Amortised Bisimulations

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Outline

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3. Amortised Bisimulations
4. Case Studies
    - Shared Messaging Communication (SMC) versus Message Passing
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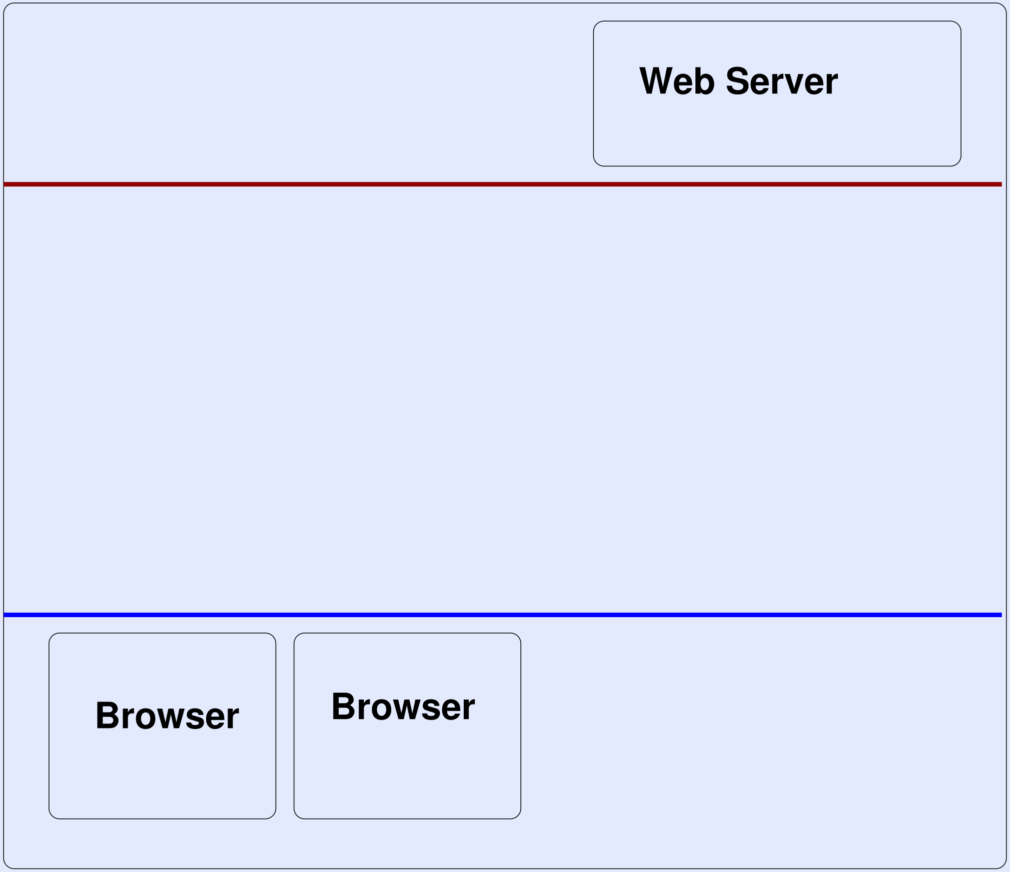
5. Conclusions

Example: Browser



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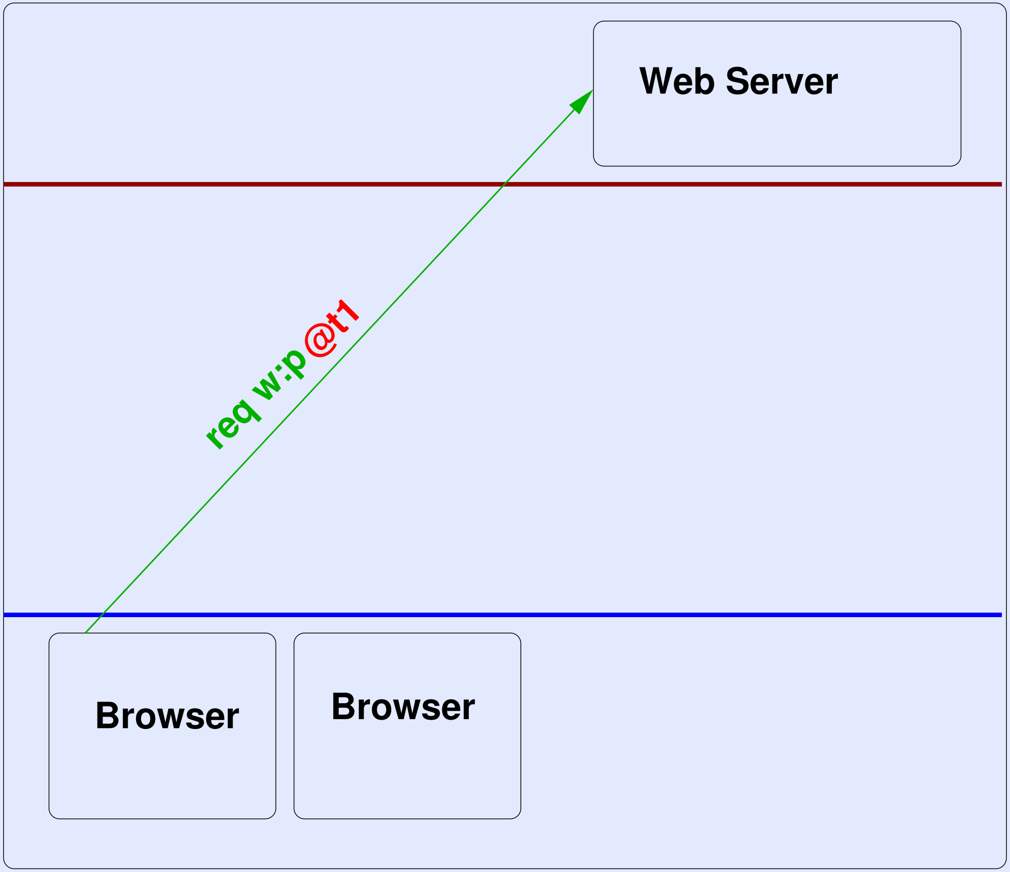

