COL333/671: Introduction to AI

Semester I, 2021

Adversarial Search

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Outline

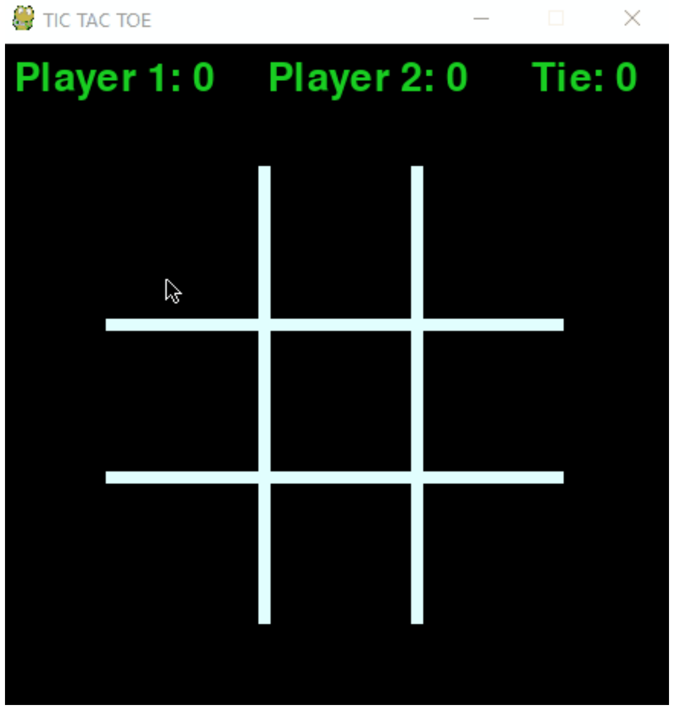
- Last Class
  - Local Search
- This Class
  - Adversarial Search
- Reference Material
  - AIMA Ch. 5 (Sec: 5.1-5.5)

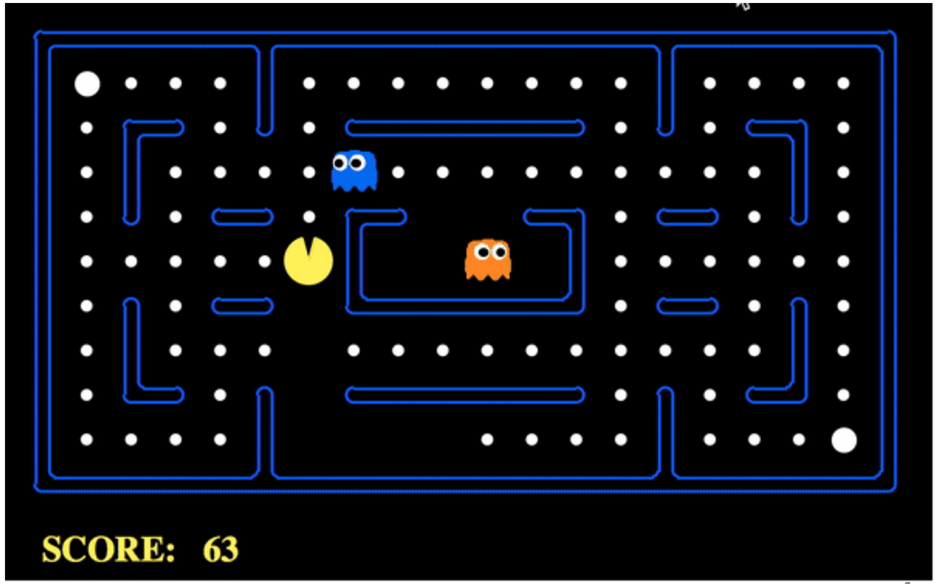
Acknowledgement

These slides are intended for teaching purposes only. Some material has been used/adapted from web sources and from slides by Doina Precup, Dorsa Sadigh, Percy Liang, Mausam, Dan Klein, Anca Dragan, Nicholas Roy and others.

Game Playing and AI

- **Games: challenging decision-making problems**
  - Incorporate the state of the other agent in your decision-making. Leads to a vast number of possibilities.
  - Long duration of play. Win at the end.
  - Time limits: Do not have time to compute optimal solutions.





Games: Characteristics

- Axes:
  - Players: one, two or more.
  - Actions (moves): deterministic or stochastic
  - States: fully known or not.

- Zero-Sum Games
  - Adversarial: agents have opposite utilities (values on outcomes)

- **Core: contingency problem**
  - The opponent's move is not known ahead of time. A player must respond with a move for every possible opponent reply.
- **Output**
  - Calculate a strategy (policy) which recommends a move from each state.

Playing Tic-Tac-Toe: *Essentially a search problem!*

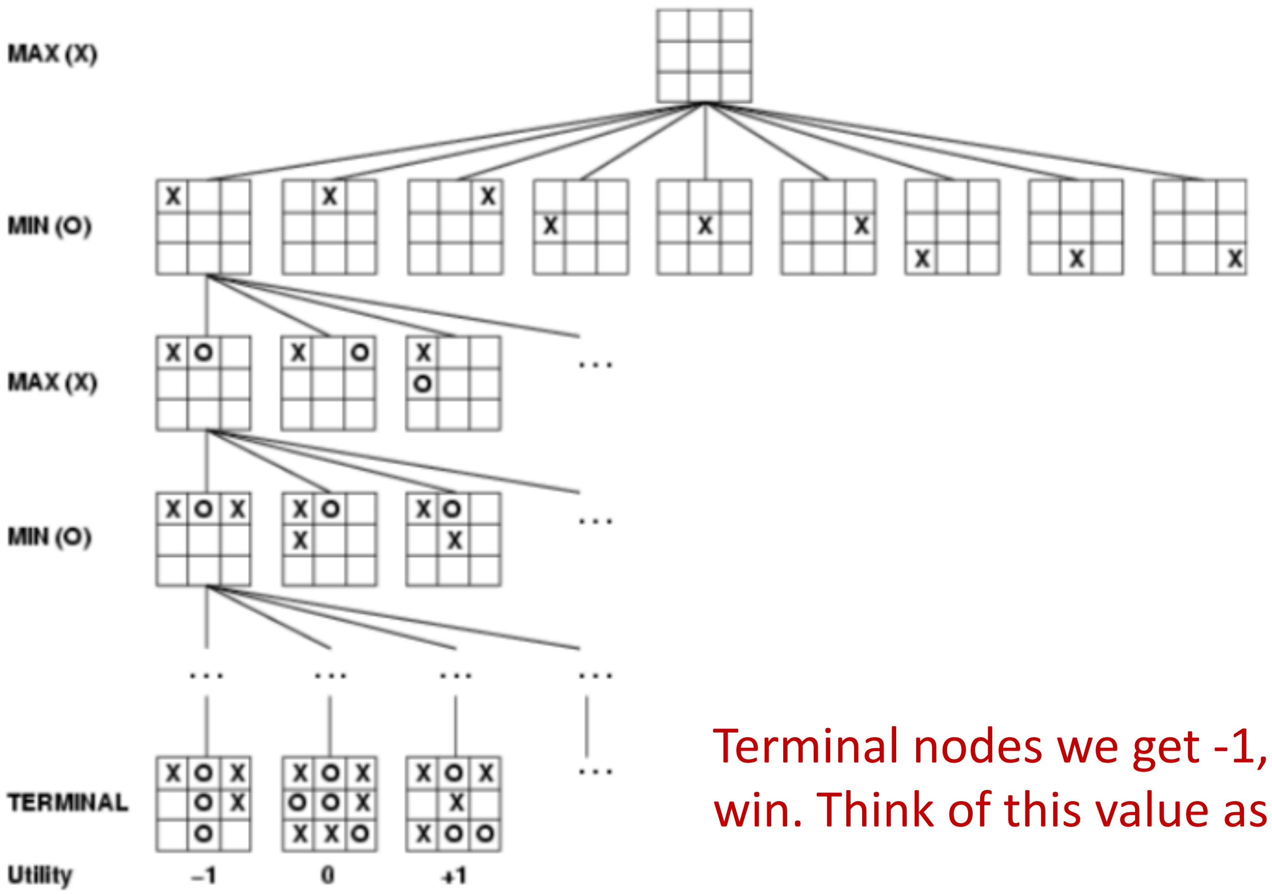
computer's turn

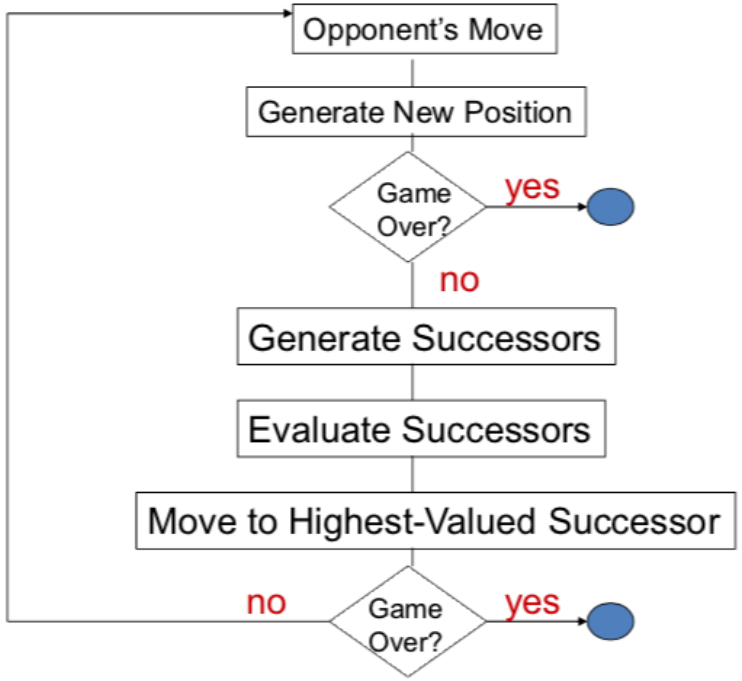
opponent's turn

computer's turn

opponent's turn

leaf nodes are evaluated

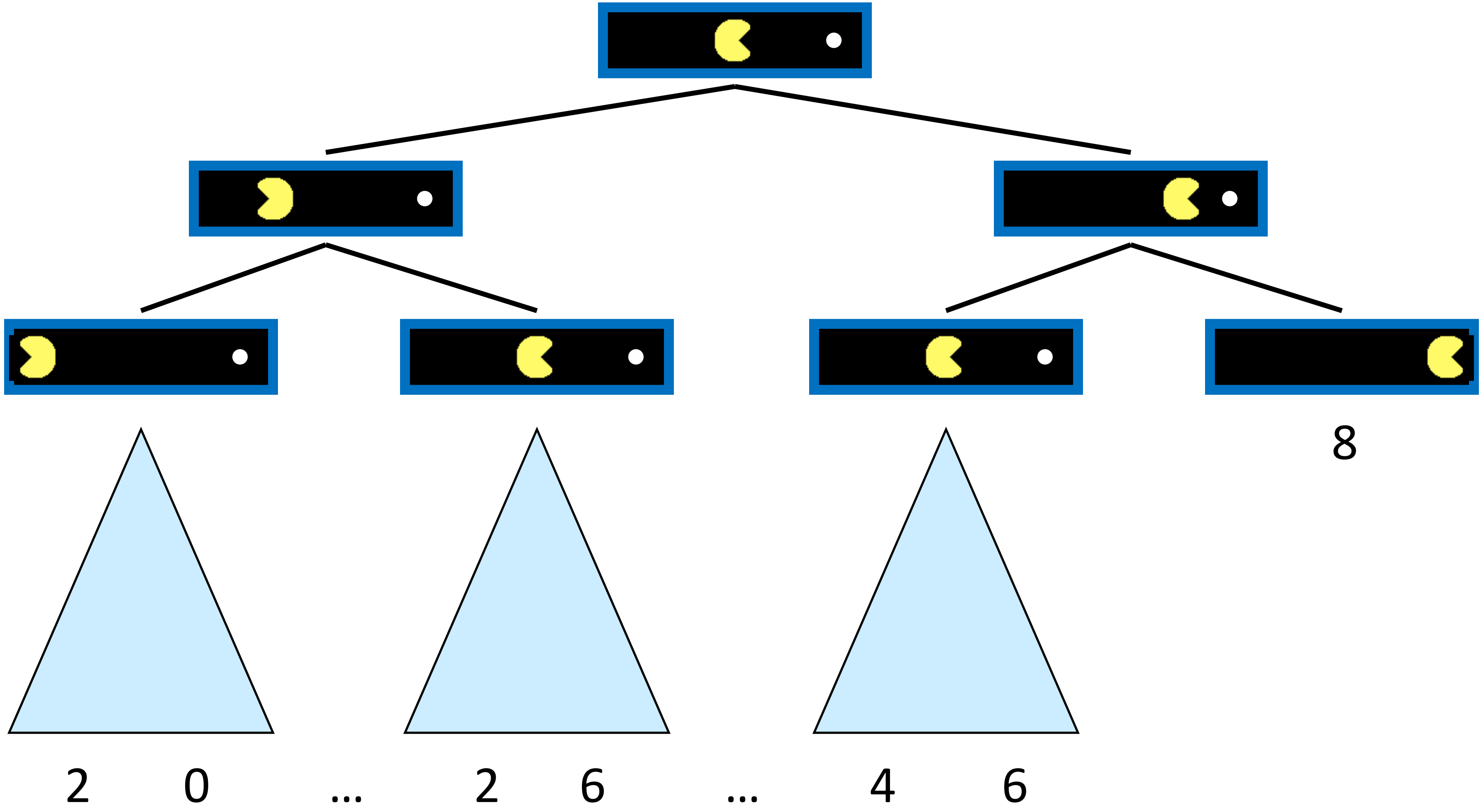




Terminal nodes we get -1, 0 or 1 for loss, tie or win. Think of this value as a "utility" of a state.

