COL864: Special Topics in AI

Semester II, 2021-22

Sate Estimation - I

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Today’s lecture

- Last Class
  - Planning Motions
- This Class
  - State Estimation
    - Recursive State Estimation
    - Bayes Filter
  - References
    - Probabilistic Robotics Ch 1 & 2
    - AIMA Ch 15 (till sec 15.3)

Acknowledgements

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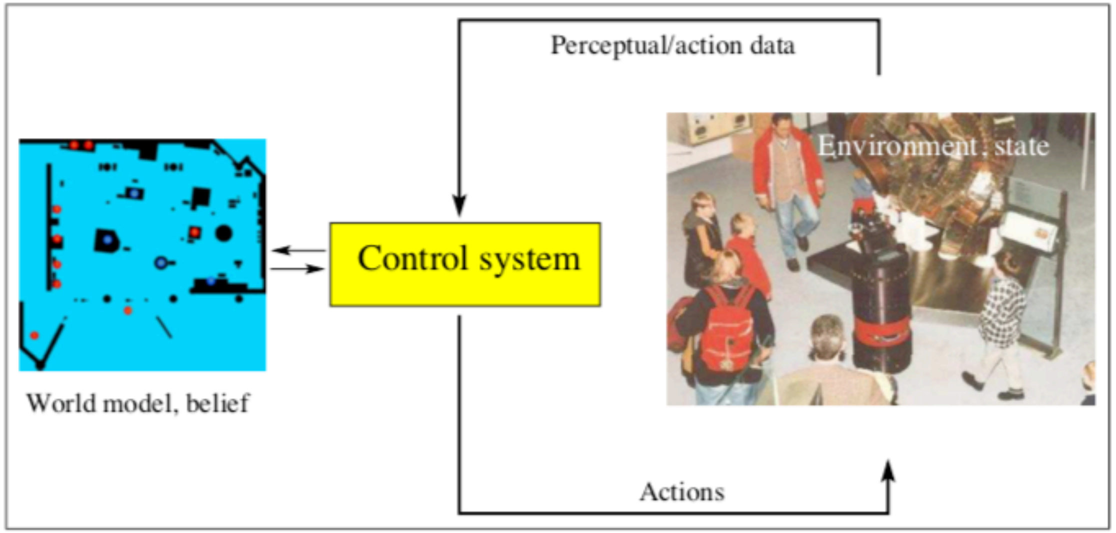
Robot Environment Interaction

- Environment or world
  - Objects, robot, people, interactions
  - Environment possesses a true internal state
- Observations
  - The agent cannot directly access the true environment state.
  - Takes observations via its sensors which are error prone.
- Belief
  - Agent maintains a belief or an estimate with respect to the state of the environment derived from observations.
  - The belief is used for decision making
- Actions
  - Agent can influence the environment through its physical interactions (actuations, motions, language interaction etc.)
  - The effect of actions may be stochastic.
  - Taking actions affects the world state and the robot's internal belief with regard to this state.

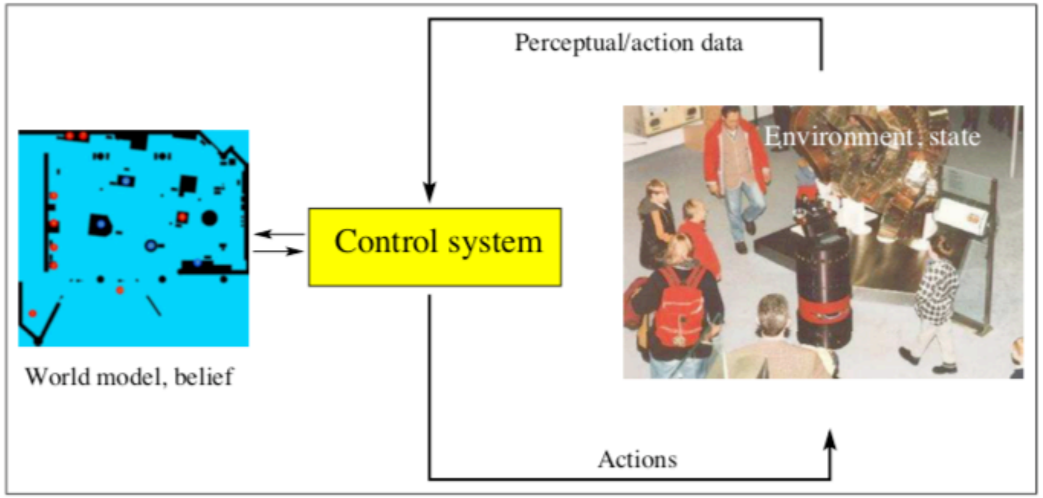


State Estimation

- Framework for estimating the state from sensor data.
- Estimating quantities that are *not directly observable.*
  - *But* can be inferred if certain quantities are available to the agent.
- State estimation algorithms
  - Compute *belief distributions* over *possible states* of the world.