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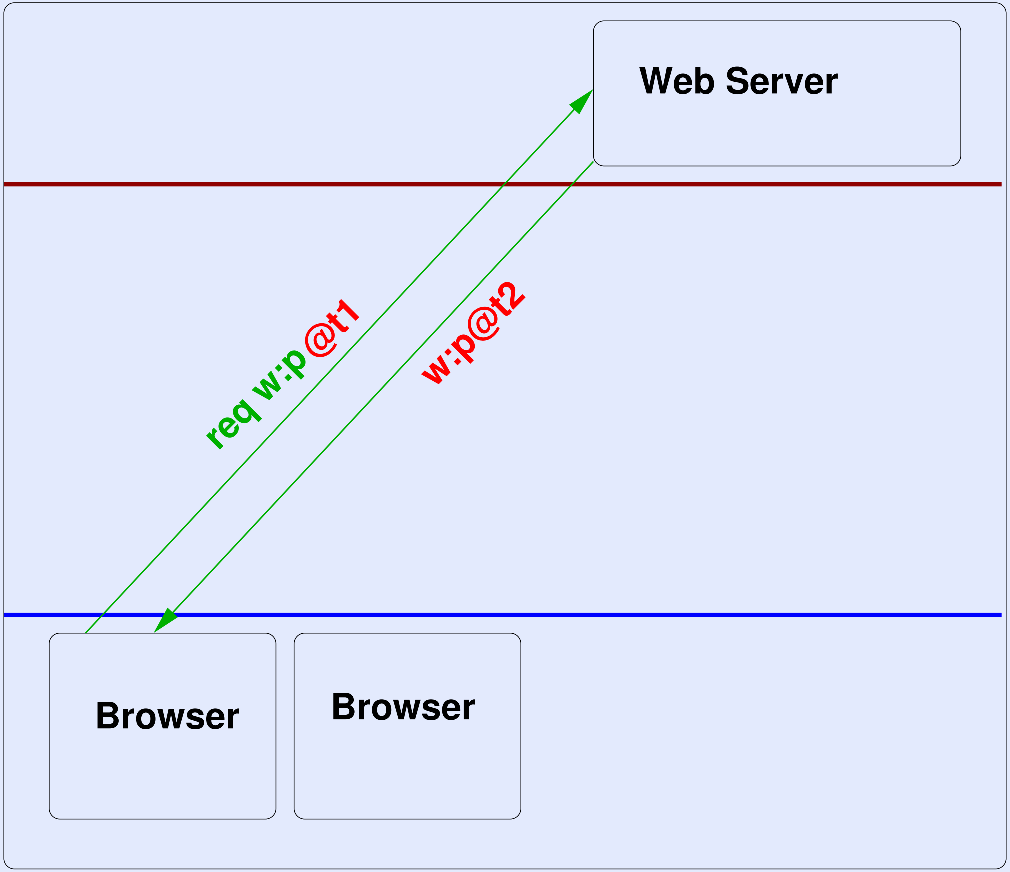

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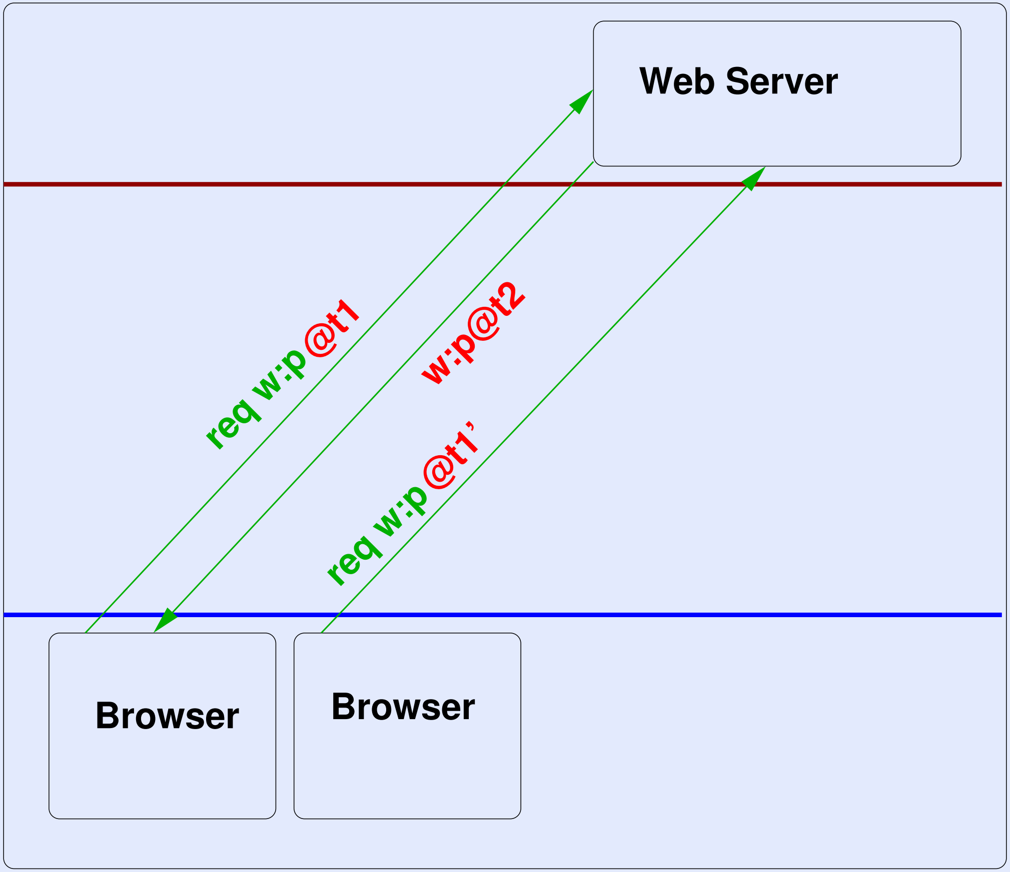

