency bugs might manifest much later than when they occur. We found that PCT missed some of the manifestations as the main thread exits prematurely at the end of the program. Thus, we artificially insert a priority change point for the main thread before it exits.

## 5. EXPERIMENTS

In this section, we describe the evaluation of our PCT scheduler on several real-world programs of varying complexity.

| Programs | Bug |  |
| :--- | :--- | :--- |
|  | Manifestation | Known? |
| Splash-FFT | Platform dependent macro <br> missing a wait leading to <br> order violations | YES |
| Splash-LU | YES |  |
| Splash-Barnes | Crash during decompression | YES |
| Pbzip2 | Internal assertion fails due <br> to a race condition | NO* $^{*}$ |
| Work Steal Queue | YES |  |
| Dryad | Use after free failing an <br> internal assertion | NO |
| IE | Javascript parse error | NO |
| Mozilla | Crash during restoration |  |

Table 1: Concurrency benchmarks and bugs. NO* indicates both known and unknown bugs.

| Programs | LOC | $d$ | $n$ | $k$ | $k_{\text {eff }}$ |
| :--- | ---: | :--- | ---: | ---: | ---: |
| Splash-FFT | 1200 | 1 | 2 | 791 | 139 |
| Splash-LU | 1130 | 1 | 2 | 1517 | 996 |
| Splash-Barnes | 3465 | 1 | 2 | 7917 | 318 |
| Pbzip2 | 1978 | 2 | 4 | 1981 | 1207 |
| Work Steal Queue | 495 | 2 | 2 | 1488 | 75 |
| Dryad | 16036 | 2 | 5 | 9631 | 1990 |
| IE | - | $1^{*}$ | 25 | 1.4 M | 0.13 M |
| Mozilla | 245172 | $1^{*}$ | 12 | 38.4 M | 3 M |

Table 2: Characteristics of various benchmarks. Here 1M means one million operations. $1^{*}$ indicates that the previously unknown bug was found while running with a bug depth of 1 .

All experiments were conducted on an quad-core Intel Xeon L5420 running at 2.50 GHz , with 16 GB of RAM running $64-$ bit Windows Server Enterprise operating system.

### 5.1 Experimental Setup

### 5.1.1 Benchmarks and Bugs

In our evaluation, we used open source applications such as Mozilla Firefox code-named Shiretoko, a commercial web browser-Internet Explorer, a parallel decompression utility - Pbzip2, three Splash benchmarks (FFT, LU, Barnes), a work stealing queue implementation [13] and a component of Dryad [16]. We used these applications as most of these were used in prior work on discovering concurrency bugs [26, 22]. Table 1 lists the manifestation of the bug in these applications and also reports whether the bug was previously known. PCT discovered all previously known bugs faster than reported in respective prior work [26, 22]. We also find new bugs in work stealing queue and Barnes benchmark that were missed by prior work. Finally, we find previously unknown bugs in Firefox and Internet Explorer.

Table 2 also lists the various properties of the benchmarks. The table lists the number of threads ( $n$ ), the total number of synchronization operations executed ( $k$ ), and the depth of the bug ( $d$ ) in the application. It also shows the effective number of operations after optimization ( $k_{\text {eff }}$ ) described in Section 4. It is interesting to note that our prototype detected the bugs in Mozilla and Internet Explorer even though, it has a large value of $k$. Moreover, Mozilla Firefox and Internet Explorer are large applications and the ability to detect bugs in these large applications demonstrates the scalability of the tool.

| Programs | Stress | Random | PCT |  |
| :--- | :---: | :---: | :---: | :---: |
|  |  | Sleep | Empirical | Bound |
| Splash-FFT | 0.06 | 0.27 | 0.50 | 0.5 |
| Splash-LU | 0.07 | 0.39 | 0.50 | 0.5 |
| Splash-Barnes | 0.0074 | 0.0101 | 0.4916 | 0.5 |
| Pbzip2 | 0 | 0 | 0.701 | 0.0001 |
| Work Steal Queue | 0 | 0.001 | 0.002 | 0.0003 |
| Dryad | 0 | 0 | 0.164 | $2 \times 10^{-5}$ |

Table 3: Empirical probability of finding the bug with various methods such as Stress, Random delay insertion methods and PCT and the worst-case bound (based on $\mathrm{n}, \mathrm{k}$ and d in Table 2).