State

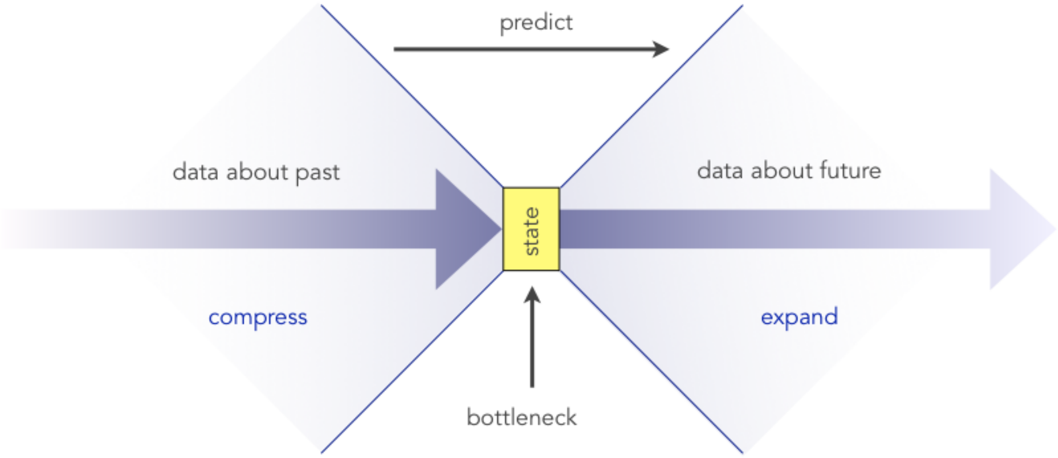
- What is typically part of the state, x?
  - Robot pose: position and orientation or kinematic state
  - Velocities: of the robot and other objects like people.
  - Location and features of surrounding objects in the environment.
  - Semantic states: is the door open or closed?
  - ……..
- What is put in the state is influenced by which task we seek to perform
  - Navigation
  - More complex example (e.g., delivery of hospital supplies)



State

- Environment is characterized by the state.
  - *"A collection of all aspects of the agent and its environment that can impact the future"*
- A sufficient statistic of the past observations and interactions required for future tasks.
- State plays and important role for decision making.

Figure courtesy Byron Boots



State: statistic of history sufficient to predict the future

Markovian assumption:

Future is independent of past given present

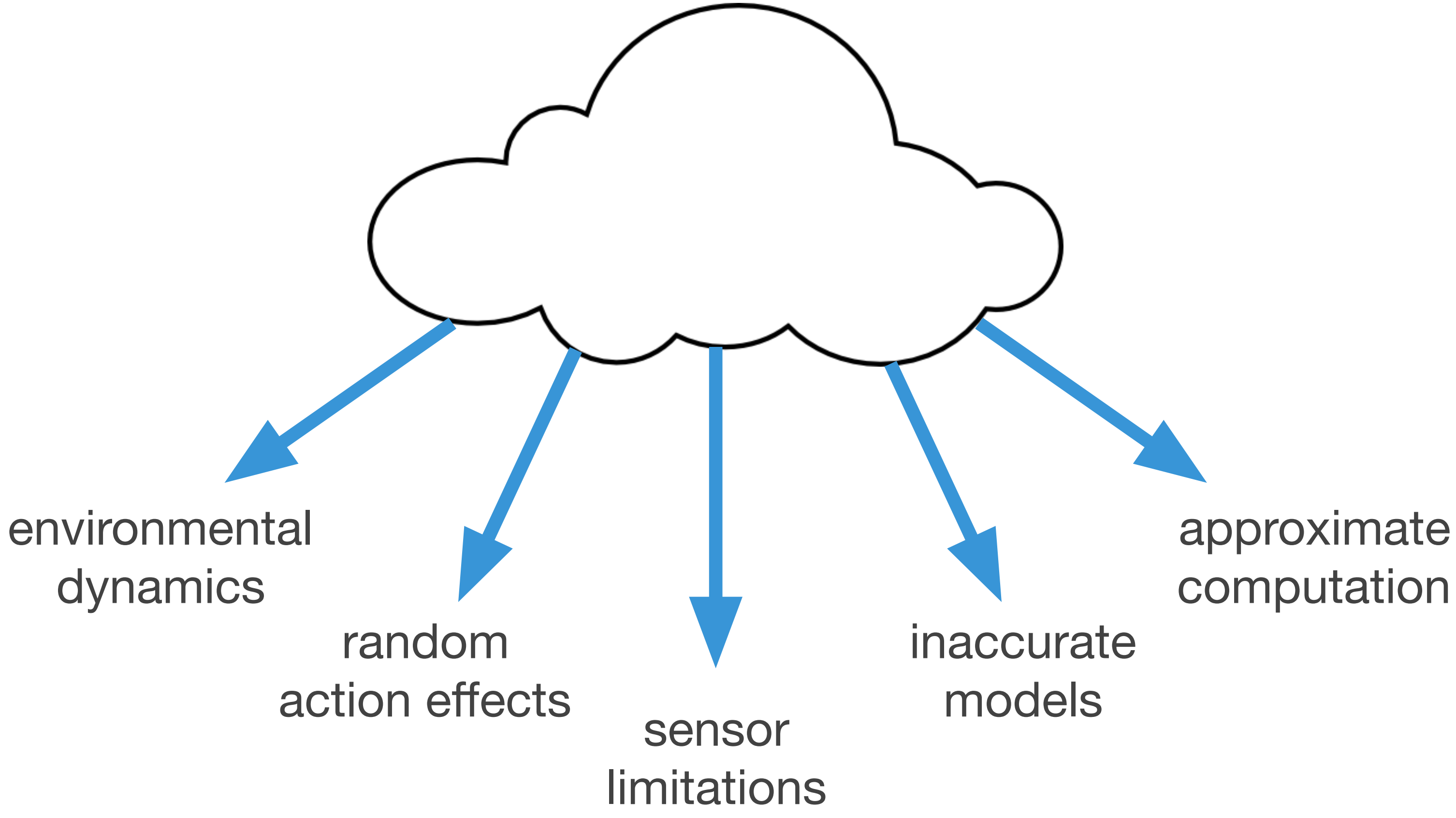
Two aspects: Sensing and Taking Actions

- **Taking Sensor Measurements**
  - Camera, range, tactile, language query etc.
  - Denote measurement data as z_t
  - Noisy observations of the true state.
  - Measurements typically add information, *decrease uncertainty.*

- **Taking Actions (or Controls)**
  - Physical interaction: robot motion, manipulation of objects, *NO_OP* etc.
  - Carry information about the change of state.
  - Source of control data: odometers or wheel encoders.
  - Denote control data as u_t
  - Actions are never carried out with absolute certainty.
  - In contrast to measurements, actions generally *increase uncertainty.*

Uncertainty

Explicitly represent uncertainty using probability theory.



Probability Recap

Independence

- X and Y are independent iff

  $P(x,y) = P(x)\,P(y)$

- $P(x \mid y)$ is the probability of x given y

  $P(x \mid y) = P(x,y) / P(y)$

  $P(x,y) = P(x \mid y)\,P(y)$

- If X and Y are independent then

  $P(x \mid y) = P(x)$

Marginalization

Discrete case

$$\sum_x P(x) = 1$$

$$P(x) = \sum_y P(x,y)$$

$$P(x) = \sum_y P(x \mid y) P(y)$$

Continuous case

$$\int p(x)\, dx = 1$$

$$p(x) = \int p(x,y)\, dy$$

$$p(x) = \int p(x \mid y) p(y)\, dy$$

Conditioning

- Law of total probability:

Marginalize out z.

$$P(x) = \int P(x, z)dz$$

$$P(x) = \int P(x \mid z)P(z)dz$$

Conditioning on the extra variables.

$$P(x \mid y) = \int P(x \mid y, z)\, P(z \mid y)\, dz$$

Conditional Independence

- X and Y are conditionally independent given Z.
- Given Z , X does not add information about Y and vice versa.

$$P(x, y | z) = P(x | z) P(y | z)$$

$$P(x | z) = P(x | z, y)$$

$$P(y | z) = P(y | z, x)$$

Bayes Rule

$$P(x,y) = P(x \mid y)P(y) = P(y \mid x)P(x)$$

$$\Rightarrow$$

$$P(x \mid y) = \frac{P(y \mid x)\ P(x)}{P(y)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}$$

$$P(x \mid y) = \frac{P(y \mid x)\ P(x)}{P(y)} = \eta\ P(y \mid x)\, P(x)$$

$$\eta = P(y)^{-1} = \frac{1}{\sum_x P(y \mid x)P(x)}$$

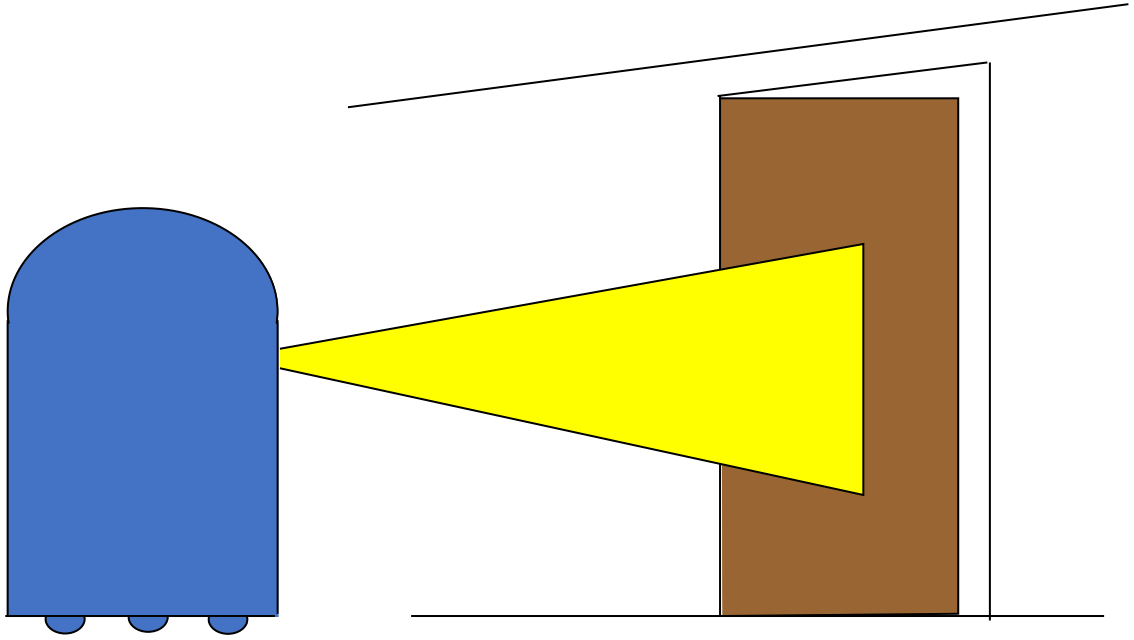
Bayes Rule with Background Knowledge

When extra information is available. Incorporate that knowledge as observations of extra variables.