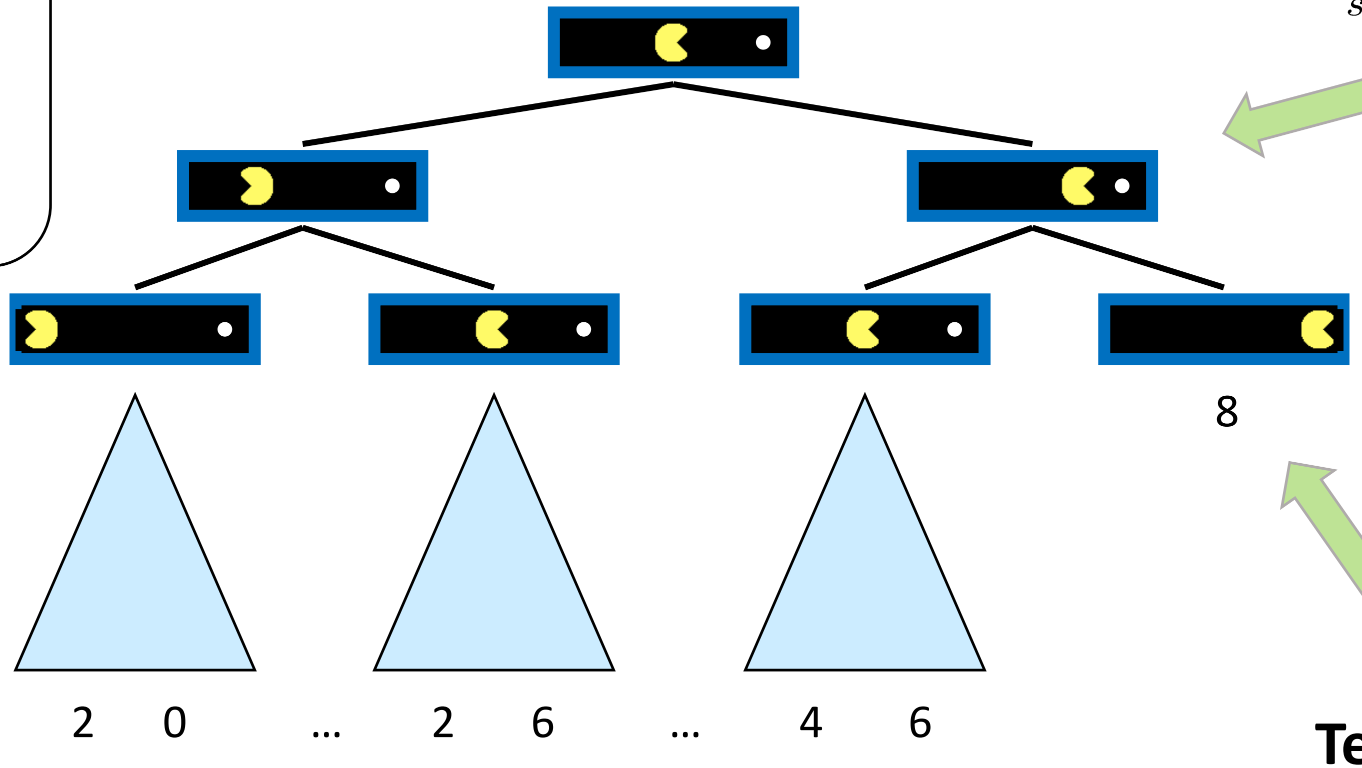
in and from Mausam

Single-Agent Trees



Computing “utility” of states to decide actions

Value of a state:
The best achievable outcome (utility) from that state



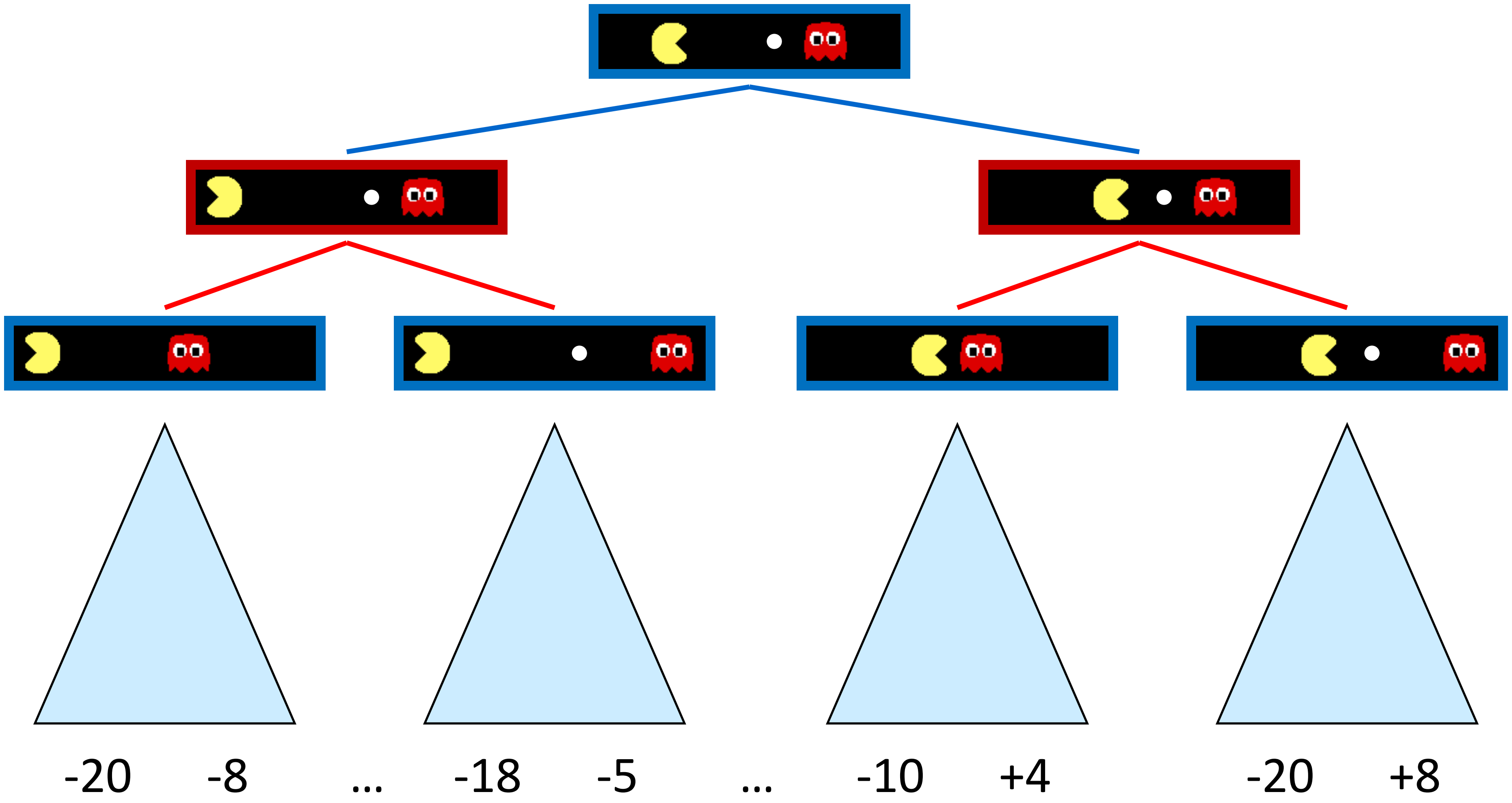
Non-Terminal States:

$$V(s) = \max_{s' \in \text{children}(s)} V(s')$$

Terminal States:

$$V(s) = \text{known}$$

Game Trees: Presence of an Adversary



The adversary's actions are not in our control. Plan as a contingency considering all possible actions taken by the adversary.

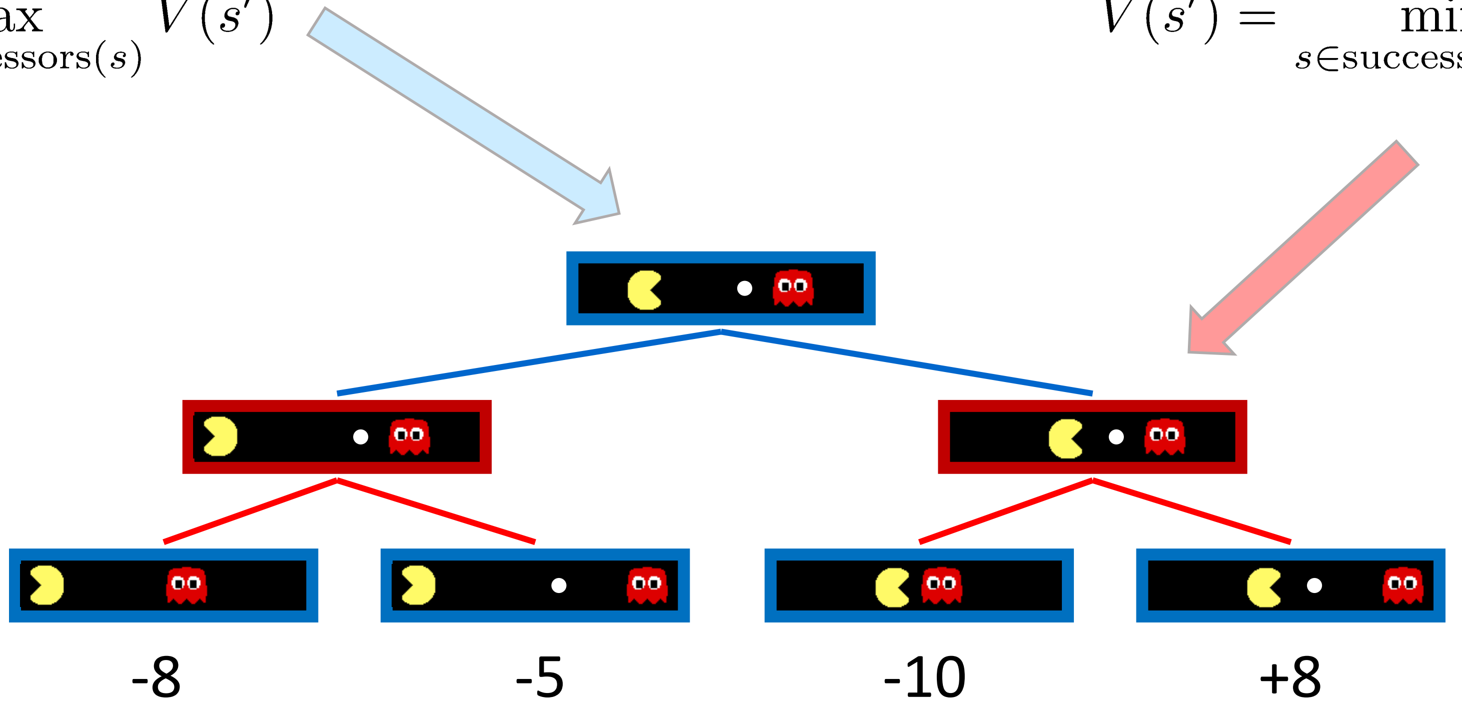
Minimax Values

States Under Agent's Control:

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

States Under Opponent's Control:

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$



Terminal States:

$$V(s) = \text{known}$$

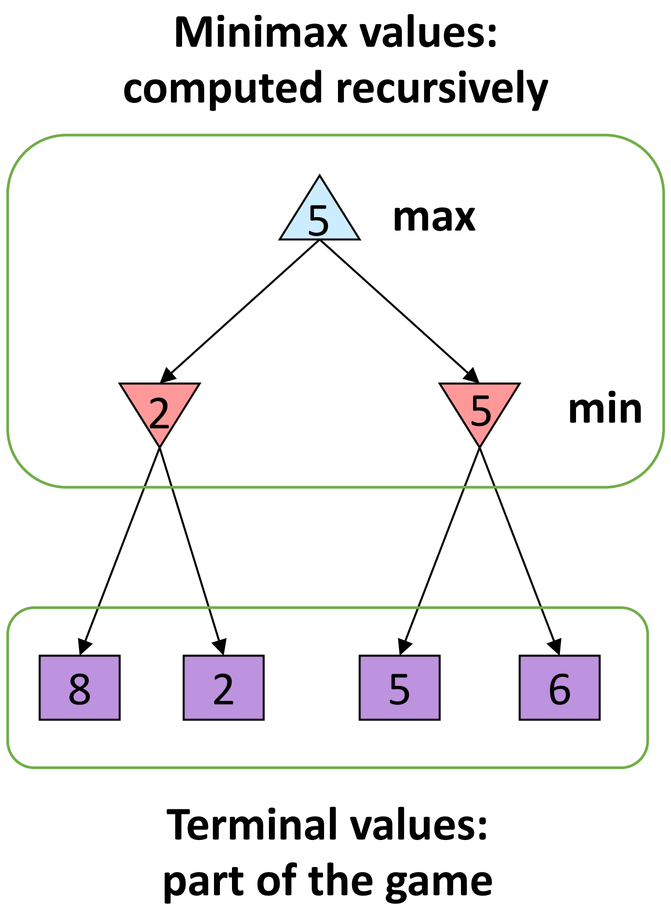
Adversarial Search (Minimax)

- Consider a deterministic, zero-sum game
  - Tic-tac-toe, chess etc.
  - One player maximizes result and the other minimizes result.

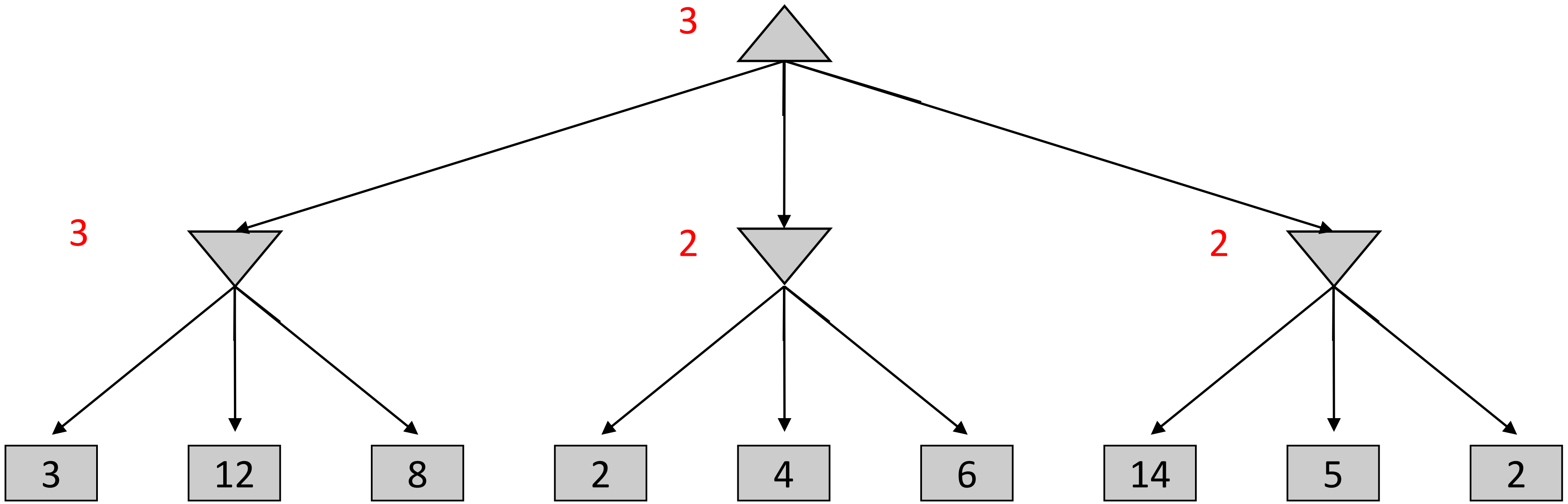
- Minimax Search
  - Search the game tree for best moves.
  - Select optimal actions that move to a position with the highest minimax value.
  - What is the minimax value?
    - It is the best achievable utility against the optimal (rational) adversary.
    - Best achievable payoff against the best play by the adversary.

Minimax Algorithm

- Ply and Move
  - Move: when action taken by both players.
  - Ply: is a half move.
- Backed-up value
  - of a MAX-position: the value of the largest successor
  - of a MIN-position: the value of its smallest successor.
- Minimax algorithm
  - Search down the tree till the terminal nodes.
  - At the bottom level apply the utility function.
  - Back up the values up to the root along the search path (compute as per min and max nodes)
  - The root node selects the action.



Minimax Example



Minimax Implementation

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, min-value(successor))
    return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, max-value(successor))
    return v
```

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Implementation

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is MIN: return min-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def min-value(state):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor))
    return v
```

Useful, when there are multiple adversaries.

Minimax Properties

- Completeness
  - Yes

- Complexity
  - Time: $O(b^m)$
  - Space: $O(bm)$

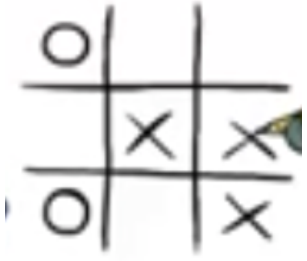
- **Requires growing the tree till the terminal nodes.**
- **Not feasible in practice for a game like Chess.**

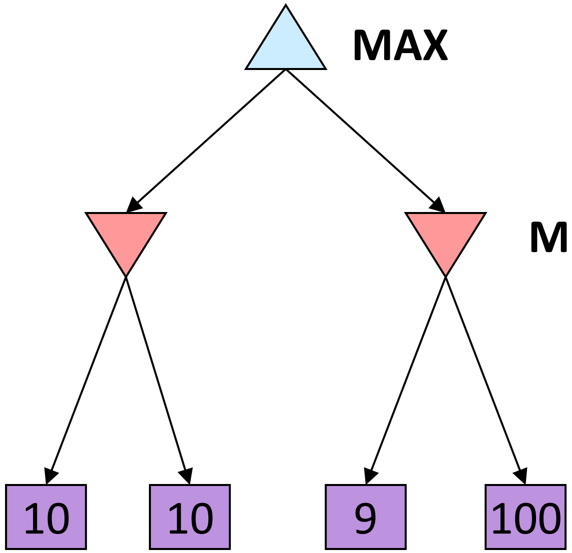
- Chess:
  - branching factor $b \approx 35$
  - game length $m \approx 100$
  - search space $b^m \approx 35^{100} \approx 10^{154}$
- The Universe:
  - number of atoms $\approx 10^{78}$
  - age $\approx 10^{18}$ seconds
  - 10^8 moves/sec x 10^{78} x 10^{18} = 10^{104}

Minimax Properties

- Optimal
  - If the adversary is playing optimally (i.e., giving us the min value)
    - Yes
  - If the adversary is not playing optimally (i.e., <u>not</u> giving us the min value)
    - No. Why? It does not exploit the opponent’s weakness against a suboptimal opponent).

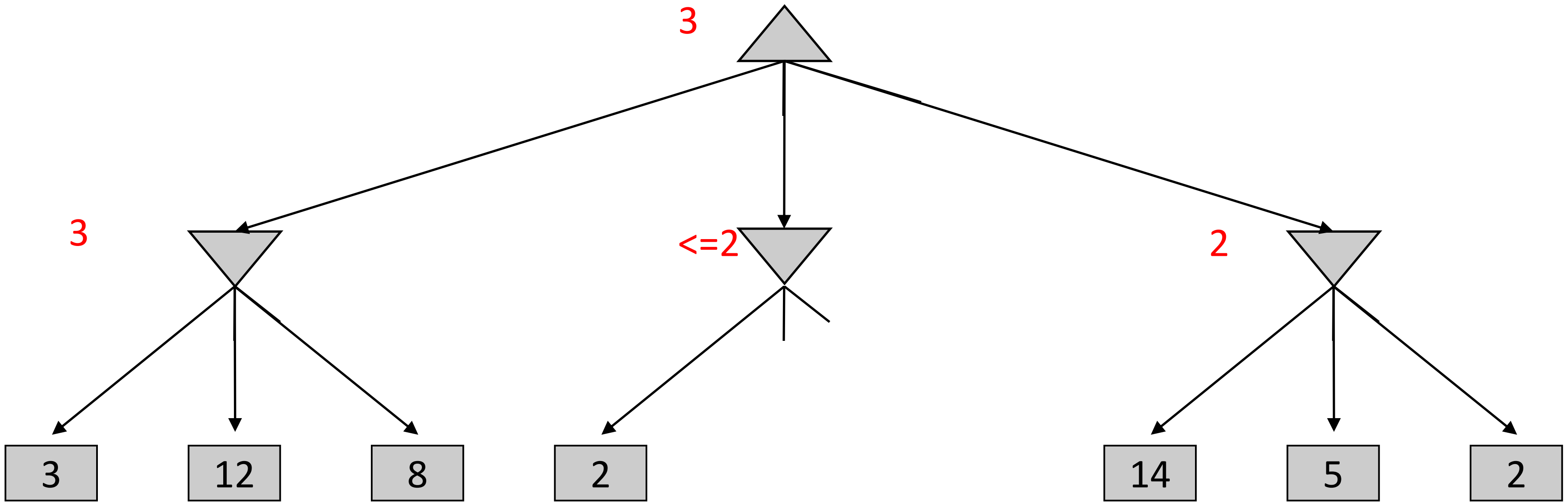
You: Cricle. Opponent: Cross





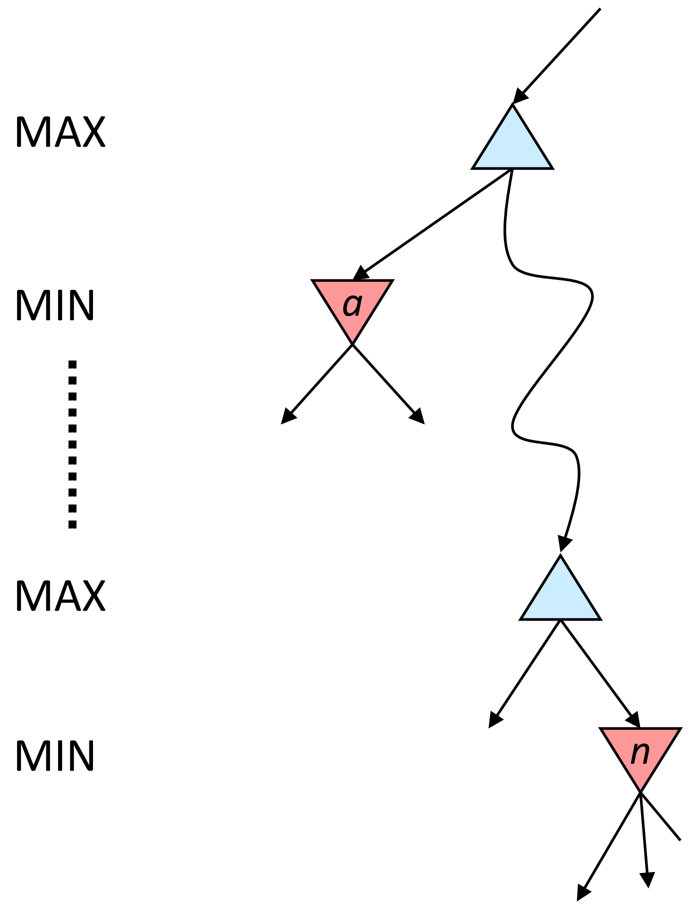
If min returns 9? Or 100?

