CSL100: Introduction to Computers and Computer Science
I semester 2013-14
Instructors: Subhashis Banerjee and S. Arun-Kumar

Assignment 3: **A clockwise spiral**

Study the figure below in which all **lattice points**[1] are traced through the arrows sarting from the origin of the (x, y)-plane.

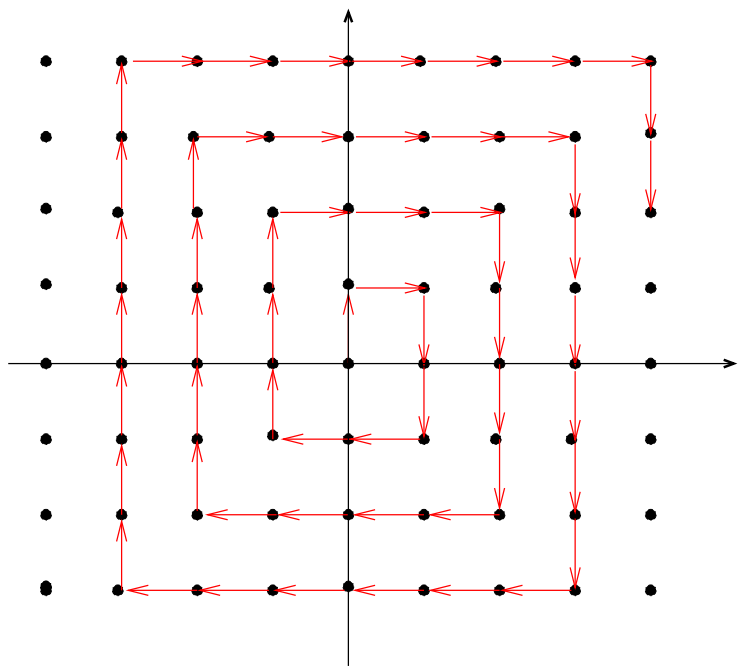

Figure 1: A clockwise spiral

Consider the following function *cspiral* which defines the clockwise-spiral of Figure 1 for a few integer valued ordered pairs in the $x - y$-plane, starting from the origin $(0, 0)$.

$$\begin{array}{lcl}
cspiral(0,0) & = & 0 \\
cspiral(0,1) & = & 1 \\
cspiral(1,1) & = & 2 \\
cspiral(1,0) & = & 3 \\
cspiral(1,-1) & = & 4 \\
cspiral(0,-1) & = & 5 \\
cspiral(-1,-1) & = & 6 \\
cspiral(-1,0) & = & 7 \\
cspiral(-1,1) & = & 8 \\
cspiral(-1,2) & = & 9 \\
cspiral(0,2) & = & 10 \\
\vdots & \vdots & \vdots
\end{array}$$

1. Define functions $nextpoint, prevpoint : \mathbb{Z}^2 \to \mathbb{Z}^2$ which for any given point on the spiral will yield the *next* and the *previous* (provided the given point is not $(0, 0)$) points on the spiral respectively.

[1] A lattice point is a point on the (x, y)-plane such that both coordinates are integers

2. Use the function *nextpoint* to define the function *cspiral* as a *linear* time algorithm which for any point $(m, n) \in \mathbb{Z}^2$ determines the value of $cspiral(m, n)$. Write in SML a function `cspiral1: int * int -> int` which implements this function *cspiral*.

3. Since the function *cspiral* is a 1-1 correspondence from the set $\mathbb{Z} \times \mathbb{Z}$ to the set $\mathbb{N}$ of non-negative integers, there exists a corresponding inverse function *laripsc* from $\mathbb{N}$ to $\mathbb{Z} \times \mathbb{Z}$, which for each whole number m, gives the integer-valued ordered pair (x, y) that it corresponds to.

   Define the two functions *cspiral*2 and *laripsc*2 as *constant* time algorithms and implement SML functions `cspiral2` and `laripsc2` respectively, such that they completely define the spiral. In particular, you should check (for a few large values) that the following identities always hold:

$$\boxed{cspiral(laripsc(m)) = m \text{ for all non-negative integers } m}$$

and

$$\boxed{laripsc(cspiral(x, y)) = (x, y) \text{ for all integers } x, y}$$