$$P(x \mid y, z) = \frac{P(y \mid x, z)\ P(x \mid z)}{P(y \mid z)}$$

Example of State Estimation

- The robot wants to estimate the state of the door as closed or open
  - Has a noisy sensor that produces measurement, z
- Estimate: P(open|z)?
  - Likelihood that the true state of the door is open given that z was measured.



$$P(z \mid open) = 0.6 \qquad P(z \mid \neg open) = 0.3$$

$$P(open) = P(\neg open) = 0.5$$

The observation z is correlated with the true state as open or not-open.
E.g., measuring a particular distance or classifying an image.

Causal vs. Diagnostic Reasoning

- P(open|z) is diagnostic reasoning
  - *Given that I observe z what is the likelihood that the door state is actually open?*
- P(z|open) is causal reasoning (can estimate by counting frequencies)
  - *Given that the door state is open what is the likelihood of getting measurement z?*
- Often causal knowledge is easier to obtain
  - Given the underlying state collect the data.
- Bayes rule enables the use of causal knowledge:

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z)}$$

Incorporating a single measurement

- Higher likelihood of observation z when the door **is** open compared to when the door is closed.
- The incorporation of the measurement *z* **raises** the probability that the door is open.

$$P(z \mid open) = 0.6 \qquad P(z \mid \neg open) = 0.3$$

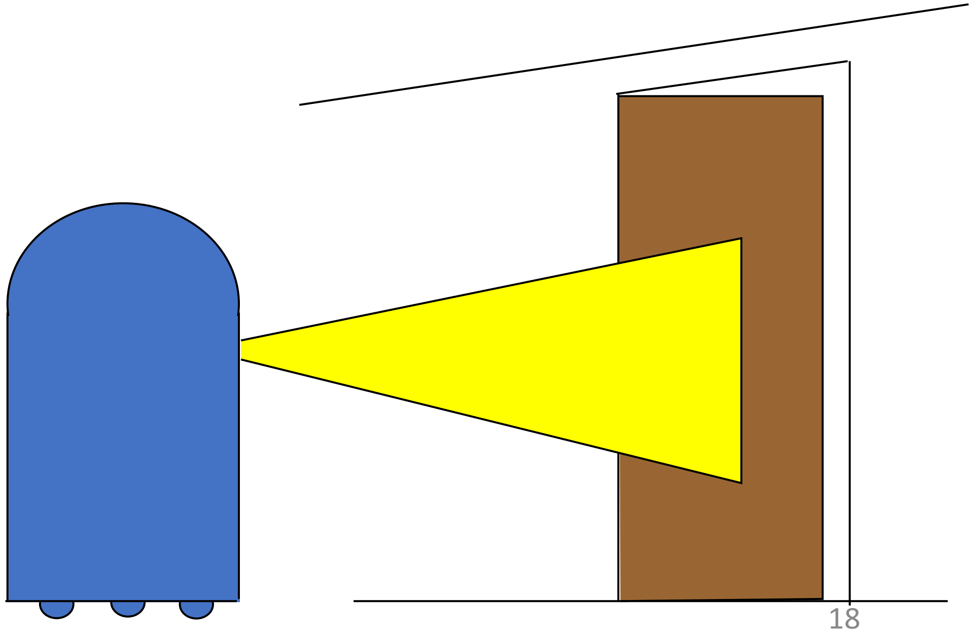
$$P(open) = P(\neg open) = 0.5$$

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z \mid open)\,p(open) + P(z \mid \neg open)\,p(\neg open)}$$

$$P(open \mid z) = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67$$

Incorporating multiple measurements

- Suppose the robot has another sensor that produces a second observation z_2
- How can we combine the measurement of the second sensor
- What is $P(open|z_1, z_2)$?
- In general, how to estimate $P(x| z_1 \ldots z_n)$



Recursive Bayesian Updating

$$P(x \mid z_1,\ldots,z_n) = \frac{P(z_n \mid x, z_1,\ldots,z_{n-1})\, P(x \mid z_1,\ldots,z_{n-1})}{P(z_n \mid z_1,\ldots,z_{n-1})}$$

Markov assumption: z_n is conditionally independent of $z_1,\ldots,z_{n-1}$ given *x*.

$$P(x \mid z_1,\ldots,z_n) = \frac{P(z_n \mid x)\, P(x \mid z_1,\ldots,z_{n-1})}{P(z_n \mid z_1,\ldots,z_{n-1})}$$

$$= \eta\, P(z_n \mid x)\, P(x \mid z_1,\ldots,z_{n-1})$$

$$= \eta_{1\ldots n} \prod_{i=1\ldots n} P(z_i \mid x)\, P(x)$$

In our causal modeling view, the world state is *causing* all the observations.