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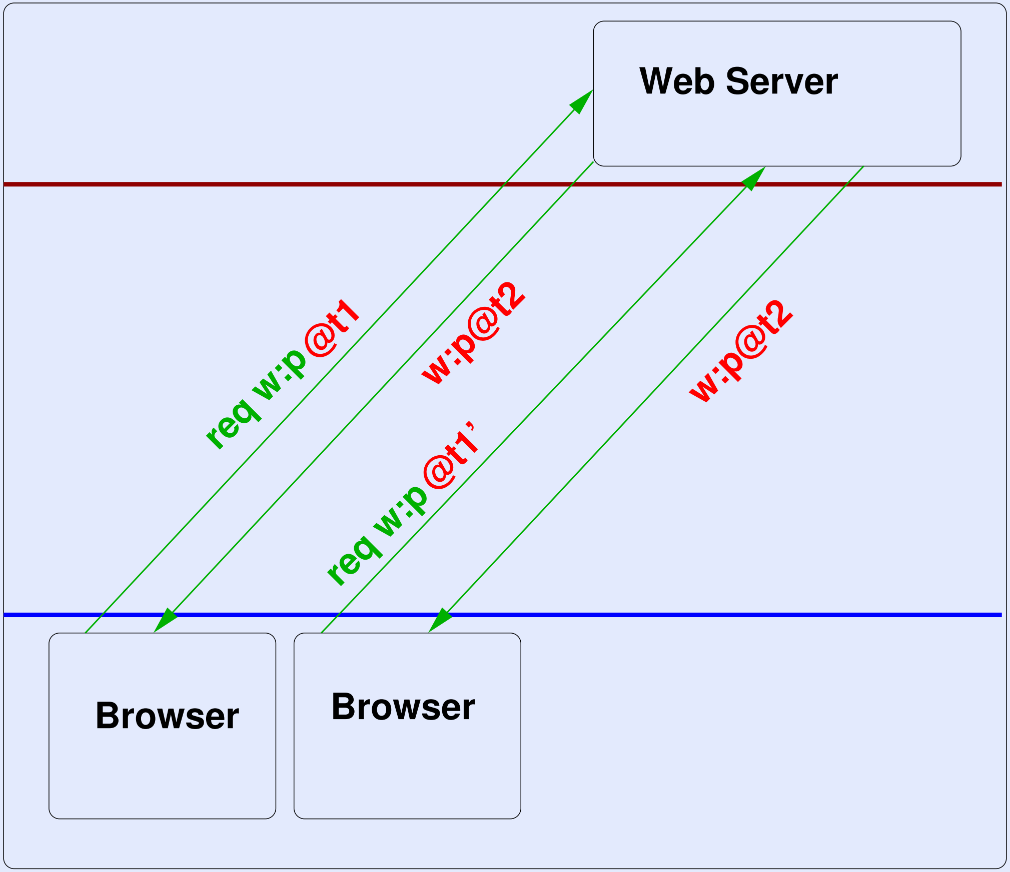
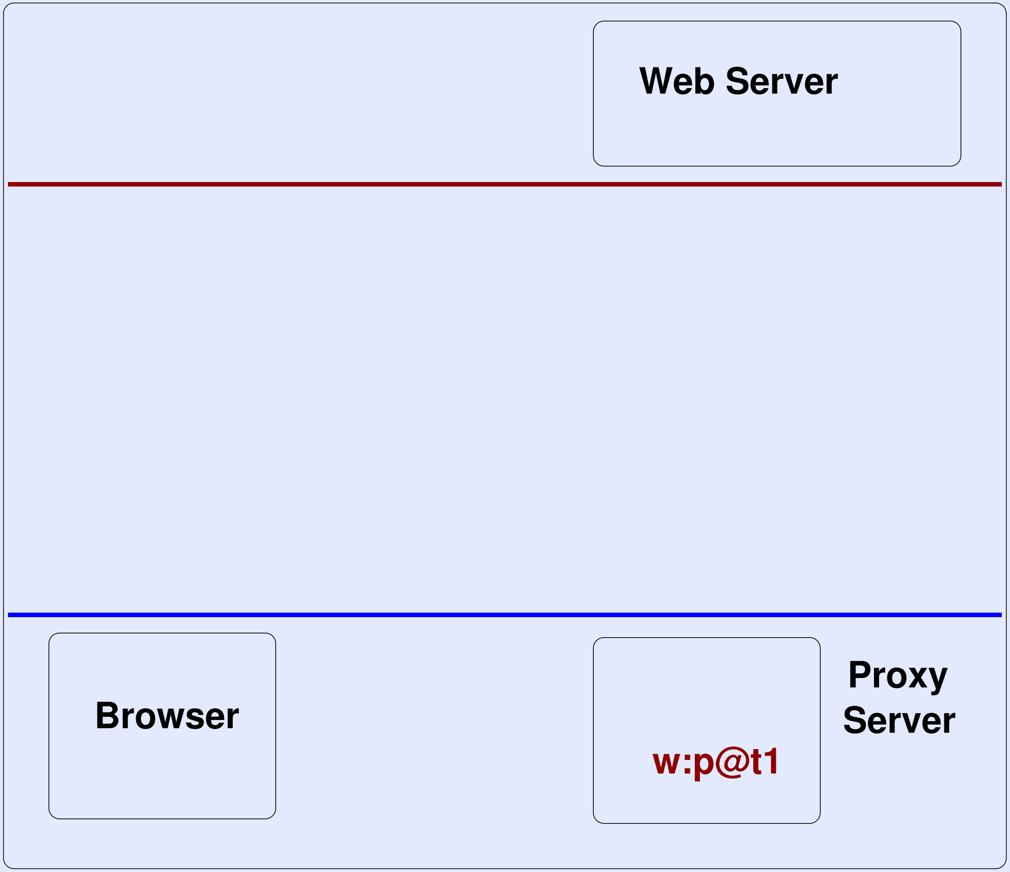


Example: Proxy Server



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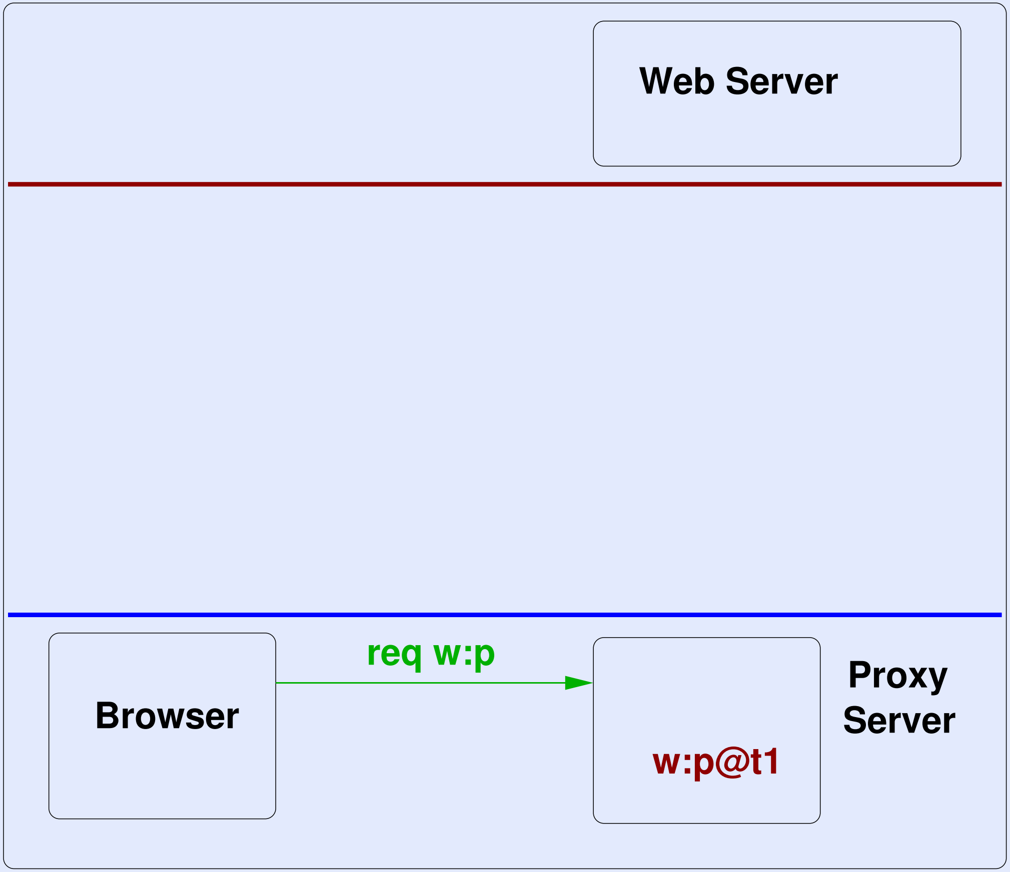

