Turing Machines As Acceptors

Example 1 $L = \{a^n b^n \mid n \geq 1\}$ ($n \geq 0$)

Q \ T	a	b	A	B	□
q_0	(q_a, A, R)			(q_2, B, R)	$(q_F, □, R)$
q_a	(q_a, a, R)	(q_b, B, L)		(q_a, B, R)	
q_b	(q_b, a, L)		(q_0, A, R)	(q_b, B, L)	
q_2'				(q_2', B, R)	$(q_F, □, R)$
q_F					

We use the following invariant property that in any ID the tape contents are of the form

$$A^* a^* B^* b^*$$

to determine the transitions.

If $n \geq 0$ then we could also include a transition

$\delta(q_0, □) = (q_F, □, R)$

Example 2. $L_2 = \{a^n b^n c^n \mid n \geq 1\}$. For this we use the following tape invariant

$$A^* a^* B^* b^* C^* c^*$$

and modify the transition table appropriately to obtain a deterministic TM T_2.

Example 3. $L_3 = \{xx \mid x \in \{a,b\}^*\}$.

In this case we could follow a nondeterministic approach to "guess" the midpoint of the string. However a deterministic TM is still possible provided we divide the problem into subproblems and tackle each subproblem one by one.

So here is the solution schema

1. Find the midpoint of the string.
2. Move the right half of the string by one cell to the right.
3. Insert a new symbol "C" to mark the separation of left half from right half.
4. Give a deterministic transition table to accept the string "xCx"

Subproblem 1,2,3. Finding the midpoint of the string and inserting a symbol "C" at the midpoint.

Solution Schema.

1. Starting from the leftmost end move to the rightmost end and replace the first "□" by "C".
2. Change the leftmost lowercase symbol to UPPERCASE and the rightmost lowercase symbol to UPPERCASE
3. Exchange the "C" with the new uppercase symbol immediately preceding it.
   The tape invariant is now something like

$$(A+B)^*.(a+b).C.(A+B)^*$$

4. Repeat steps 2–3 till the tape contents become

$$(A+B)^*C(A+B)^*$$

5. Now change all upper-case letters except C to lowercase (if necessary).