Necessary to examine all values in the tree?



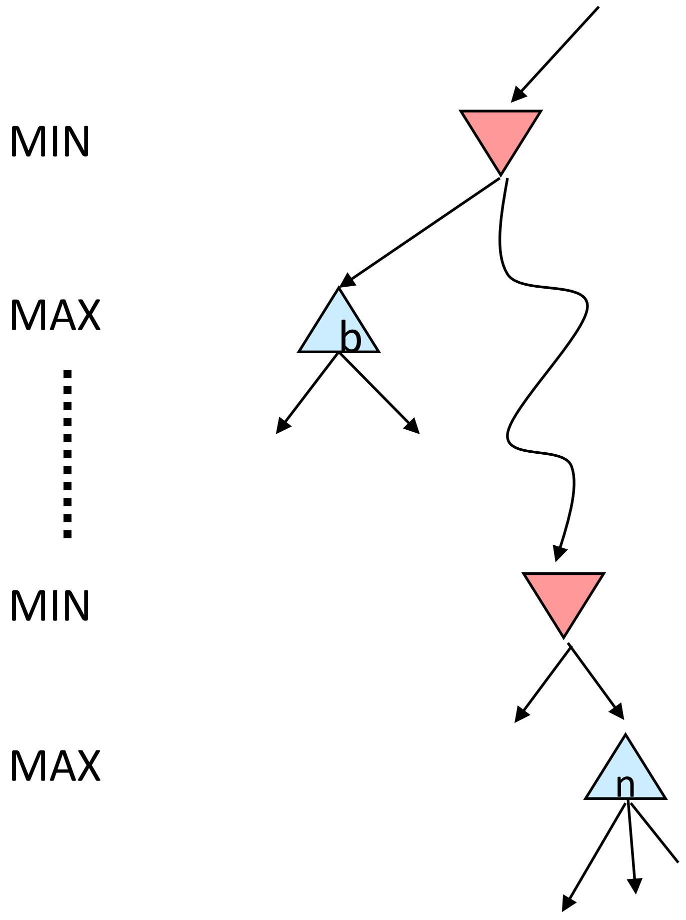
Alpha-Beta Pruning: General Idea

- **General Configuration (MIN version)**
  - Consider computing the MIN-VALUE at some node *n*, examining *n*'s children
  - *n*'s estimate of the childrens' min is reducing.
  - Who can use *n*'s value to make a choice? MAX
  - Let *a* be the best value that MAX can get at any choice point along the current path from the root
  - If the value at *n* becomes worse than *a*, MAX will not pick this option, so we can stop considering *n*'s other children (any further exploration of children will only reduce the value further)

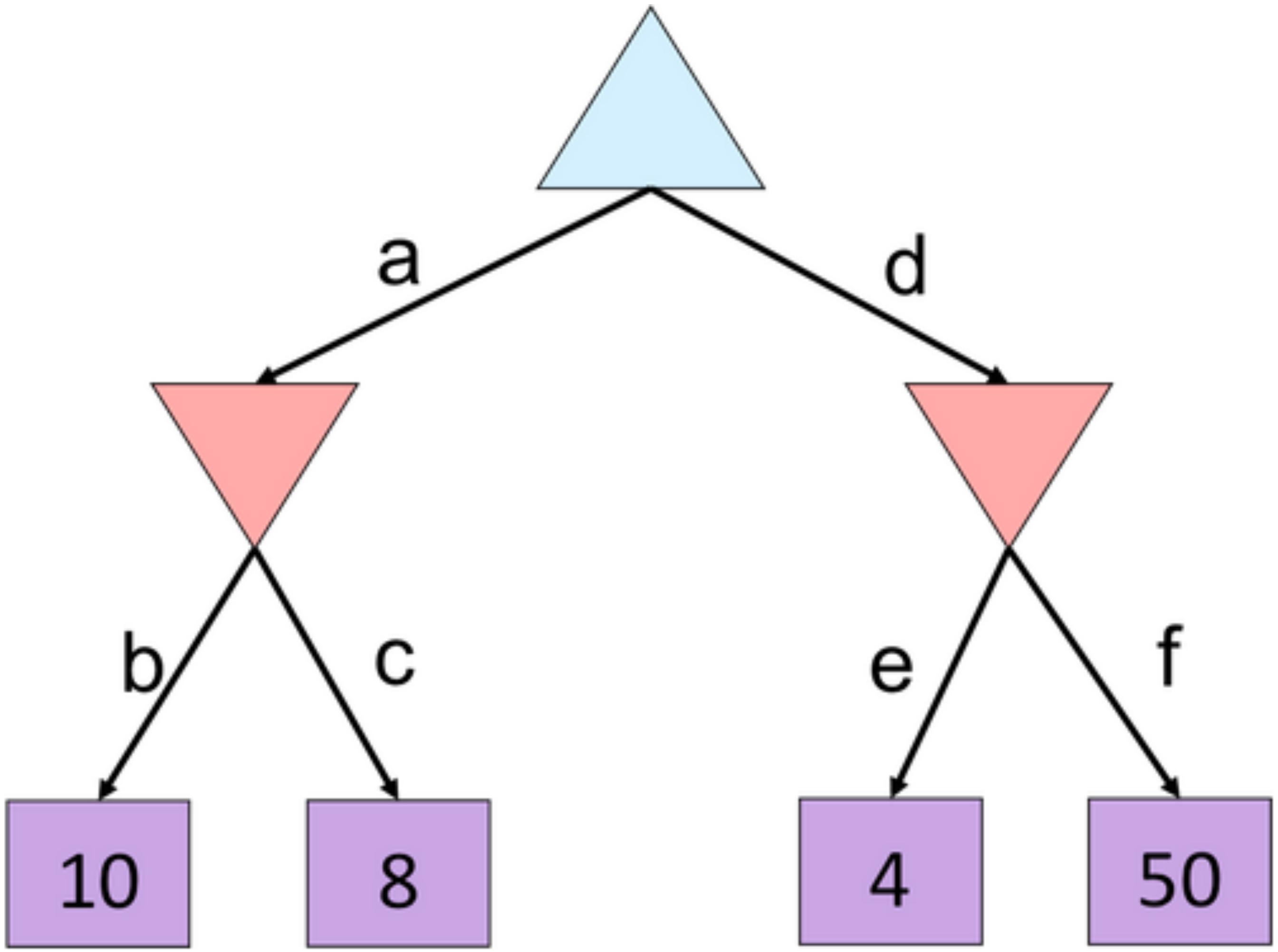


Alpha-Beta Pruning: General Idea

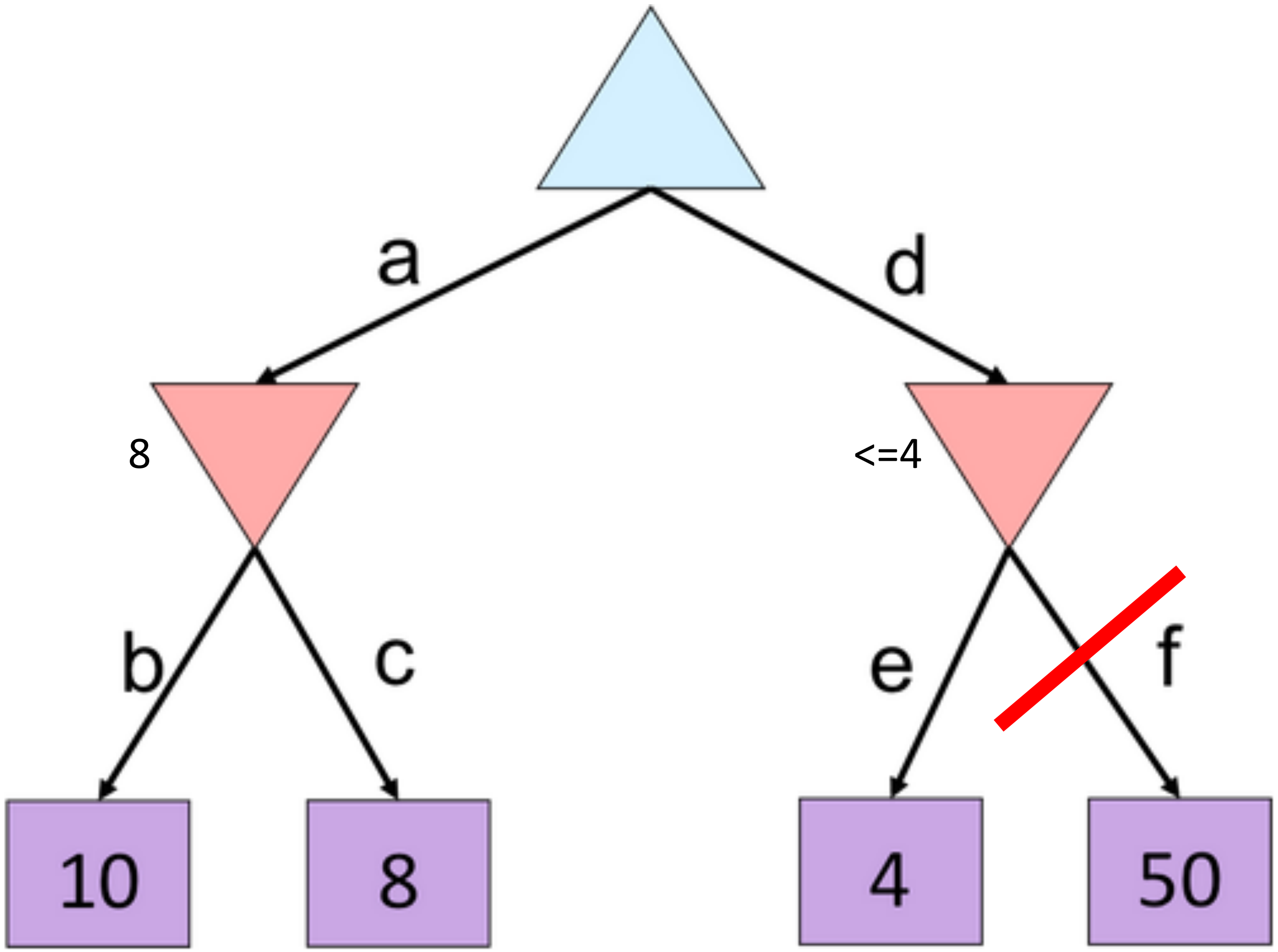
- **General Configuration (MAX version)**
  - Consider computing the MAX-VALUE at some node *n*, examining *n*'s children
  - *n*'s estimate of the childrens' min is increasing.
  - Who can use *n*'s value to make a choice? MIN
  - Let *b* be the lowest (best) value that MIN can get at any choice point along the current path from the root
  - If the value at *n* becomes higher than *b*, MIN will not pick this option, so we can stop considering *n*'s other children (any further exploration of children will only increase the value further)