Incorporating second sensor measurement

- Higher likelihood of observation z when the door is **not** open compared to when the door is open.
- The inclusion of the second measurement z_2 **lowers** the probability for the door to be open.

$$P(z_2 \mid open) = 0.5 \qquad P(z_2 \mid \neg open) = 0.6$$

$$P(open \mid z_1) = 2/3 \qquad P(\neg open \mid z_1) = 1/3$$

$$P(open \mid z_2, z_1) = \frac{P(z_2 \mid open)\, P(open \mid z_1)}{P(z_2 \mid open)\, P(open \mid z_1) + P(z_2 \mid \neg open)\, P(\neg open \mid z_1)}$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2} \cdot \frac{2}{3} + \frac{3}{5} \cdot \frac{1}{3}} = \frac{5}{8} = 0.625$$

Sensor Model

- Sensor model
  - What is the likelihood of obtaining this sensor measurement given the true state?
  - A conditional distribution over observations given the true state. Generative Model.
  - Observations or measurements can be considered as the noisy projection of the state

$$p(z_t|x_t)$$

Action Model

- Action or Motion model
  - How the actions or controls change the state of the world?
  - Incorporate the outcome of an action u into the current “belief”, we use the conditional distribution.
  - Specifies how does the state change by application of the action (from the state, x_{t-1} to the state, x_t by executing the action, u_t).

$$p(x_t | x_{t-1}, u_t)$$

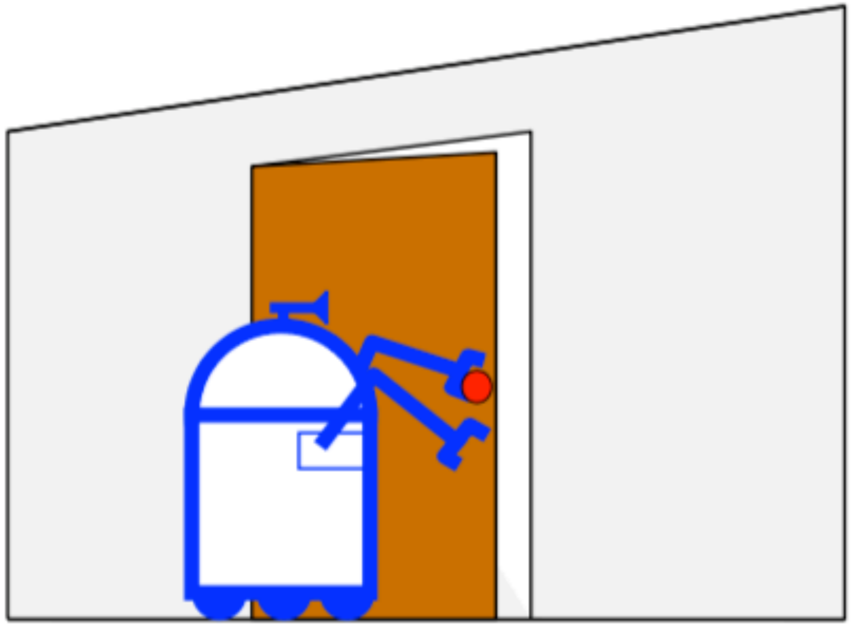
Belief over the world state

- Belief
  - Expresses the agent's internal knowledge about the state of an aspect of the world.
  - Note: we do not know the true state.
  - The belief estimated from the sensor measurement data and the actions taken till now.

$$Bel(x_t) = p(x_t | z_{1:t}, u_{1:t})$$

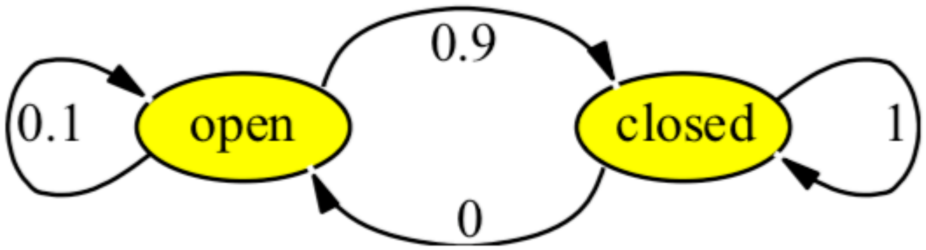
Example: Incorporating action effects

Example: Closing the door



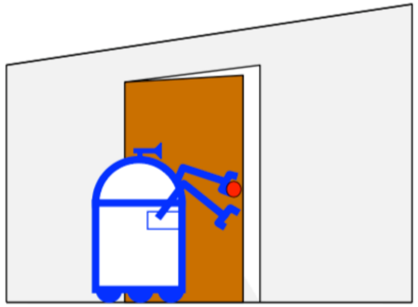
Probabilistic effects

$P(x|u,x')$ for u = "close door":



If the door is open, the action "close door" succeeds in 90% of all cases.