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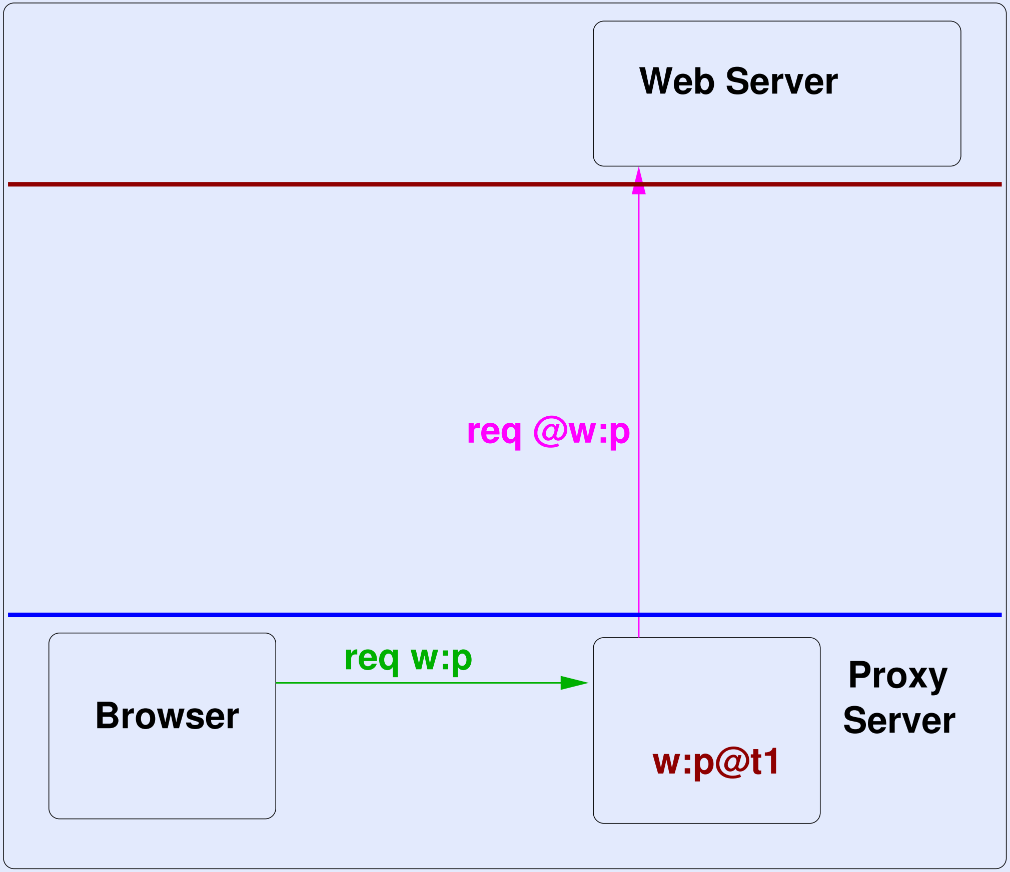

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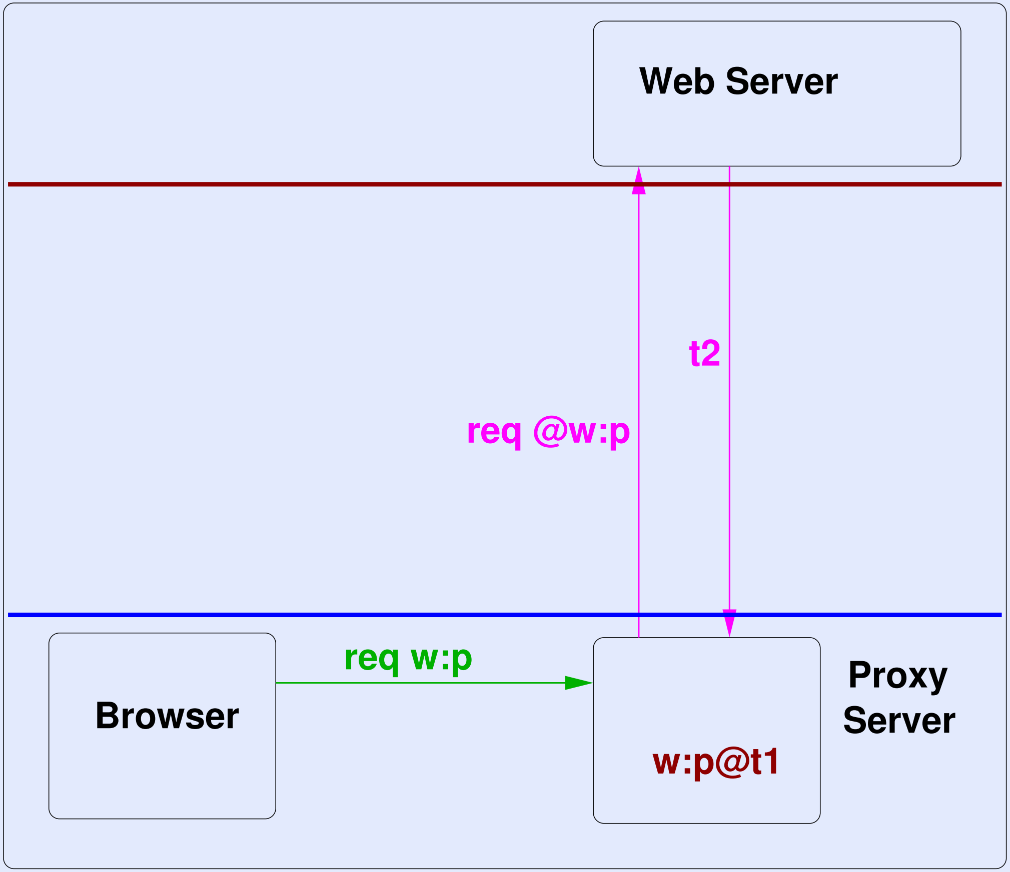

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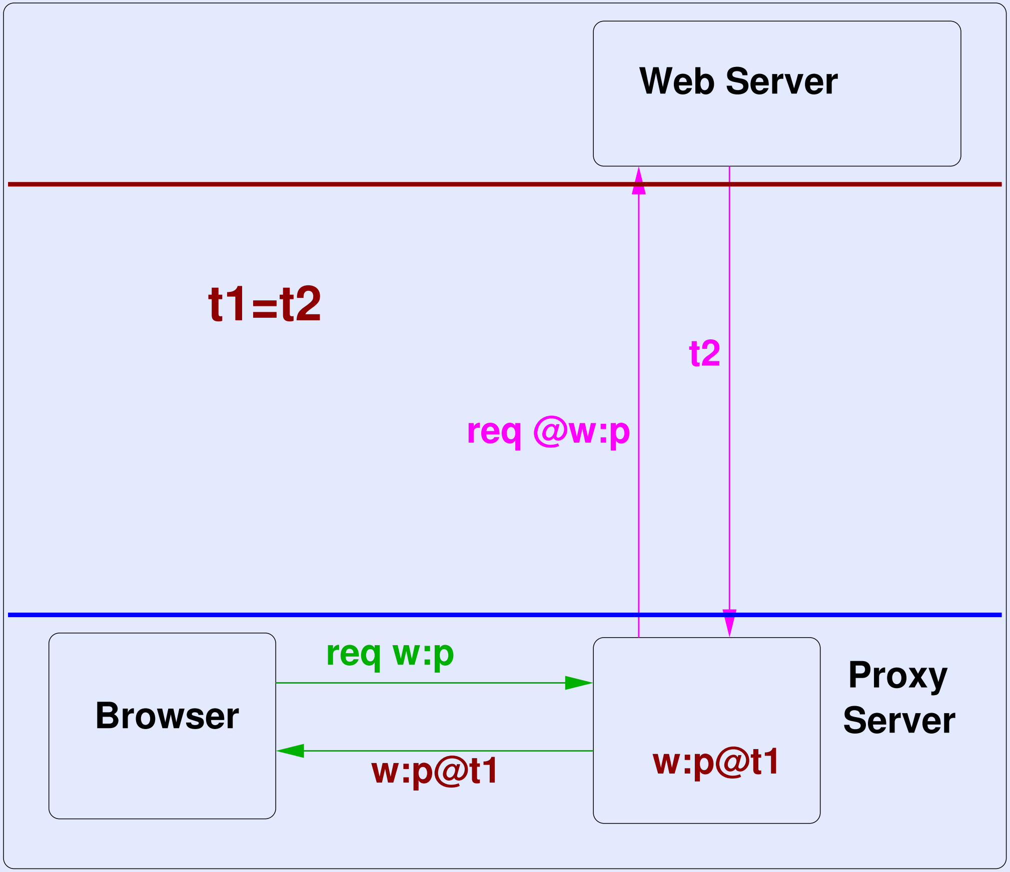