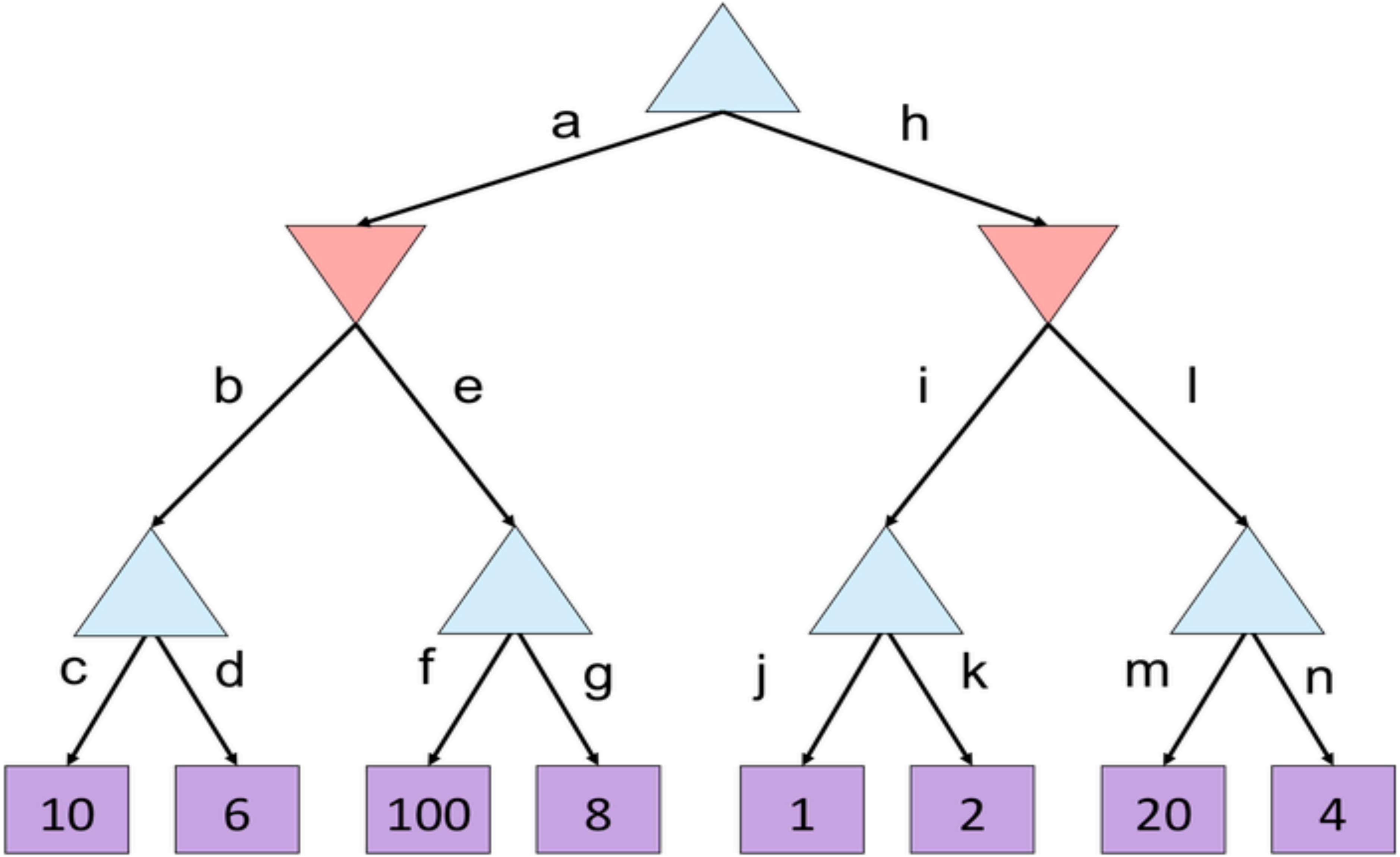
Pruning: Example



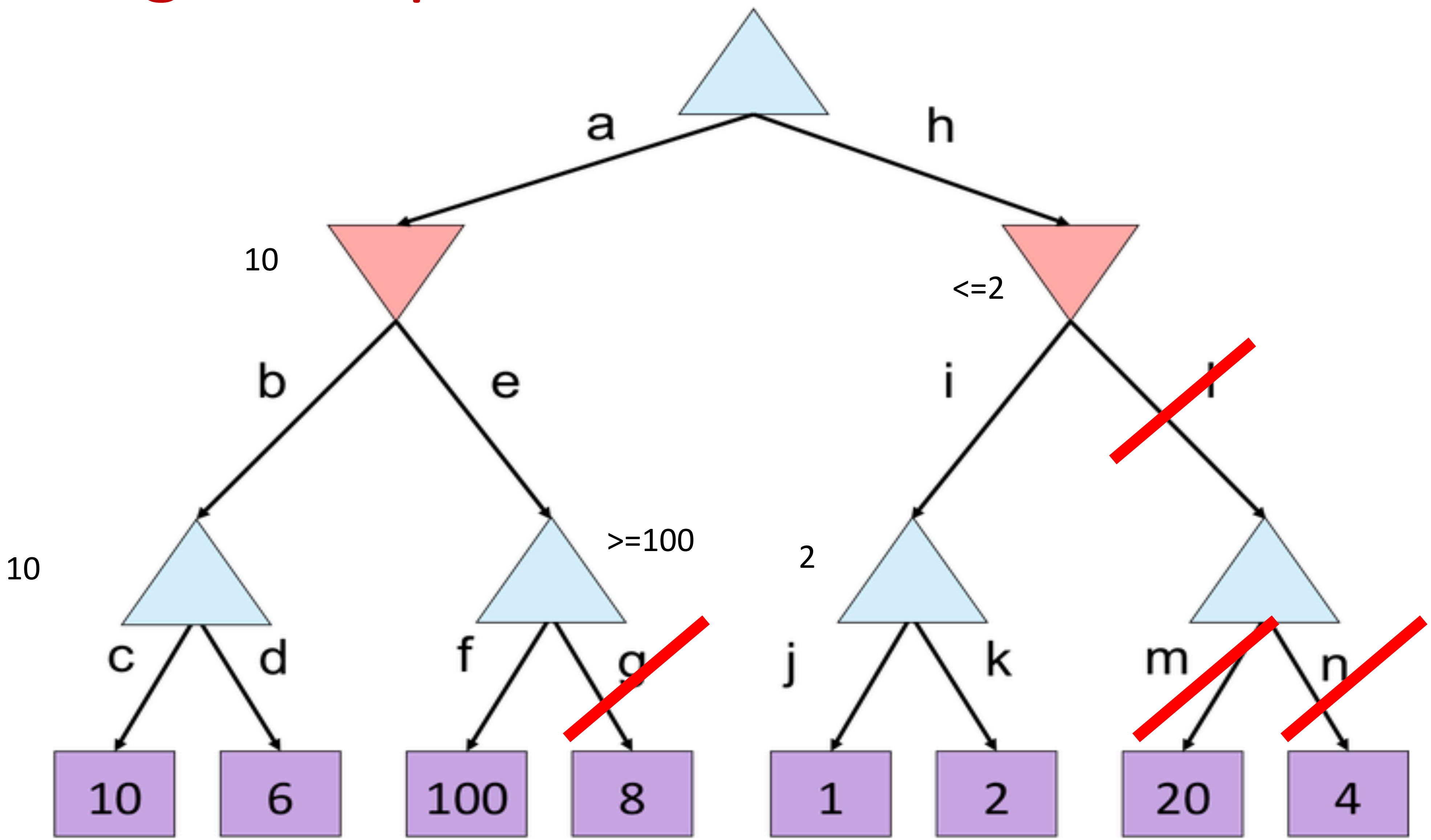
Pruning: Example



Pruning: Example



Pruning: Example



Alpha-Beta Implementation

α: MAX's best option on path to root
β: MIN's best option on path to root

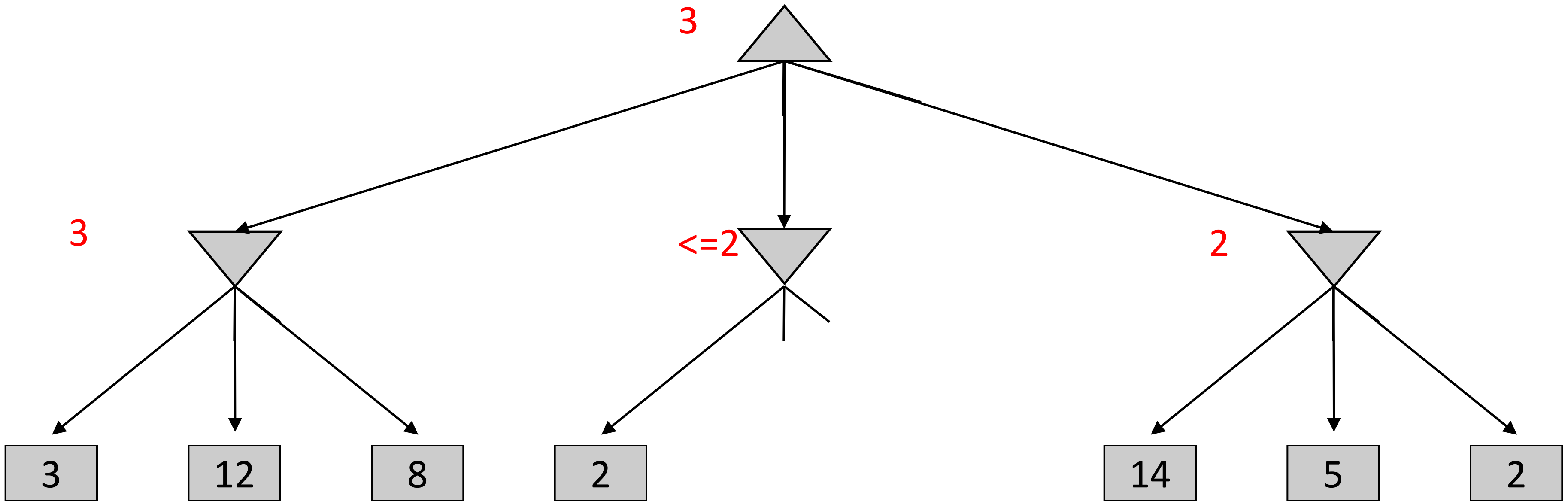
```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α return v
        β = min(β, v)
    return v
```

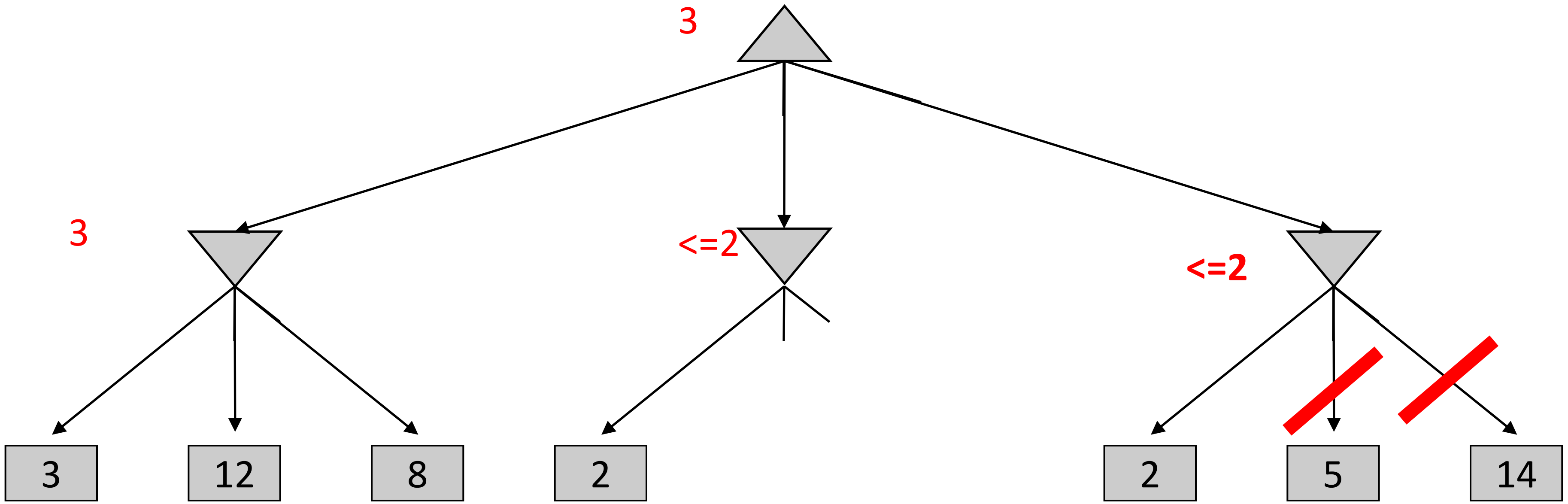
Alpha-Beta Pruning - Properties

1. Pruning has **no effect** on the minimax value at the root.
   - Pruning does not affect the final action selected at the root.
2. A form of **meta-reasoning** (computing what to compute)
   - Eliminates nodes that are irrelevant for the final decision.