### 5.1.2 Comparing Other Techniques

As a point of comparison, we also ran the benchmarks in Table 3 with our stress testing infrastructure. Our stress infrastructure ran all these benchmarks under heavy load a million times. We made a honest, good-faith effort to tune our stress infrastructure to increase its likelihood of finding bugs. Our effort is reflected by the fact that our stress infrastructure detected the known bugs in the benchmarks with a higher probability and a lot quicker than prior stress capabilities reported in literature [26].

As another interesting comparison, we implemented a scheme that introduces random sleeps at synchronization operations with a certain probability [2]. The experiments are sensitive to the particular probability of sleep. Again, we made a good-faith effort to find the configuration that works best for our benchmarks. We experimentally discovered a probability of $1 / 50$ performed reasonable well in detecting bugs.

These two reflect the state of the art concurrency testing techniques that we are able to recreate in our setting. Heuristicdirected testing [26, 17] and systematic scheduling [23] require sophisticated analysis and we are currently unable to perform quantitative experiments with these techniques. We compare with these techniques qualitatively.

### 5.2 Effectiveness

### 5.2.1 Comparison with worst-case bound

Apart from discovering known and unknown bugs, to evaluate whether our prototype attains the theoretical guarantees discussed in Section 3, we calculated empirical probabilities of detecting the bug with our prototype. In this experiment, our prototype ran each application one million times with each run having a different random seed. The relative frequency of occurrence of the bug in these runs represents the empirical probability. Table 3 reports the empirical and the expected theoretical probability of finding the bug with PCT. The theoretical probability is computed using the bound $1 / n k^{d-1}$ obtained in Section 3. We report the empirical probabilities only for applications where it was feasible to do one million runs.

Table 3 reveals that our prototype meets and exceeds the theoretical worst-case bound. For Barnes, LU and FFT that have 2 threads, our implementation finds the bug with a probability approximately half. For the Barnes benchmark, PCT slightly misses the worst-case bound. This is the effect of priority perturbations introduced to guarantee starvation-
freedom, as discussed in Section 4.

For benchmarks with a 2 depth bug, namely Pbzip2, Work Steal Queue, and Dryad, our implementation is orders of magnitude better than the worst-case bound. The Pbzip2 bug, shown in Fig. 2 requires an extra constraint that the main thread does not prematurely die before the error manifests. Since PCT guarantees this by default (Section 4), PCT finds the bug as if it was a depth 1 bug. Both Work Steal Queue and Dryad demonstrate the effect of optimizations that reduce $k$. We study our results with the Work Steal Queue in detail in Section 5.3.

### 5.2.2 Comparison to other techniques

Table 3 summarizes our experiments comparing PCT with stress, synchronization based random sleeps. Our sophisticated stress infrastructure has trouble finding bugs of depth one which are trivially caught by PCT. Note, that our stress infrastructure detects bugs of in FFT, LU and Barnes much more successfully than reported in prior literature [26]. Stress simply does not find bugs of higher depth.

Our random sleep scheme detects bugs in FFT, LU, Barnes way quicker than stress. It also detected the 2-edge bug in work stealing queue albeit with a low probability. However it runs out of luck with Pbzip and Dryad for all probabilities we experimented with. To summarize, PCT scheduler discovers bugs with a higher probability and much more quickly than the state of the art stress infrastructure and other competitor schemes such as random sleep.

