

## COL869: Special Topics in Concurrency

March 20, 2017

Home-work 4

Deadline Fri 31 Mar 2017

1. Consider the mutual exclusion problem of the unmanned railway crossing for which a specification  $S0$  and an implementation  $I000$  involving the (parallel) composition of processes  $T0, G0, C0$  (along with a restriction on all internal actions) was given. Let  $Iijk$  denote the state of the implementation involving processes  $Ti, Gj$  and  $Ck$  as appropriate.
  - (a) Prove that  $S0 \approx I000$  by constructing a weak bisimulation and proving that it is indeed a weak bisimulation.
  - (b) For each state  $Sm$  in the specification which of the states  $Iijk$  is observationally congruent to it. Prove your result.
  - (c) We had assumed in our design that the Train could communicate with the Gate. However (at least in India) the Gate is usually lowered and raised using a motor attached to the controller and usually the Controller also controls the Gate. Modify the implementation so that the Train and the Gate only communicate with the Controller and never with each other.
    - i. Prove the correctness of your modified implementation
    - ii. Prove that in the initial states the specification and implementation are observationally congruent to each other.
2. For all  $N > 0$  prove that  $FIFO(\varepsilon) \approx PIPE(\perp^{N+2})$ 
  - (a) by constructing a weak bisimulation  $\mathcal{R}$  such that  $FIFO(\varepsilon)\mathcal{R}PIPE(\perp^{N+2})$  and
  - (b) proving that  $\mathcal{R}$  is actually a weak bisimulation
  - (c) Characterize the subset of pairs of states  $\langle FIFO(s), PIPE(t) \rangle \in \mathcal{R}$  for which  $FIFO(s) \approx^+ PIPE(t)$ . Prove your result.
3. Prove that the relation  $\approx^+$  is the largest congruence contained in  $\approx$ . (Hint1: Assume  $\approx^c \subseteq \approx$  is a congruence. Now prove that  $P \approx^c Q$  implies  $P \approx^+ Q$ . Hint2: You don't have to take Hint1 if you don't find it convenient.)
4. Prove that
  - (a)  $P \approx Q$  implies for all  $a \in Act$ ,  $a.P \approx^+ a.Q$ .
  - (b)  $P + \tau.P \approx^+ \tau.P$
  - (c) For all  $a \in Act$ ,  $a.(P + \tau.Q) \approx^+ a.(P + \tau.Q) + a.Q$
5. Prove that  $P \approx Q$  if and only if one of the following conditions holds.
  - (a)  $P \approx^+ Q$
  - (b)  $\tau.P \approx^+ Q$
  - (c)  $P \approx^+ \tau.Q$