Alpha-Beta Pruning – Order of nodes matters



Alpha-Beta Pruning – Order of nodes matters



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2. A form of **meta-reasoning** (computing what to compute)
    - Eliminates nodes that are irrelevant for the final decision.
3. The alpha-beta search cuts the largest amount off the tree when we examine the **best move first**
    - However, best moves are typically **not** known. Need to make estimates.

Alpha-Beta Pruning – Order of nodes matters

```
                4                        MAX
       +--------------+--------------+
       2              3              4          MIN
   +----+----+    +----+----+    +----+----+
   6    4    2    7    5    3    8    6    4     MAX
 +--+ +--+ +--+ +-+-+ +--+ +--+ +--+ +--+ +--+--+
 6  5 4  3 2 1 1 3 7  4  5 2  3 8  2 1 6 1  2  4
```

If the nodes were indeed encountered as "worst moves first" – then no pruning is possible

```
                4                        MAX
       +--------------+--------------+
       4              3              2          MIN
   +----+----+    +----+----+    +----+----+
   4    6    8    3    x    x    2    x    x     MAX
 +--+ +--+ +--+ +--+              +-+-+
 4  2 6  x 8  x 3  2              1 2 1
```

If the nodes were encountered as "best moves first" – then pruning is possible

Note: In reality, we don't know the ordering.

Slide adapted from Prof. Mausam

Alpha-Beta Pruning - Properties

1. Pruning has **no effect** on the minimax value at the root.
    - Pruning does not affect the final action selected at the root.
2. A form of **meta-reasoning** (computing what to compute)
    - Eliminates nodes that are irrelevant for the final decision.
3. The alpha-beta search cuts the largest amount off the tree when we examine the **best move first**
    - Problem: However, best moves are typically **not** known.
    - Solution: Perform iterative deepening search and evaluate the states.
4. Time Complexity
    - **Best ordering - $O(b^{m/2})$.** Can double the search depth for the same resources.
    - On average – $O(b^{3m/4})$ if we expect to find the min or max after b/2 expansions.

Minimax for Chess

- Chess:
  - branching factor $b \approx 35$
  - game length $m \approx 100$
  - search space $b^m \approx 35^{100} \approx 10^{154}$
- The Universe:
  - number of atoms $\approx 10^{78}$
  - age $\approx 10^{18}$ seconds
  - 10^8 moves/sec x 10^{78} x 10^{18} = 10^{104}

Alpha-Beta for Chess

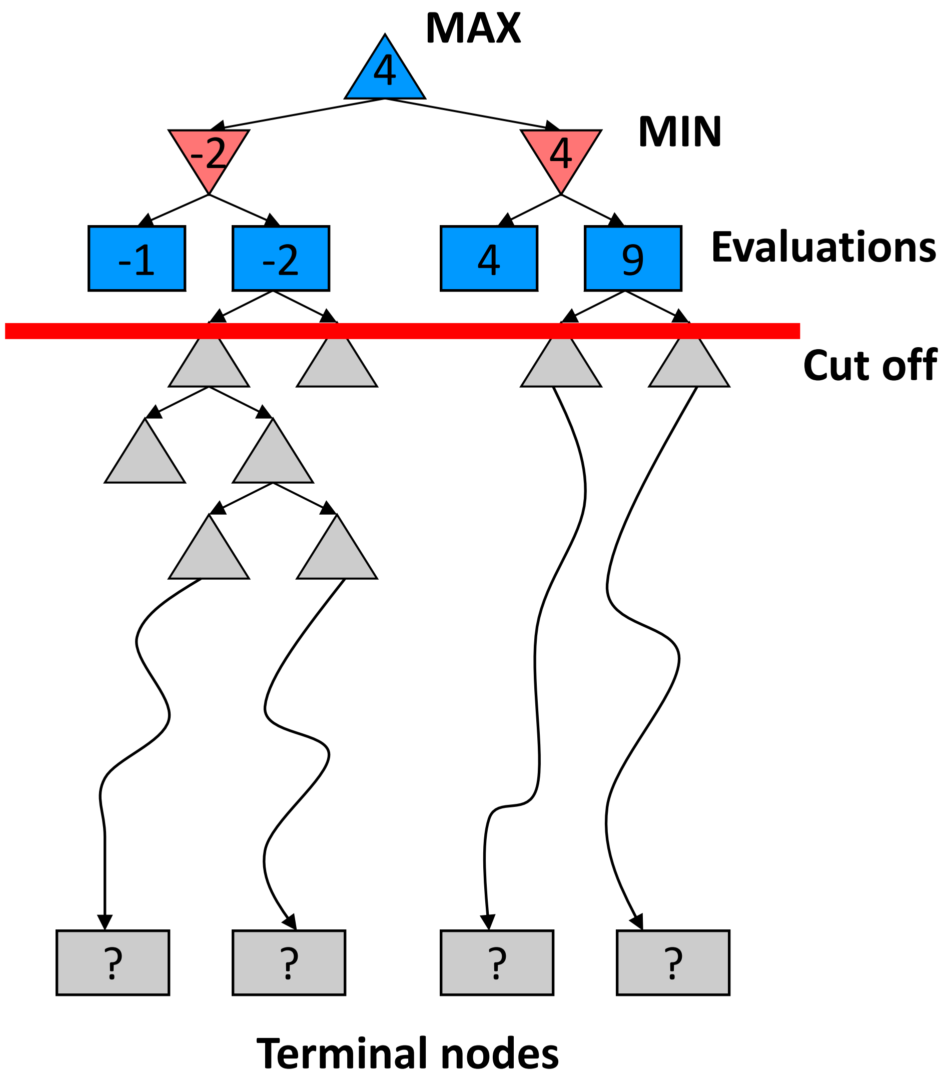
- Chess:
  - branching factor $b \approx 35$
  - game length $m \approx 100$
  - search space $b^{m/2} \approx 35^{50} \approx 10^{77}$

Cutting-off Search

- Problem (Resource costraint):
  - Minimax search: full tree till the terminal nodes.
  - Alpha-beta prunes the tree but still searches till the terminal nodes.
  - We can’t search till the terminal nodes.

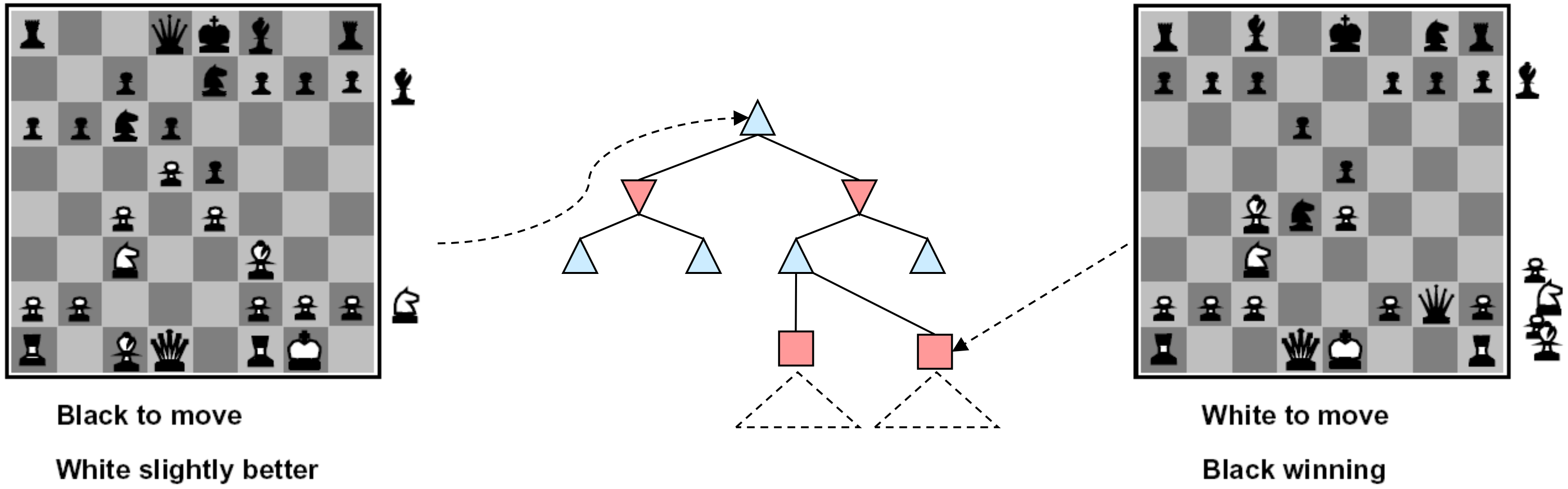
- Solution:
  - Depth-limited Search (H-Minimax)
  - Search only to a limited depth (cutoff) in the tree
  - **Replace the terminal utilities with an evaluation function for non-terminal positions.**

$$
\text{H-MINIMAX}(s,d) = \begin{cases} \boxed{\text{EVAL}(s) \qquad \text{if CUTOFF-TEST}(s,d)} \\ \max_{a \in Actions(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if PLAYER}(s) = \text{MAX} \\ \min_{a \in Actions(s)} \text{H-MINIMAX}(\text{RESULT}(s,a), d+1) & \text{if PLAYER}(s) = \text{MIN}. \end{cases}
$$



Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search.
- Estimate the chances of winning.



- Ideal function: returns the actual **minimax** value of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

- e.g. $f_i(s)$ = (number of pieces of type i), each weight w_i etc.

Evaluation Functions and Alpha-Beta

- Evaluation functions are always imperfect.

- Value at a min-node will only keep going down. Once value of min-node lower than better option for max along path to root, can prune

- Evaluation function as a guidance for pruning
  - IF evaluation function provides upper-bound on value at min-node, and upper-bound already lower than better option for max along path to root THEN can prune

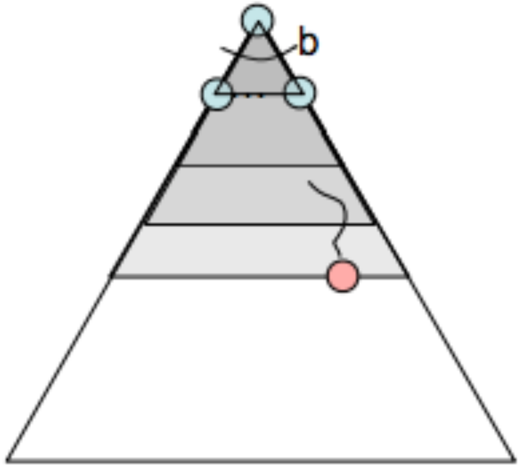
Determining “good” node orderings

- The ordering of nodes helps alpha-beta pruning.
  - Worst ordering $O(b^m)$. Best ordering $O(b^{m/2})$.

```
                4                    MAX
      +---------------+---------------+
      4               3              2         MIN
   +----+----+    +----+----+    +----+----+
   4    6    8    3    x    x    2    x    x    MAX
 +--+ +--+ +--+ +--+             +-+-+
 4 2  6 x  8 x  3 2              1 2 1
```

- How to find good orderings
  - Problem: we only know them when we evaluate the nodes.