CHESS [23] finds the Work Steal Queue and the Dryad bug after (approx.) 200 and 1000 runs of the program respectively. The PCT scheduler detects the same bug in the $6^{\text {th }}$ and $35^{t h}$ run of the program respectively. We were unable to run CHESS on other benchmarks. As the next section shows, PCT scales much better if one increases the number of threads, while we expect CHESS to perform exponentially worse.

In comparison to CTrigger [26], PCT finds the bug in the benchmarks common to both well within the first three iterations. This is more efficient as we do not require a separate profiling run required by CTrigger.

### 5.3 Work Steal Queue Case Study

As a case study, we discuss the impact of increasing the number of synchronization operations and the number of threads with the PCT scheduler for the applications in Table 3. Due to space constraints, we report the behavior only with the work stealing queue program. Other benchmarks show similar behavior.

Work Steal Queue program implements a queue of work items and was originally designed for the Cilk multithreaded programming system [13]. In this queue, there are two kinds of threads namely victims and stealers. Victim pushes and pops from the tail of the queue and a stealer steals from the head of the queue. The application uses sophisticated lock-free synchronization to protect the queue. To study the impact of increased threads, we increased the number of stealers and the number of items being pushed and popped.


Figure 9: Probability of finding bugs with an increase in the number of threads and number of items.

### 5.3.1 Effect of Execution Size on PCT

Figure 9 shows the probability of finding the bug with our prototype implementing the PCT scheduler with an increase in the number of threads and the number of operations for the work stealing queue program. In this experiment, we increase $n$ by increasing the number of stealers. (The program requires that there is exactly one victim.) Each stealer does two steal attempts, while the victim pushes and pops a specified number of items. We increase $k$ by increasing the number of items.

Figure 9 reveals that the probability of finding the bug actually increases with the increase in the number of threads. Moreover, the probability of finding the bug is same irrespective of the number of operations. We were initially surprised by this result as we expected PCT to perform a lot worse with an increase $n$ and $k$ as the theory predicts a probability of $1 / n k$ for finding this depth 2 bug.

However, as the number of threads and the operations increase, there are many opportunities to find the bug. PCT needs to find the race condition when any one of the stealer is interfering while attempting to steal any of the items. These events, however, are not completely independent during the execution and thus we are not able to theoretically justify the result in Fig. 9.

### 5.3.2 PCT vs Stress

Figure 10 shows the probability of finding the bug with the PCT scheduler and Stress testing with an increase in the number of threads for the work stealing queue program. In this experiment, the number of stealers was varied from 2 to 64 with total number of items being pushed and popped by another thread at a constant four items. As discussed earlier, the probability of PCT increases with the number of threads. However, the interesting thing to note, even with the sophisticated stress testing framework, the probability of detecting the bug with stress is low and is highly nondeterministic. Irrespective of the system load, PCT Scheduler has the same probability of detecting the bug when given the same random seed. Even though the inefficacy of stress has been highly discussed, it is important to note that the stress does detect bugs in practice $[26,23]$. The primary


Figure 10: Probability of finding bugs with PCT and Stress with increase in threads and number of threads


Figure 11: Coverage of various thread events with the specified number of program runs with PCT and Stress for the work stealing queue program.
problem with stress is with reproduction of the bug.

### 5.3.3 Interleaving Coverage

To evaluate the coverage of thread events with PCT, we instrumented the work stealing queue program with twenty events, fourteen events in the main thread which does the pushes and the pops and six events in the stealer thread. There were a total of 168 unique event pairs possible in this setting. Figure 11 plots the cumulative percentage of the events covered as the program is run specified number of times. The horizontal axis represents the number of times the program was run (in logarithmic scale). We restrict the horizontal axis to the 8192 runs as stress did not explore any new event pair beyond those already explored in the new runs after that and PCT eventually explored all the event pairs. Figure 10 showed that stress had a low probability of catching the bug. Figure 11 shows that stress does not cover more than $20 \%$ of the event pairs, few of which result in a bug. Thus, stress's inability/ineffectiveness to detect the bug is highly correlated with the event pairs not covered. The ability to cover almost all the event pairs enables PCT to detect the bug.