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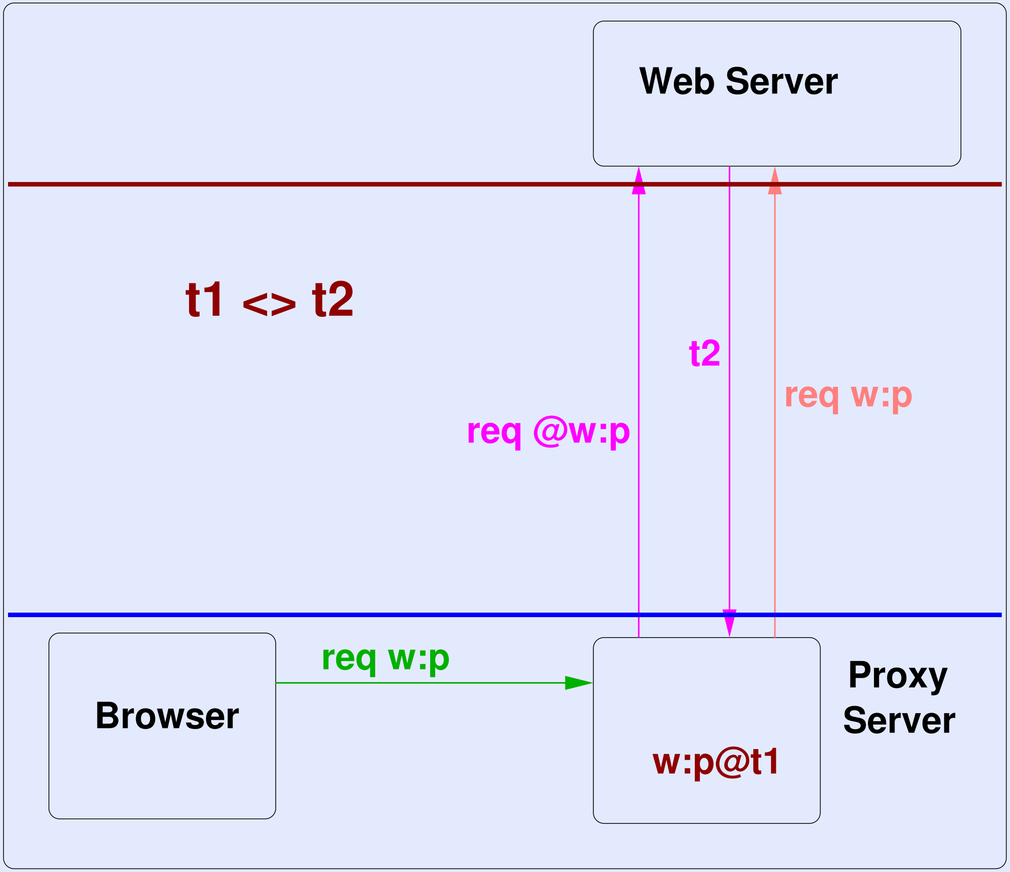

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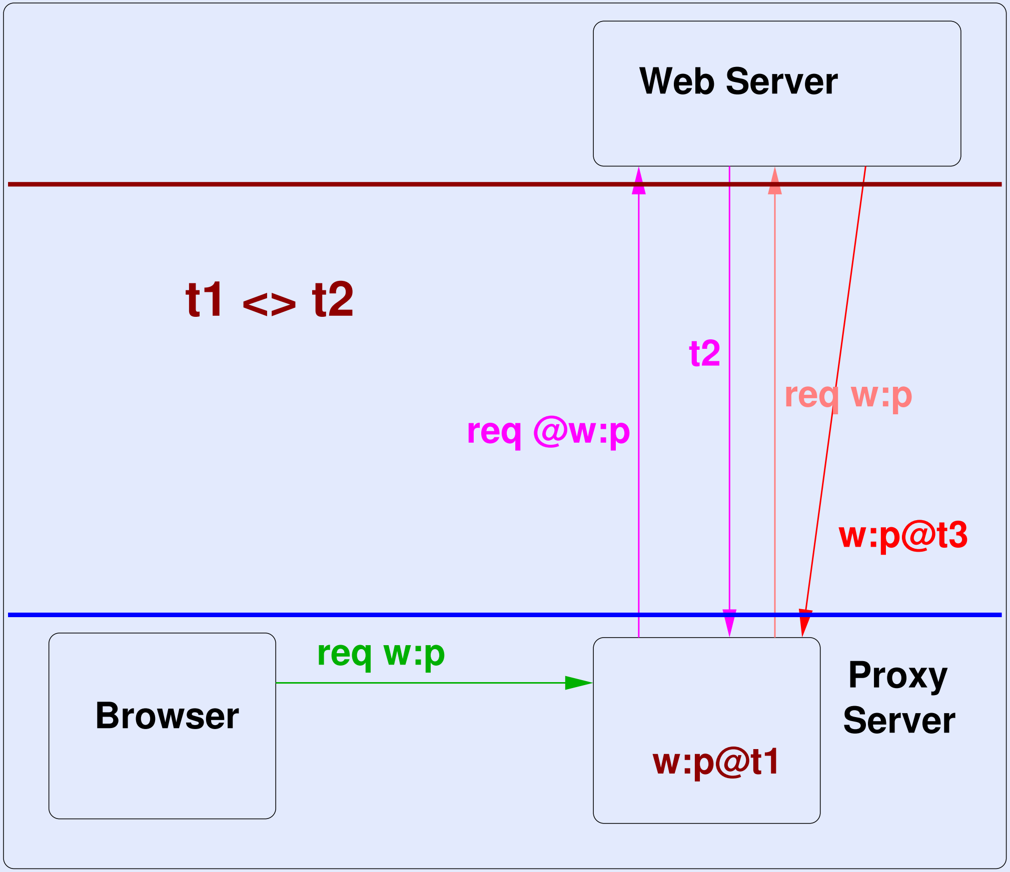