- One approach – iterative deepening to determine evaluations for nodes
  - What if we can do iterative deepening to a certain depth. Use the evaluation function at the set depth and then compute the values for the nodes in the tree that is generated.
  - Next time, use the evaluations of the previous search to order the nodes. Use them for pruning.
  - Use evaluations of the previous search for order.

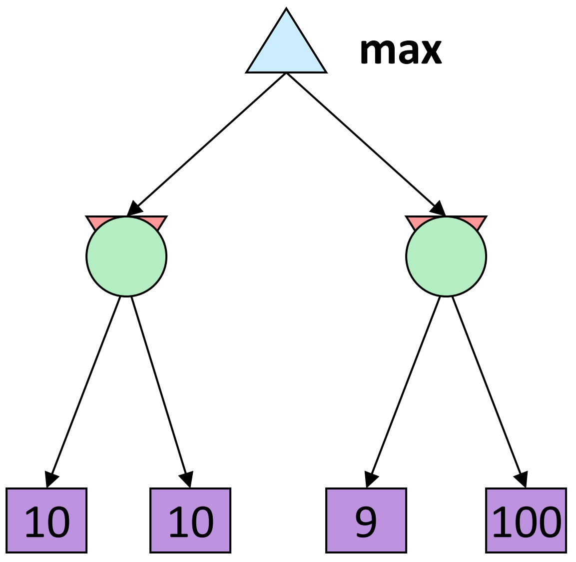


Incorporating Chance: Expectimax Search

- When the result of an action is not known.

- Incorporate a notion of chance
  - Include chance nodes
    - Unpredictable opponents: the ghosts move randomly in Pacman.
    - Explicit randomness: rolling dice by a player in a game.

- Expectimax search:
  - At chance nodes the outcome is uncertain
  - Calculate the ***expected utilities:*** weighted average (expectation) of children



Expectimax Search

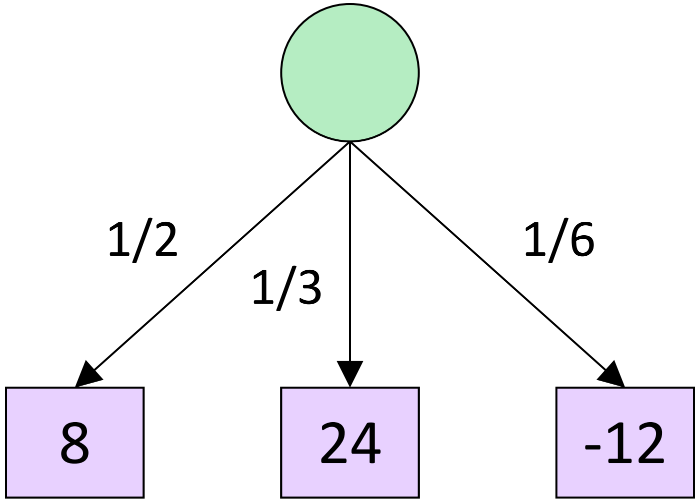
```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

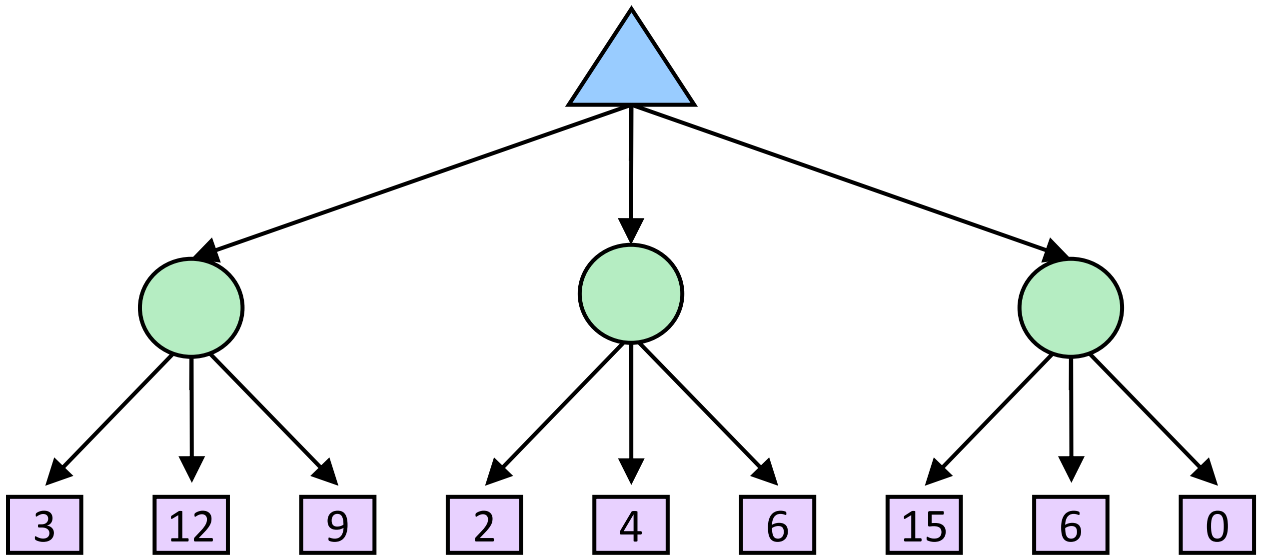
Expectimax Search

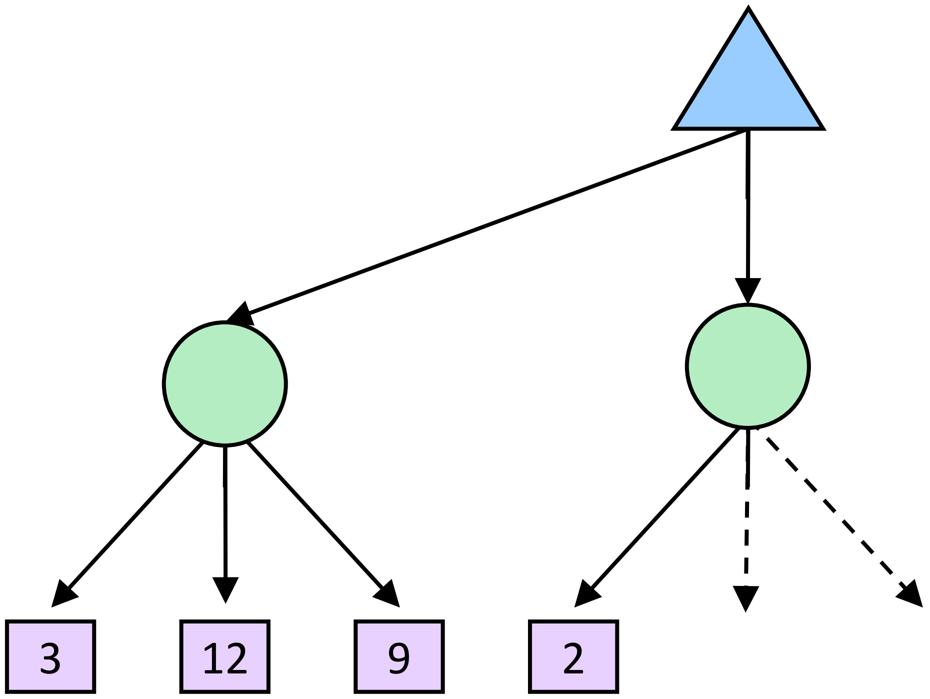
```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```



$$v = (1/2)\ (8) + (1/3)\ (24) + (1/6)\ (-12) = 10$$

Expectimax Search

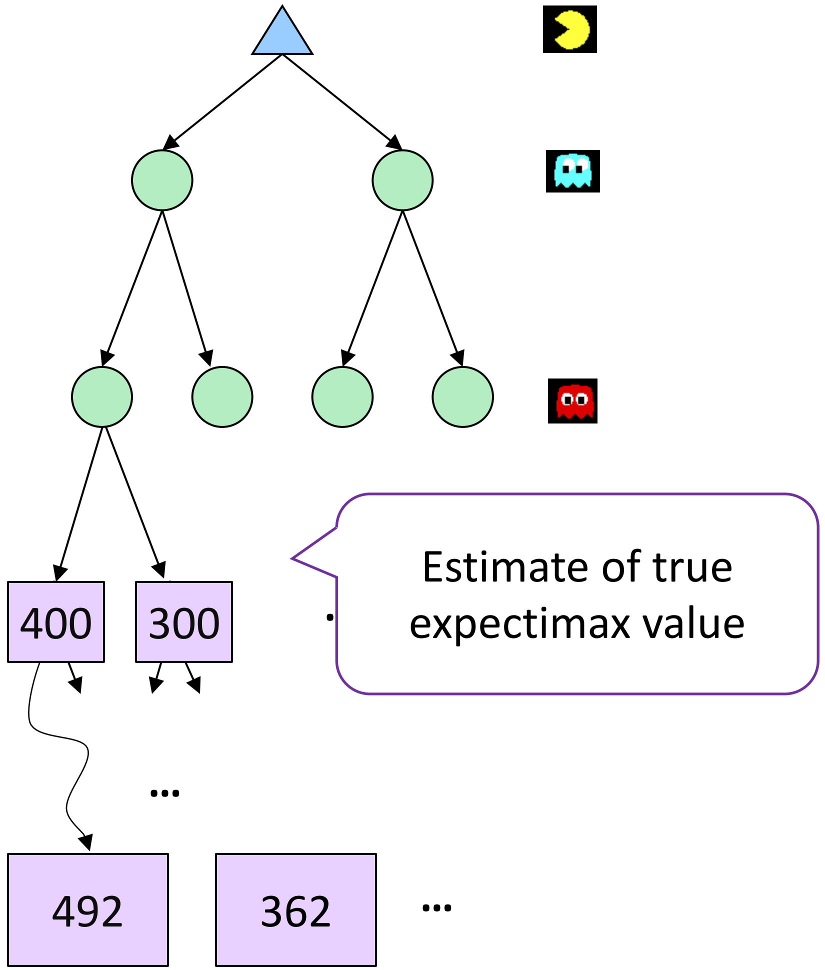




Can we perform pruning?