## 6. RELATED WORK

Our work is closely related with concurrency verification and testing techniques that aim to find bugs in programs. We classify prior work and compare our work below.

Dynamic Scheduling: PCT is related to techniques that control the scheduler to force the program along buggy schedules. Stress testing is a commonly used method where the program is subjected to heavy load in the presence of artificial noise created by both running multiple unrelated tests simultaneously and by inserting random sleeps, thread suspensions, and thread priority changes. In contrast, PCT uses a disciplined, mathematically-random priority mechanism for finding concurrency bugs. This paper shows that PCT outperforms stress theoretically and empirically.

Prior approaches for generating randomized schedules [9, 27] insert random sleep at synchronization points and use heuristics based on various coverage criteria, concurrency bug patterns and commutativity of thread actions to guide the scheduler [11]. These approaches involve a random decision at every scheduling point. As shown in Section 2, even simple concurrency bugs can frustrate these techniques. In stark contrast, PCT uses a total of $n+d-1$ random decisions, together for the initial thread priorities and $d-1$ priority change points. Our key insight is that these small but calculated number random decisions are sufficient for effectively finding bugs.

Researchers have also proposed techniques that actively look for concurrency errors $[26,17]$. They use sophisticated analysis, either by running profiling runs that detect suspicious non-serializable access patterns [26], or use static or dynamic analysis to find potential deadlocks [17]. In a subsequent phase these techniques heuristically perturb the OS scheduler guided by the prior phase. Our technique does not require prior analysis but it still is comparable in bug-finding power of these techniques. For instance, for the SPLASH2 benchmarks used in the former technique [26], PCT finds the bug in the first few runs far less time than required for the profiling runs in the previous approach. However, our technique is orthogonal to these approaches and identifying potential buggy locations can improve PCT as well.

Systematic Exploration: Model checking techniques [14, 23] systematically explore the space of thread schedules of a given program in order to find bugs. These techniques can prove the absence of errors only after exploring the state space completely. In contrast, PCT provides a probabilistic guarantee after every run of the program. With respect to bug-finding capability, we have compared PCT with the CHESS tool [23] on two benchmarks. PCT finds bugs much faster than CHESS in both cases.

CHESS uses a heuristic of exploring schedules with fewer number of preemptions. By default, CHESS explores executions nonpreemptively except at few chosen steps where it inserts preemptions. In contrast, PCT is a priority based scheduler and can introduce arbitrary number of preemptions in the presence of blocking operations even if the bug depth is small. For instance, when a low-priority thread wakes up a higher-priority thread, a priority-based scheduler preempts the former to scheduler the latter.

Concurrency Bug Detection: Our approach is also related to numerous hardware and software techniques that find concurrency errors dynamically but without exercising any scheduler control $[29,10,12,21]$. PCT's goal is to direct the scheduler towards buggy schedules and requires that these bugs are detected by other means, either by a program assertion or with the use of these concurrency bug detection engines.

## 7. CONCLUSION

This paper describes PCT, a randomized algorithm for concurrency testing. PCT uses a disciplined schedule-randomization technique to provide efficient probabilistic guarantees of finding bugs during testing. We evaluate an implementation of PCT for x86 executables on demonstrate its effectiveness in finding several known and unknown bugs in large-scale software.

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[^0]:    ${ }^{1}$ For exposition, we assume that the bug depth parameter $d$ is provided as an input by the user. In practice, our tool chooses $d$ automatically from an appropriate random distribution.

[^1]:    ${ }^{2}$ In theory, $d$ can be as large as $k$. In this case, our bound (as required) is worse than $1 / n^{k}$. However, we consider a depth $k$ bug a practical impossibility, especially for modern software that is built from a large number of loosely-coupled components.