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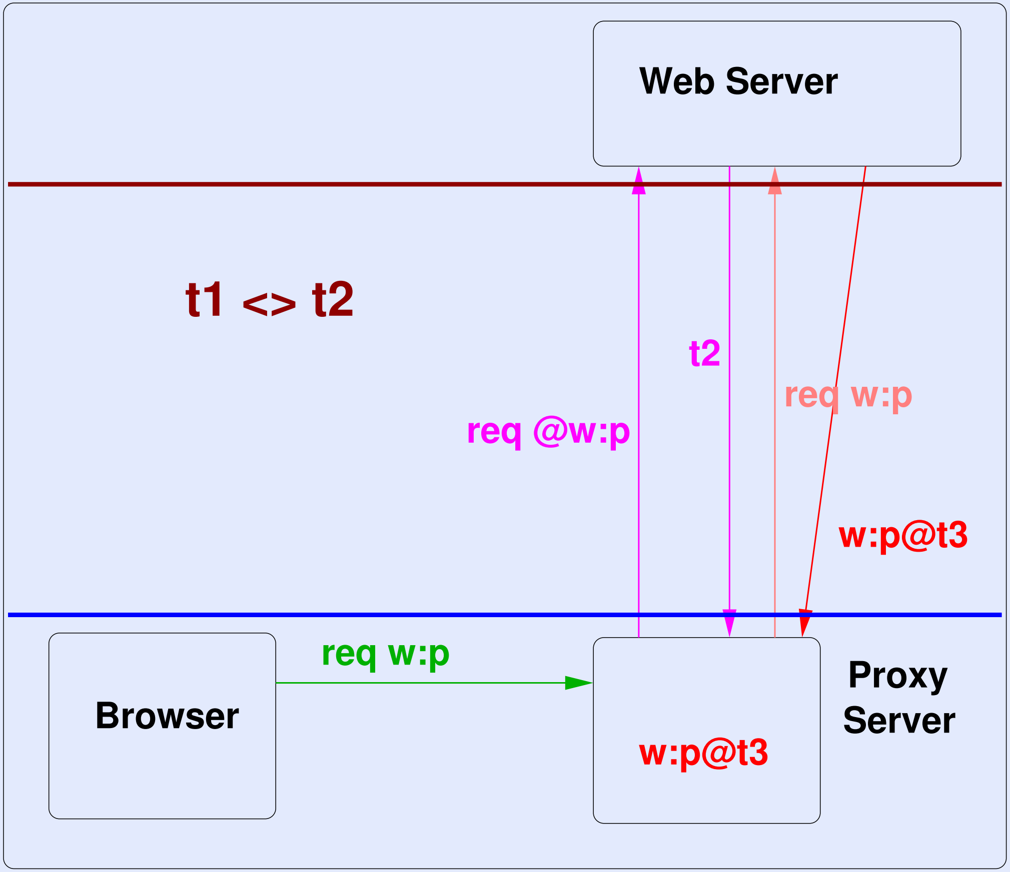

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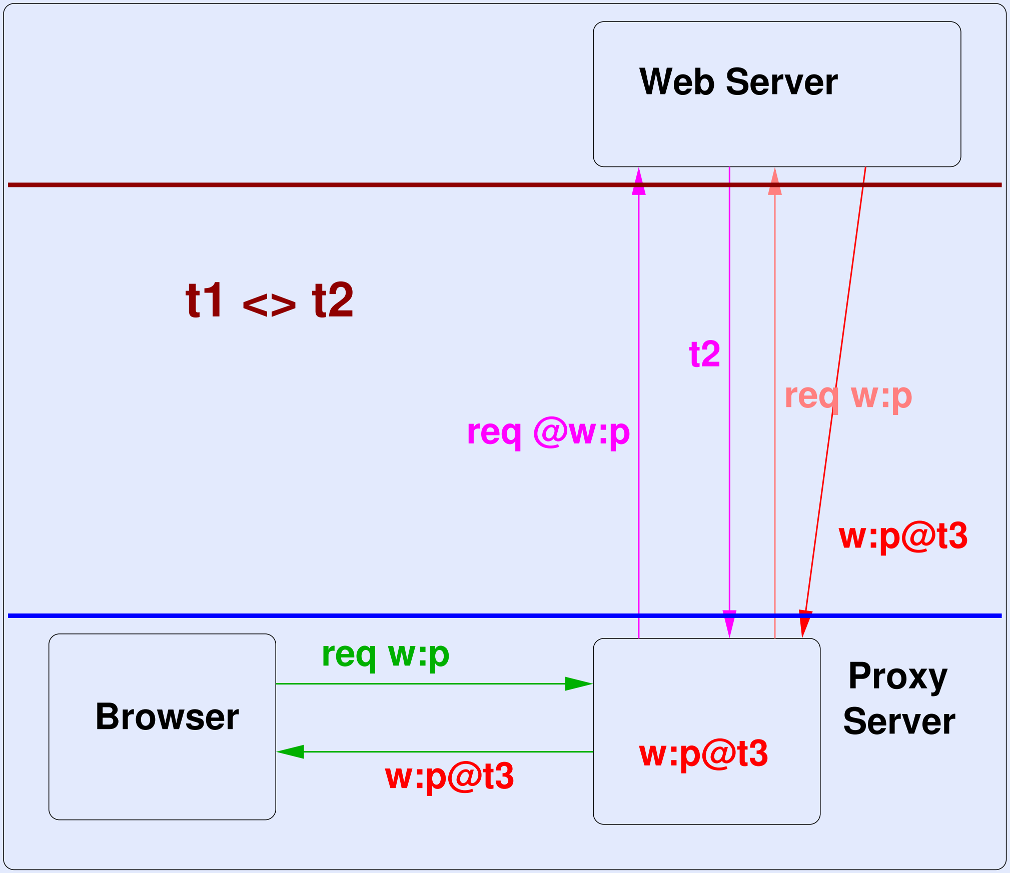

Motivation

A semantic notion for nondeterministic and nonterminating concurrent systems to verify

- "functional" correspondence
- comparison of costs in the long run!

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A semantic notion for nondeterministic and nonterminating concurrent systems to verify

- "functional" correspondence
- comparison of costs in the long run!

Classical notion for functional equivalence:

- bisimilarity, observation equivalence [Park,Milner]
- testing equivalence [deNicola & Hennessy]
- ⋮

Functional Correspondence

R is a ρ-bisimulation, $\rho \subseteq Act_\tau \times Act_\tau$,
if for all $(p, q) \in R$:

1. If $p \xrightarrow{a} p'$ then
   $\exists q', b\ [a \rho b$ and $q \xrightarrow{b} q'$ and $(p', q') \in R]$,
2. if $q \xrightarrow{b} q'$ then
   $\exists p', a\ [a \rho b$ and $p \xrightarrow{a} p'$ and $(p', q') \in R]$,

where $a, b \in Act_\tau$.

ρ functional correspondence relation

Example: library

functional correspondence ρ

central	local
$go\_to\_lib_{cen}$	$go\_to\_lib_{loc}$
$access\_book_{cen}$	$access\_book_{loc}$
$access\_book_{cen}$	$reserve\_book$

$$\begin{aligned}
Central_Lib &\stackrel{df}{=} go_to_lib_{cen}.At_Central_Lib \\
At_Central_Lib &\stackrel{df}{=} access_book_{cen}.Central_Lib
\end{aligned}$$

$$\begin{aligned}
Local_Lib &\stackrel{df}{=} go_to_lib_{loc}.At_Local_Lib \\
At_Local_Lib &\stackrel{df}{=} access_book_{loc}.Local_Lib \\
&\quad + reserve_book.Local_Lib
\end{aligned}$$

Example: library with costs

functional correspondence ρ

cost	central	local	cost
2	$go\_to\_lib_{cen}$	$go\_to\_lib_{loc}$	1
1	$access\_book_{cen}$	$access\_book_{loc}$	2
1	$access\_book_{cen}$	$reserve\_book$	4

$$Central\_Lib \stackrel{df}{=} go\_to\_lib_{cen}.At\_Central\_Lib$$
$$At\_Central\_Lib \stackrel{df}{=} access\_book_{cen}.Central\_Lib$$

$$Local\_Lib \stackrel{df}{=} go\_to\_lib_{loc}.At\_Local\_Lib$$
$$At\_Local\_Lib \stackrel{df}{=} access\_book_{loc}.Local\_Lib + reserve\_book.Local\_Lib$$

Amortised Bisimulations

=functional correspondence + amortised costs:

$(R_i)_{i\in\mathbb{N}}$ is an amortised ρ-bisimulation,
if for all i, for all $(p,q) \in R_i$:

1. If $p \xrightarrow{a} p'$ then
   $\exists q', b\ [a\rho b$ and $q \xrightarrow{b} q'$ and $(p',q') \in R_{i+c_b-c_a}]$,
2. if $q \xrightarrow{b} q'$ then
   $\exists p', a\ [a\rho b$ and $p \xrightarrow{a} p'$ and $(p',q') \in R_{i+c_b-c_a}]$,

where $a, b \in Act_\tau$.