Example: The Resulting Belief



Marginalizing out the outcome of actions

Continuous case:

$$P(x \mid u) = \int P(x \mid u, x')P(x')dx'$$

Discrete case:

$$P(x \mid u) = \sum P(x \mid u, x')P(x')$$

$$\begin{aligned}
P(closed \mid u) &= \sum P(closed \mid u, x')P(x') \\
&= P(closed \mid u, open)P(open) \\
&\quad + P(closed \mid u, closed)P(closed) \\
&= \frac{9}{10} * \frac{5}{8} + \frac{1}{1} * \frac{3}{8} = \frac{15}{16}
\end{aligned}$$

$$\begin{aligned}
P(open \mid u) &= \sum P(open \mid u, x')P(x') \\
&= P(open \mid u, open)P(open) \\
&\quad + P(open \mid u, closed)P(closed) \\
&= \frac{1}{10} * \frac{5}{8} + \frac{0}{1} * \frac{3}{8} = \frac{1}{16} \\
&= 1 - P(closed \mid u)
\end{aligned}$$

Incorporating Measurements

- Bayes rule

$$P(x \mid z) = \frac{\boxed{P(z \mid x)}\, P(x)}{P(z)} = \frac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}$$

Bayes Filter

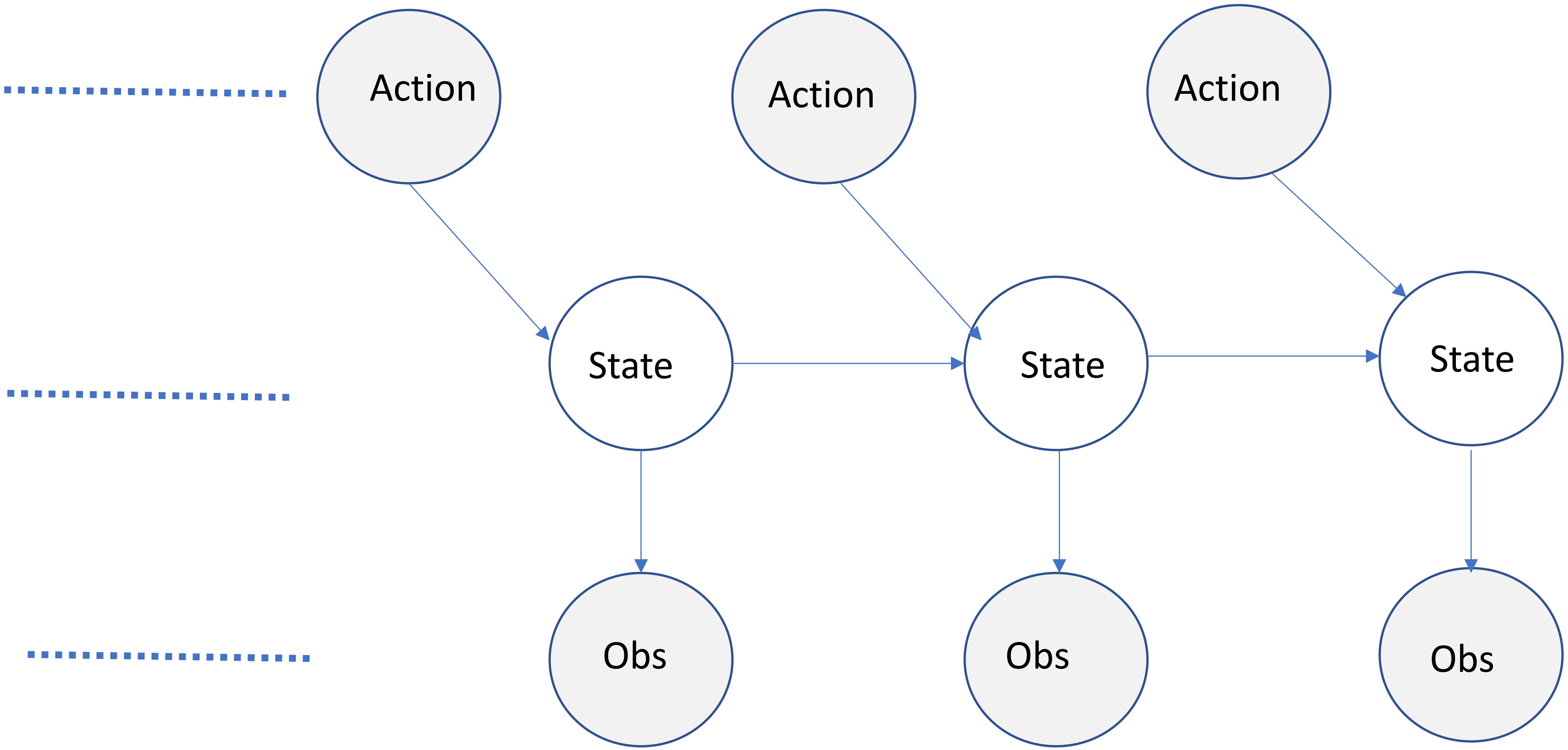
- **Given:**
  - Stream of observations *z* and action data *u:* $d_t = \{u_1, z_2 \ldots, u_{t-1}, z_t\}$
  - Sensor model $p(z_t | x_t)$
  - Action model $p(x_t | x_{t-1}, u_t)$
  - Prior probability of the system state *P(x).*

- **What we want to estimate?**
  - The state at time t
  - A belief or the posterior over the state: $Bel(x_t) = P(x_t \mid u_1, z_2 \ldots, u_{t-1}, z_t)$

Intuitively: Given all observations collected by the agent till time t and all the actions taken by the agent till time t, what is our estimate over its state?