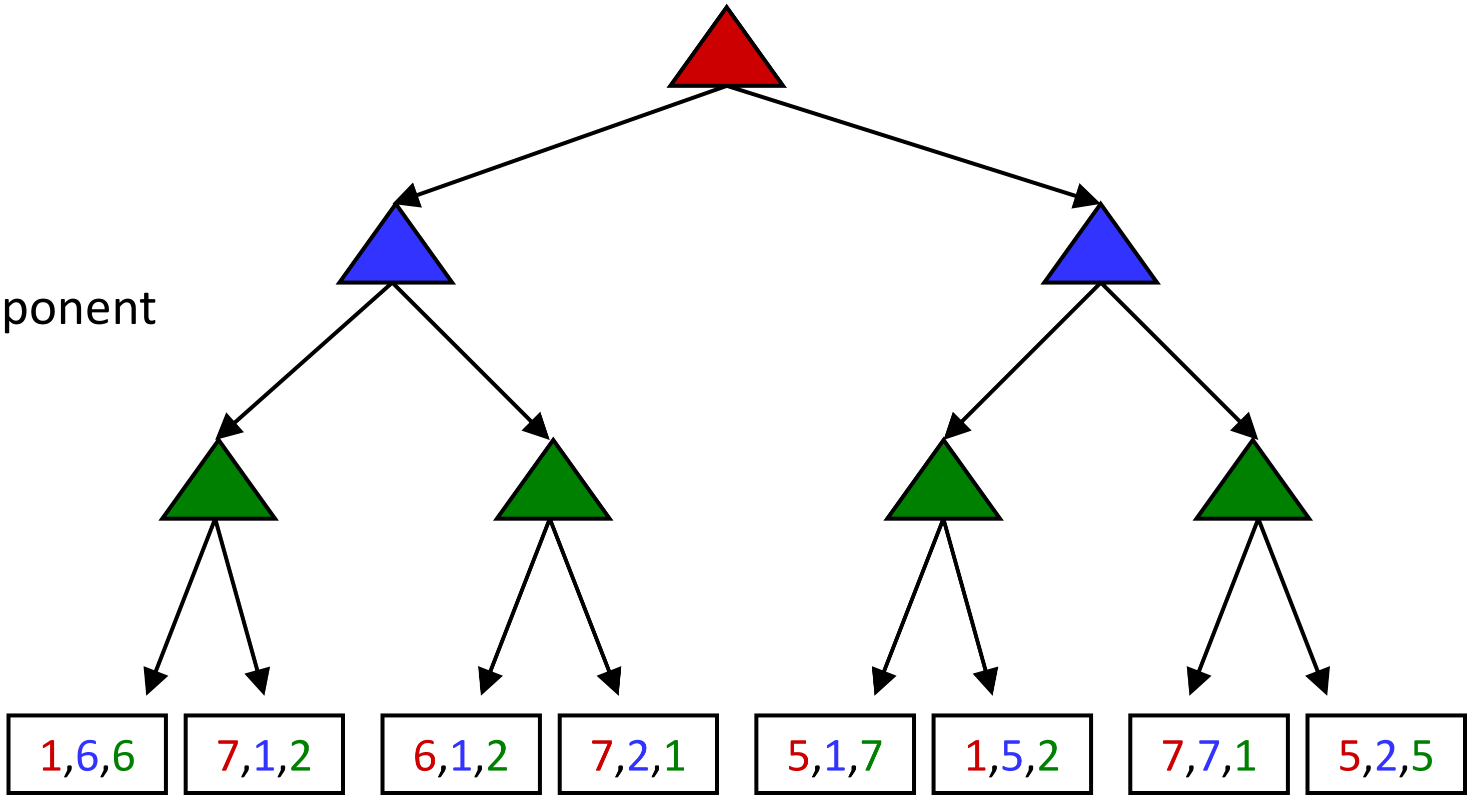
Depth-Limited Expectimax

- Depth-limit can be applied in Expectimax search.
- Use heuristics to estimate the values at the depth limit.

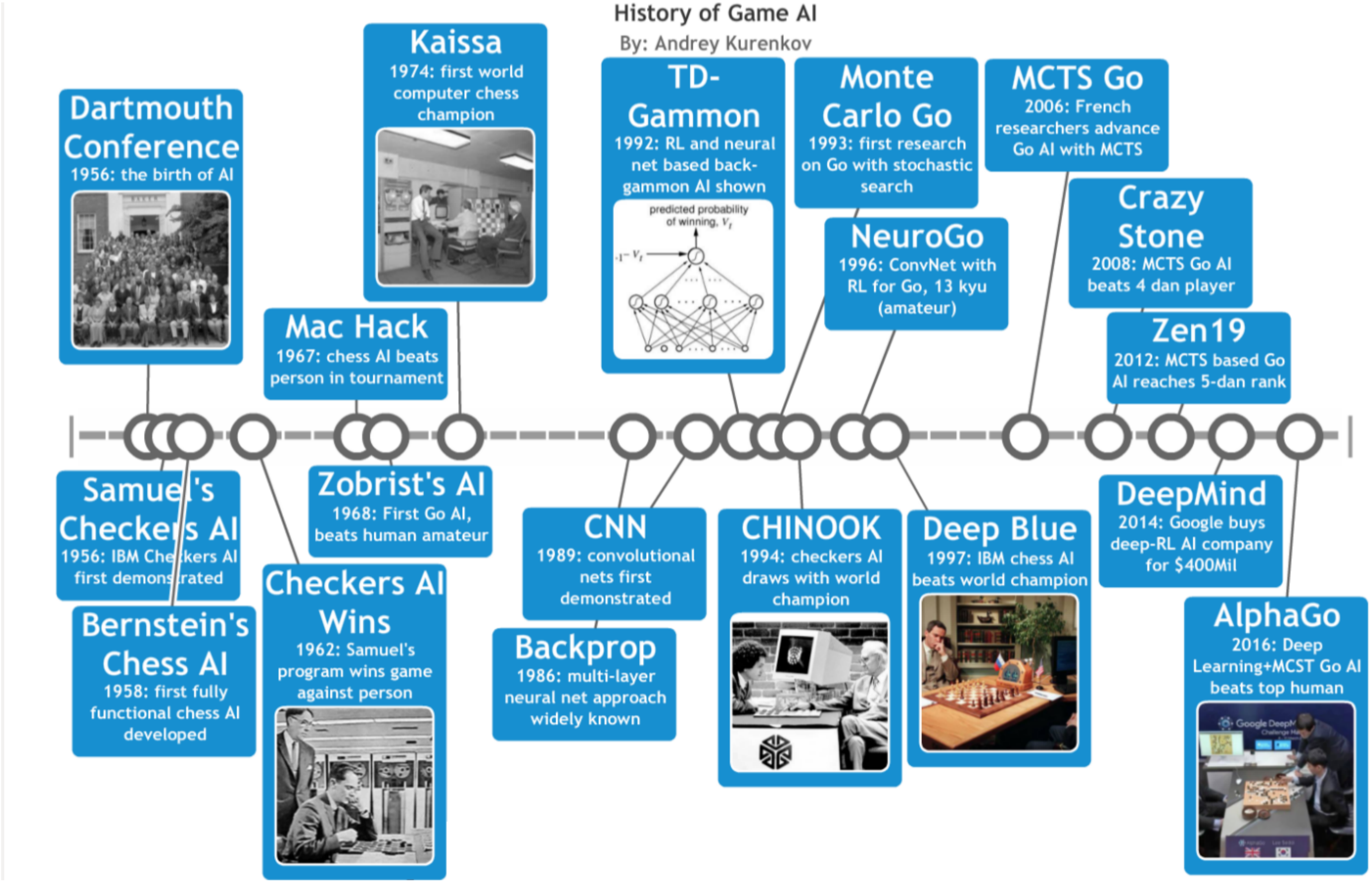


Multiple players and other games

- Other games: non zero-sum, or multiple players
- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component



“Games are to AI as grand prix is to automobile design”
Games viewed as an indicator of intelligence.



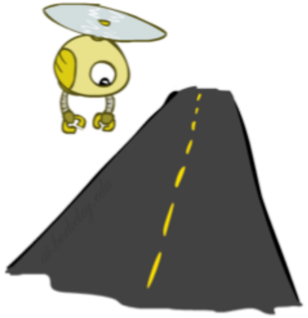
Probabilities (Recap)

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

- Example: Traffic on freeway
  - Random variable: T = whether there's traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

- Some laws of probability:
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one

- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
  - Methods for reasoning and updating probabilities later.



0.25

0.50

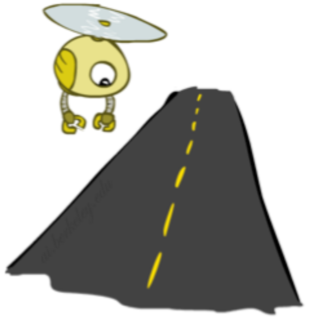
0.25

Expectations (Recap)

- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes

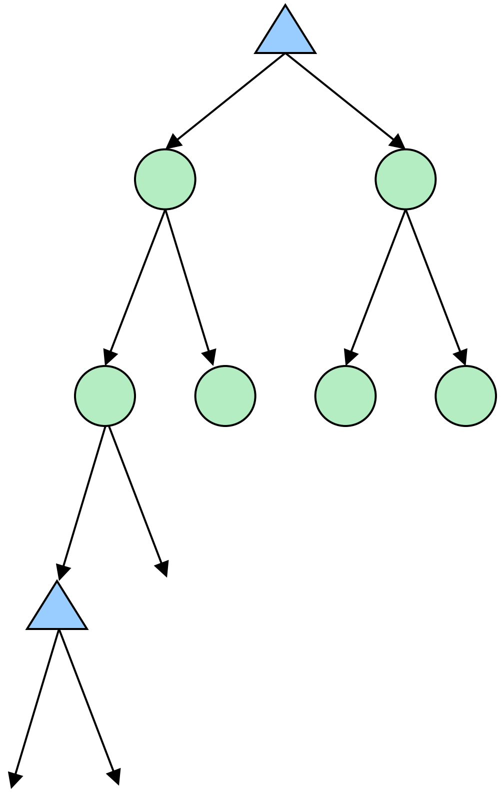
- Example: How long to get to the airport?

Time:	20 min		30 min		60 min		
	x	+	x	+	x	⇨	35 min
Probability:	0.25		0.50		0.25		



Probabilities for Expectimax

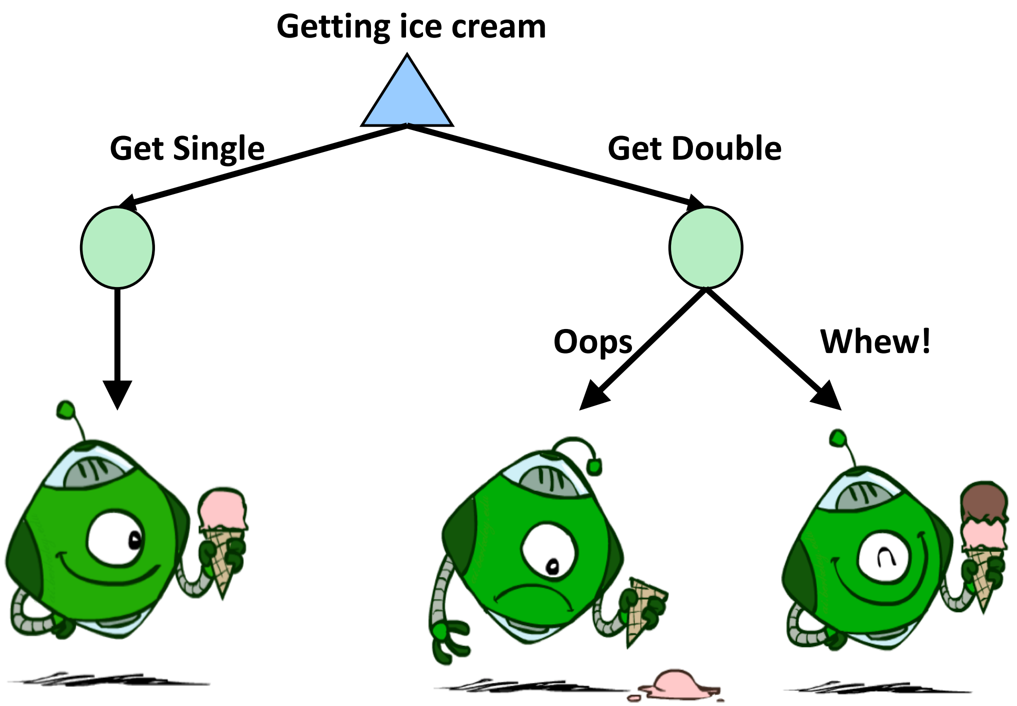
- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation. The model might say that adversarial actions are likely.
- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes (later formal ways).



Utilities and Decision-making

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences

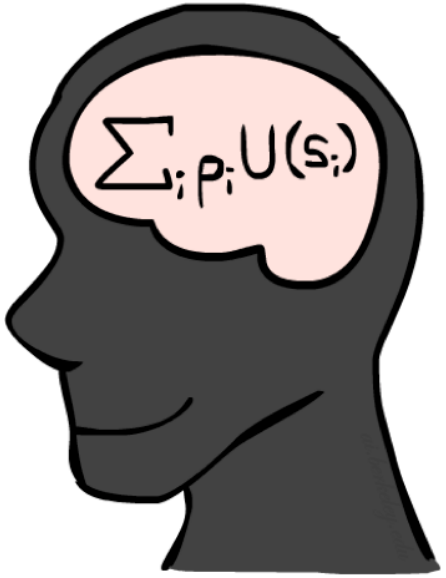
- Providing utilities
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals



- We specify the utilities for a task, let the behaviour emerge from the action.

Maximum Expected Utility

- Maximum expected utility (MEU) principle:
  - Choose the action that maximizes expected utility
  - The agent can be in several states, each with a probability distribution. Utilities map states to a value. Compute the expectation.
- We try to build models that maximize the expected utility.



$$U([p_1, S_1;\ \ldots\ ;\ p_n, S_n]) = \sum_i p_i U(S_i)$$