$p \prec_i^\rho q$ if for some amortised ρ-bisimulation $(R_i)_{i\in\mathbb{N}}$ such that $(p,q) \in R_i$.

Index i: credit counter, R_i: the i-slice of $(R_i)_{i\in\mathbb{N}}$.

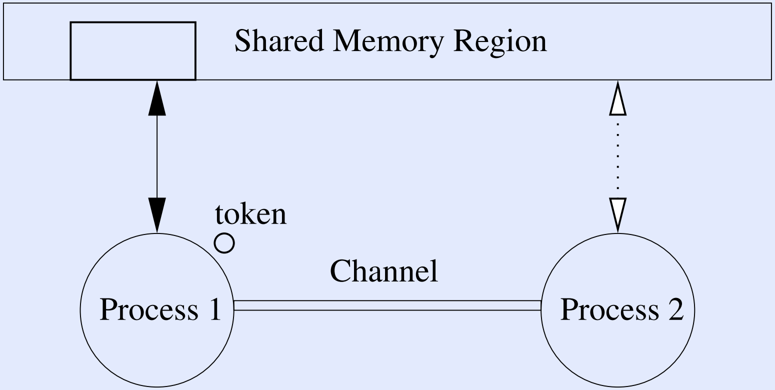
Some Basic Properties:

1. $\bigcup(\prec_i^\rho)_{i\in\mathbb{N}}$ ρ-bisimulation
2. $(\prec_i^\rho)_{i\in\mathbb{N}}$ component-wise largest strong amortised ρ-bisimulation
3. $\prec_i^\rho \subseteq \prec_{i+1}^\rho$
4. $\sim \circ \prec_i^\rho = \prec_i^\rho = \prec_i^\rho \circ \sim$
5. ρ reflexive: $\prec_i^\rho$ reflexive and $\sim \subseteq \prec_i^\rho$
6. ρ transitive: $\prec_i^\rho \circ \prec_j^\rho \subseteq \prec_{i+j}^\rho$
7. congruence properties

Shared Messaging Communication

[Kiran/Jayram/Rao/Nandy03]

- communication primitive: message passing
- new: shared memory regions



Experimental results:

SMC more efficient than convential message passing

A Message Passing Process in CCS

$$MP\_Process \stackrel{df}{=} \tau.Send\_Message + \tau.Receive\_Message$$

$$Send\_Message \stackrel{df}{=} \sum_{k \in \mathbb{N}-\{0\}} (\overline{sm}.)^k MP\_Process$$

$$Receive\_Message \stackrel{df}{=} \sum_{k \in \mathbb{N}-\{0\}} (rm.)^k MP\_Process$$

Shared Messaging Communication in CCS

$$
\begin{aligned}
SMC\_Process &\stackrel{df}{=} \tau.Request\_Token \\
&\quad + \quad \tau.Receive\_Token \\
\\
Request\_Token &\stackrel{df}{=} gum.Compose\_Token \\
Compose\_Token &\stackrel{df}{=} \sum_{k \in I\!N - \{0\}} (\overline{cps}.)^k Send\_Token \\
Send\_Token &\stackrel{df}{=} \overline{st}.SMC\_Process \\
\\
Receive\_Token &\stackrel{df}{=} rt.Consume\_Token \\
Consume\_Token &\stackrel{df}{=} \sum_{k \in I\!N - \{0\}} (csm.)^k Usage\_Over \\
Usage\_Over &\stackrel{df}{=} \overline{uo}.SMC\_Process
\end{aligned}
$$

Correspondence Between SMC and MP

cost	SMC	MP	cost
1	gum	τ	0
1	$\overline{uo}$	τ	0
1	$\overline{st}$	τ	0
1	rt	τ	0
0	$\overline{cps}$	$\overline{sm}$	2
0	csm	rm	2

$$SMC\_Process \xrightarrow{\tau} \xrightarrow{gum} \xrightarrow{\overline{cps}} \cdots \xrightarrow{\overline{cps}} \xrightarrow{\overline{st}} SMC\_Process$$

$$MP\_Process \xrightarrow{\tau} \xrightarrow{\varepsilon} \underbrace{\xrightarrow{\overline{sm}} \cdots \xrightarrow{\overline{sm}}}_{k \text{ times}} \xrightarrow{\varepsilon} MP\_Process$$

Silent Transitions

$\varepsilon\rho\, a_1 \ldots a_n$ if $\tau\rho\, a_1, \ldots, \tau\rho\, a_n$

matching of transitions:

functional equiv.	functional corresp.
$\xrightarrow{a}$	$\xrightarrow{a}$
$\stackrel{\varepsilon}{\Longrightarrow}\xrightarrow{\hat{a}}\stackrel{\varepsilon}{\Longrightarrow}$	$\stackrel{u}{\Longrightarrow}\xrightarrow{\hat{b}}\stackrel{v}{\Longrightarrow}$ where $a\rho\, b$, $\varepsilon\rho\, u$, $\varepsilon\rho\, v$

weak amortised bisimulations

$(R_i)_{i \in \mathbb{N}}$ is a weak amortised ρ-bisimulation,
if for all i, for all $(p, q) \in R_i$:

1. If $p \xrightarrow{a} p'$ then
   $\exists q', b, u, v$ [$a \rho b$ and $\varepsilon \rho \, uv$ and
   $$q \overset{u\hat{b}v}{\Longrightarrow} q' \text{ and } (p', q') \in R_{i + c_{ubv} - c_a}],$$
2. if $q \xrightarrow{b} q'$ then
   $\exists p', a, u, v$ [$a \rho b$ and $uv \rho \, \varepsilon$ and
   $$p \overset{u\hat{a}v}{\Longrightarrow} p' \text{ and } (p', q') \in R_{i + c_b - c_{uav}}],$$

where $a, b \in Act_\tau$, $u, v \in Act_\tau^*$.

$p \preccurlyeq_i^\rho q$ if for some weak amortised ρ-bisimulation $(R_i)_{i \in \mathbb{N}}$ such that $(p, q) \in R_i$.