In essence

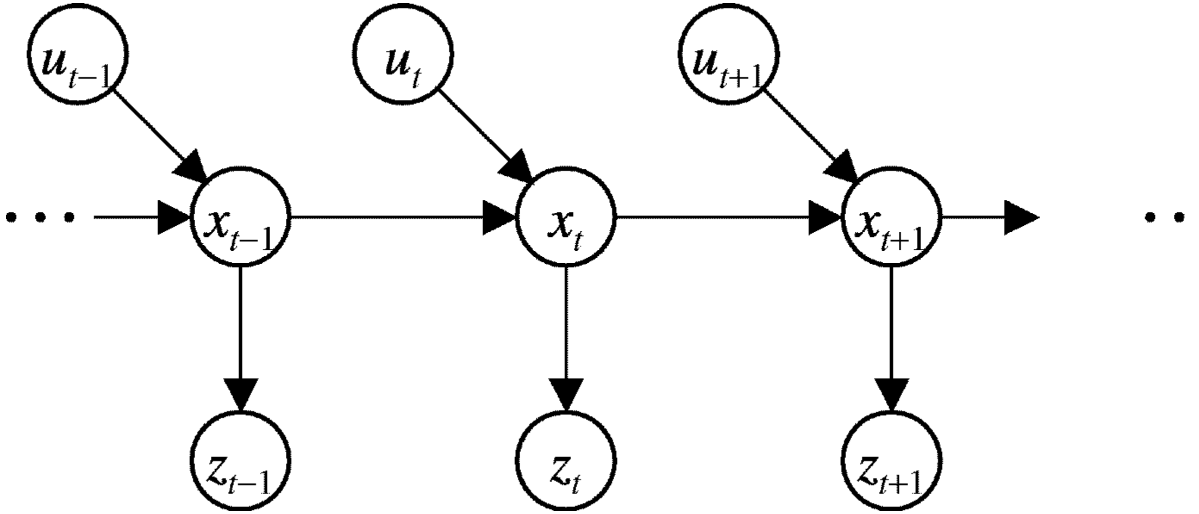
We estimate the state of the agent via measurements and knowledge of what actions were taken. Bayes Filter provides a recursive way to estimate the likelihood given the conditional independence assumptions.



Formally, a Generative Model

Assumptions

- Static world
- Independent noise
- Perfect model, no approximation errors
- Markov assumption (once you know the state the past actions and observations do not affect the future).



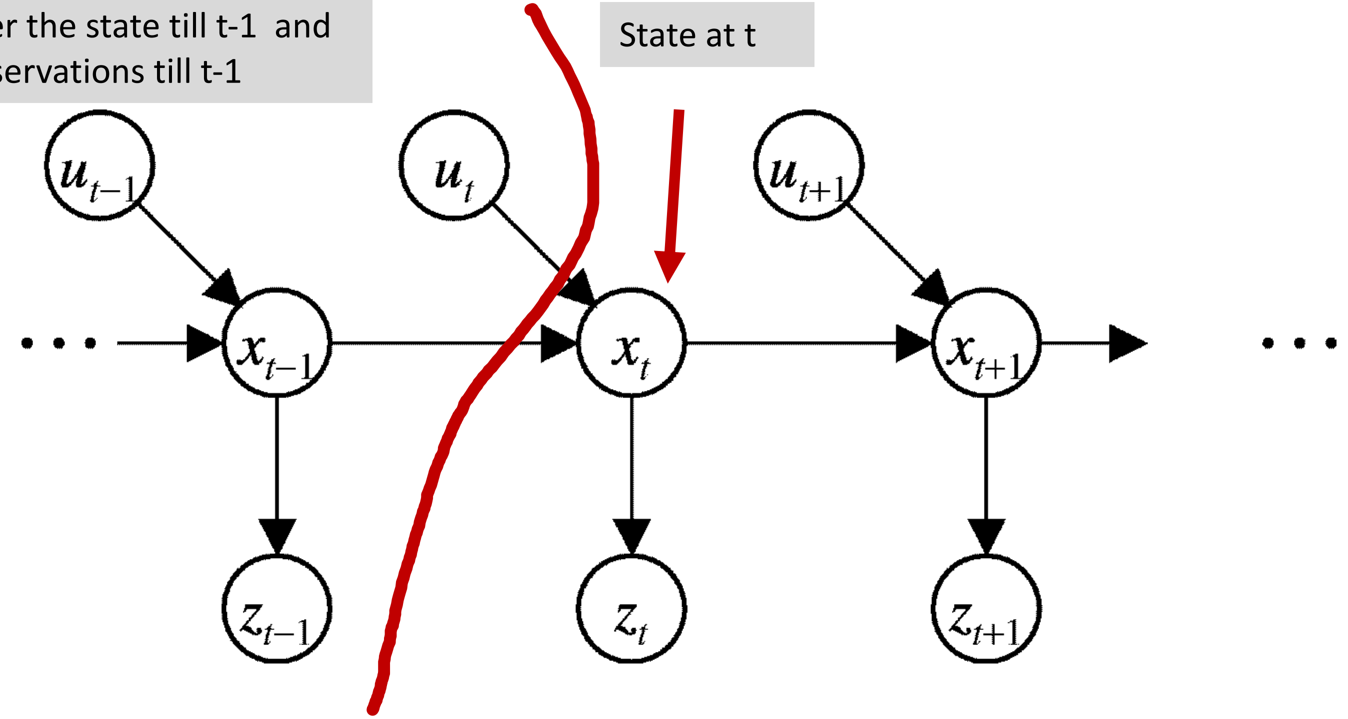
$$p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$$

$$p(x_t \mid x_{1:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Generative Model

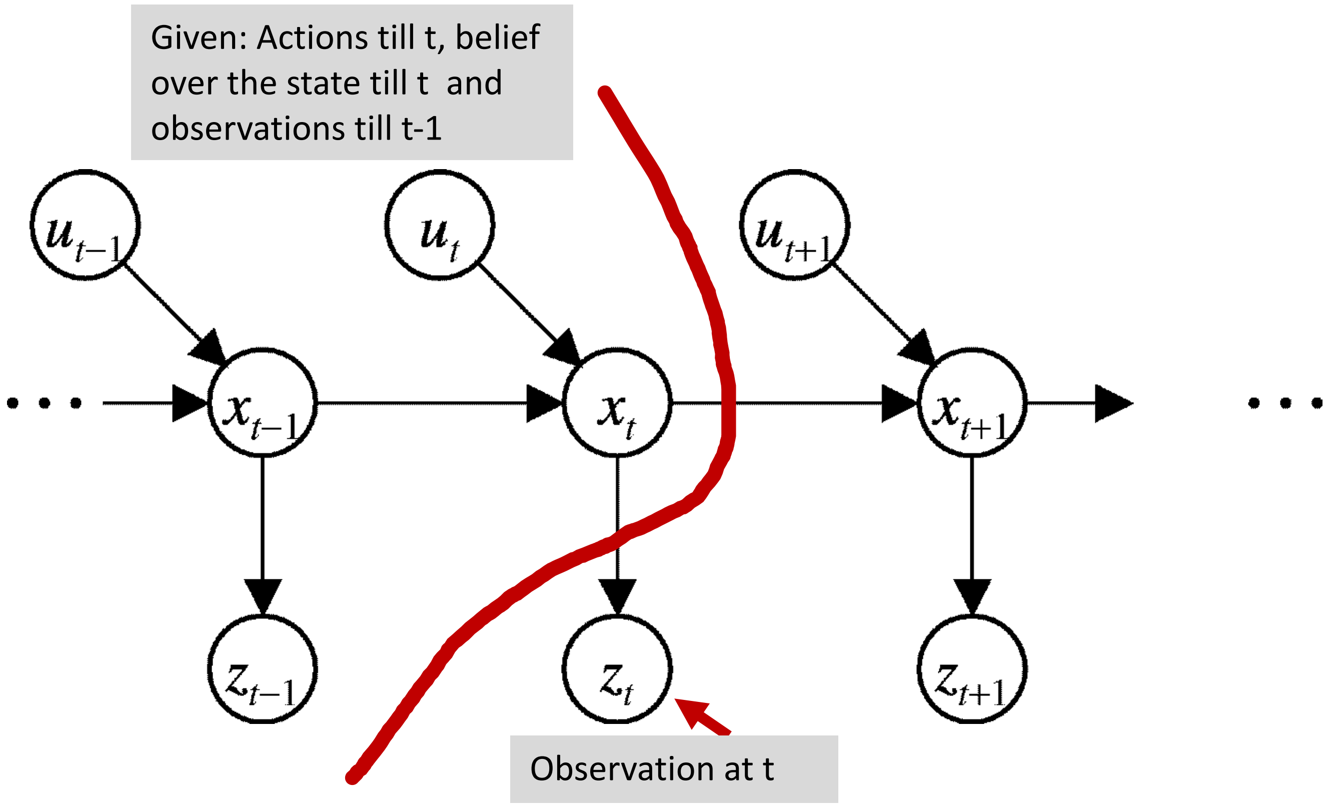
Given: Actions till t, belief over the state till t-1 and observations till t-1

State at t



$$p(x_t \mid x_{1:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$

Generative Model



$$p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$$

Bayes Filters

z = observation
u = action
x = state

$$Bel(x_t) = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

Bayes

$$= \eta \; P(z_t \mid x_t, u_1, z_1, \ldots, u_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Markov

$$= \eta \; P(z_t \mid x_t) \; P(x_t \mid u_1, z_1, \ldots, u_t)$$

Total prob.

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1}) P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

Markov

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; P(x_{t-1} \mid u_1, z_1, \ldots, u_t) \; dx_{t-1}$$

$$= \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

Bayes Filters Algorithm

1. **Algorithm Bayes_filter** (*Bel(x), d*):
2. n=0
3. **If *d* is a perceptual data item *z* then**
4. For all x do
5. $$Bel'(x) = P(z \mid x) Bel(x)$$
6. $$\eta = \eta + Bel'(x)$$
7. For all x do
8. $$Bel'(x) = \eta^{-1} Bel'(x)$$
9. **Else if *d* is an action data item *u* then**
10. For all x do
11. $$Bel'(x) = \int P(x \mid u, x') \, Bel(x') \, dx'$$
12. **Return *Bel'(x)***

$$Bel(x_t) = \eta \; P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) \; Bel(x_{t-1}) \; dx_{t-1}$$

Bayes Filter: Takeaways

- Bayes filters are a probabilistic tool for estimating the state of with observations acquired over time.
  - Target is to obtain the belief over the current state given past actions and observations.
  - Estimate this distribution in a recursive manner.
  - Update using actions
  - Update using measurements.
- Bayes rule allows us to compute probabilities that are difficult to determine otherwise.
- Under the Markov assumption, recursive Bayesian updating can be used to efficiently combine evidence.