$SMC \preccurlyeq_1^{\rho} MP$

$(SMC\_Process, MP\_Process) \in R_1$
for weak amortised bisimulation $(R_i)_{i \in \mathbb{N}}$:

	$SMC\_Process$	$MP\_Process$	condition on i	
1.	$SMC\_Process$	$MP\_Process$	$i = 2j+1,$	$j \geq 0$
2.	$Request\_Token$	$Send\_Message$	$i = 2j+1,$	$j \geq 0$
3.	$Receive\_Token$	$Receive\_Message$	$i = 2j+1,$	$j \geq 0$
4.	$Compose\_Token$	$Send\_Message$	$i = 2j,$	$j \geq 0$
5.	$(\overline{cps}.)^k Send\_Token$	$(\overline{sm}.)^k MP\_Process$	$i = 2j,$	$j \geq 1$
6.	$Consume\_Token$	$Receive\_Message$	$i = 2j,$	$j \geq 0$
7.	$(csm.)^k Usage\_Over$	$(rm.)^k MP\_Process$	$i = 2j,$	$j \geq 1$
8.	$Send\_Token$	$MP\_Process$	$i = 2j,$	$j \geq 1$
9.	$Usage\_Over$	$MP\_Process$	$i = 2j,$	$j \geq 1$

Web Access

Without a proxy server:

$$D\_System \stackrel{df}{=} D\_Client$$

$$\begin{aligned} D\_Client &\stackrel{df}{=} \overline{d\_request\_page}.D\_Client' \\ D\_Client' &\stackrel{df}{=} d\_serve\_page.D\_Client \end{aligned}$$

With a proxy server:

$$P\_System \stackrel{df}{=} (P\_Client \mid Proxy) \backslash ProxyInt$$

$$\begin{aligned} P\_Client &\stackrel{df}{=} \overline{p\_request\_page}.P\_Client' \\ P\_Client' &\stackrel{df}{=} p\_serve\_page.P\_Client \end{aligned}$$

P_System and D_System: Correspondence & Costs

cost	P_System	D_System	cost
u_1	$\overline{drh}$	$\overline{drp}$	w_1
u_2	dsh	dsp	w_2
w_1	$\overline{drp}$	τ	0
w_2	dsp	τ	0
w_1	$\overline{irp}$	$\overline{drp}$	w_1
w_2	dsp	dsp	w_2

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P_System and D_System: Behaviours

round without updating cache:

$$P\_System \xrightarrow{\tau} \xrightarrow{\overline{drh}} \xrightarrow{dsh} \xrightarrow{\tau} P\_System$$
$$D\_System \xrightarrow{\varepsilon} \xrightarrow{\overline{drp}} \xrightarrow{dsp} \xrightarrow{\varepsilon} D\_System$$

updating-round:

$$P\_System \xrightarrow{\tau} \xrightarrow{\overline{drh}} \xrightarrow{dsh} \xrightarrow{\tau} \xrightarrow{\overline{drp}} \xrightarrow{dsp} \xrightarrow{\tau} P\_System$$
$$D\_System \xrightarrow{\varepsilon} \xrightarrow{\overline{drp}} \xrightarrow{dsp} \xrightarrow{\varepsilon} \xrightarrow{\varepsilon} \xrightarrow{\varepsilon} \xrightarrow{\varepsilon} D\_System$$

Client Behaviour

the decision maker:

$$DM \stackrel{df}{=} \bar{a}.(DM \mid \underbrace{\bar{b}.\cdots\bar{b}.}_{n\text{times}}0)$$

$\longrightarrow$ *the number of updates (b's) is never higher than n times the number of no-updates (a's).*

$$P\_System \stackrel{df}{=} (P\_Client \mid Proxy \mid DM) \backslash ProxyInt \cup \{a, b\}$$

The Proxy Server

$$
\begin{aligned}
Proxy_{start} &\stackrel{df}{=} p\_request\_page.First\_Copy \\
First\_Copy &\stackrel{df}{=} \overline{i\_request\_page}.Request\_Sent \\
\\
Proxy &\stackrel{df}{=} p\_request\_page.Client\_Wait \\
Client\_Wait &\stackrel{df}{=} \overline{d\_request\_header}.Check\_Update \\
Check\_Update &\stackrel{df}{=} d\_serve\_header.Decide \\
Decide &\stackrel{df}{=} a.No\_Update + b.Update \\
Update &\stackrel{df}{=} \overline{d\_request\_page}.Request\_Sent \\
No\_Update &\stackrel{df}{=} \overline{p\_serve\_page}.Proxy \\
Request\_Sent &\stackrel{df}{=} d\_serve\_page.Cached \\
Cached &\stackrel{df}{=} \overline{p\_serve\_page}.Proxy
\end{aligned}
$$

$P\_System \preccurlyeq^{\rho} D\_System$

	P_System			D_System	condition on i
1.	P_Client	$Proxy_{start}$	$\Delta(0)$	D_Client	$i = 0$
2.	P_Client'	$First\_Copy$	$\Delta(0)$	D_Client	$i = 0$
3.	P_Client'	$Request\_Sent$	$\Delta(0)$	D_Client'	$i = 0$
4.	P_Client	$Proxy$	$\Delta(m)$	D_Client	$i \geq m \cdot u$
5.	P_Client'	$Client\_Wait$	$\Delta(m)$	D_Client	$i \geq m \cdot u$
6.	P_Client'	$Check\_Update$	$\Delta(m)$	D_Client'	$i \geq m \cdot u + v_1$
7.	P_Client'	$Decide$	$\Delta(m)$	D_Client	$i \geq m \cdot u + v$
8.	P_Client'	No_Update	$\Delta(m)$	D_Client	$i \geq m \cdot u$
9.	P_Client'	$Update$	$\Delta(m)$	D_Client	$i \geq m \cdot u + w$
10.	P_Client'	$Request\_Sent$	$\Delta(m)$	D_Client	$i \geq m \cdot u + w_2$
11.	P_Client'	$Cached$	$\Delta(m)$	D_Client	$i \geq m \cdot u$

Theorem (Web Access)

If $n \leq \frac{v}{u}$ then

$$P\_System \preccurlyeq^{\rho} D\_System$$

where

- $u \, (= u_1 + u_2)$ additional costs of one update
- $v \, (= w_1 + w_2 - u)$ costs saved by a no-update
- n given by the decision maker

Related Work

Integration of Costs

1. Weighted Automata
   edit-distance computation, string alignment
   Mohri
2. Priced Timed Automata
   states and transitions carry costs
   Behrmann, Larsen, Bouyer,...
3. performance preorder bisimulation based
   Gorrieri, Corradini,...
4. Indexed Bisimulations
   (metric on actions induced by costs) Ying

$\longrightarrow$ no negative costs, no amortisation

$\longrightarrow$ 1.,2.: mimimum-cost computation

$\longrightarrow$ 3.: no quantitative values

$\longrightarrow$ 4.: index = degree of similarity with respect to differences in costs

Concluding Remarks

$$\text{amortised bisimularity} = \underbrace{\text{functional correspondence}}_{\text{qualitative}} + \underbrace{\text{cost evaluation}}_{\text{quantitative}}$$

- basic properties for $\prec_i^{\rho}$ hold
- *amortised faster-than preorder* [Lüttgen/Vogler05] characterized
- Future Work:
  - more case studies and theoretical results to prove the robustness of the chosen set-up

Congruence Properties

Given $p \prec_i^\rho q$ and $r \prec_j^\rho s$.

1. $a.p \prec_k^\rho b.q$ if $a \rho b$ and $k \geq i + c_a - c_b \geq 0$
2. $p + r \prec_k^\rho q + s$ for $k \geq \max\{i, j\}$
3. $p \mid r \prec_k^\rho q \mid s$ for $k \geq i + j$
4. $p[f] \prec_k^\rho q[f]$ for $k \geq i$
5. $p \setminus a \prec_k^\rho q \setminus a$

Note: priced actions $(c_a > 0)$ without a complement.

btp,btc