Fuzzy Logic

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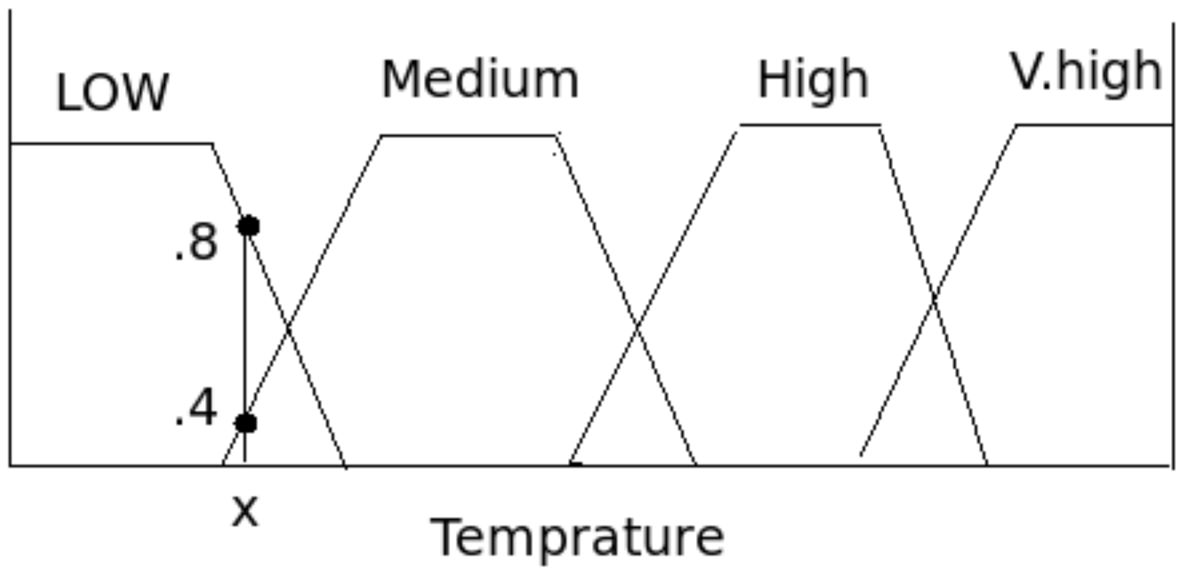
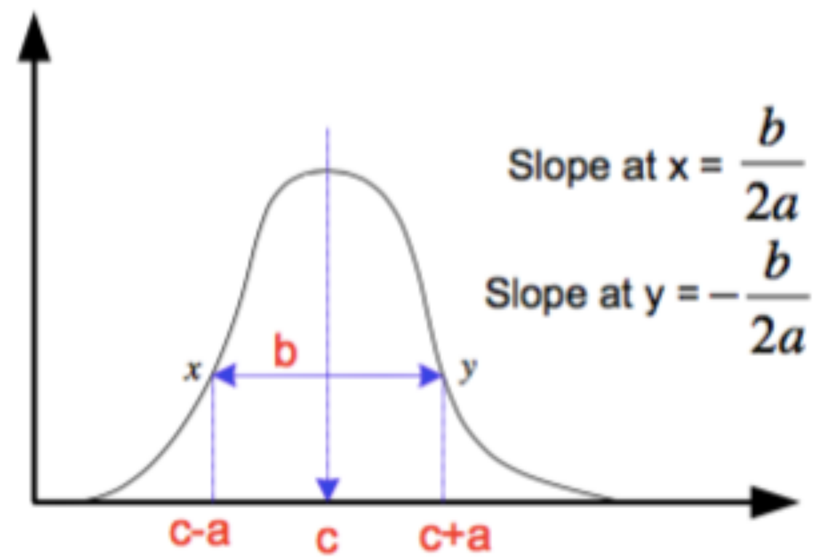
Membership Function



Membership Function

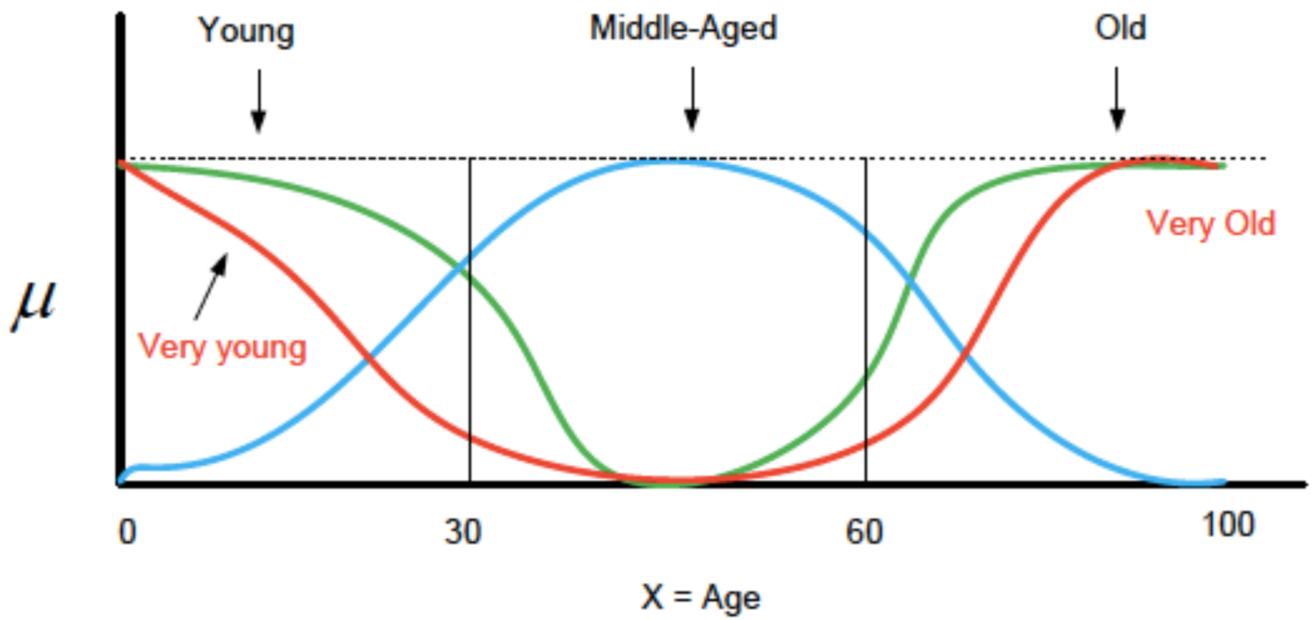
Bell (Cauchy) MF

$$\text{bell}(x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$



Membership Function

Linguistic variables and values



$$\mu_{young}(x) = bell(x, 20, 2, 0) = \frac{1}{1+(\frac{x}{20})^4}$$

$$\mu_{old}(x) = bell(x, 30, 3, 100) = \frac{1}{1+(\frac{x-100}{30})^6}$$

$$\mu_{middle-aged} = bell(x, 30, 60, 50)$$

Fuzzy Operations

Caretsian Product ($A \times B$):

$$\mu_{A \times B}(x, y) = min\{\mu_A(x), \mu_B(y)\}$$

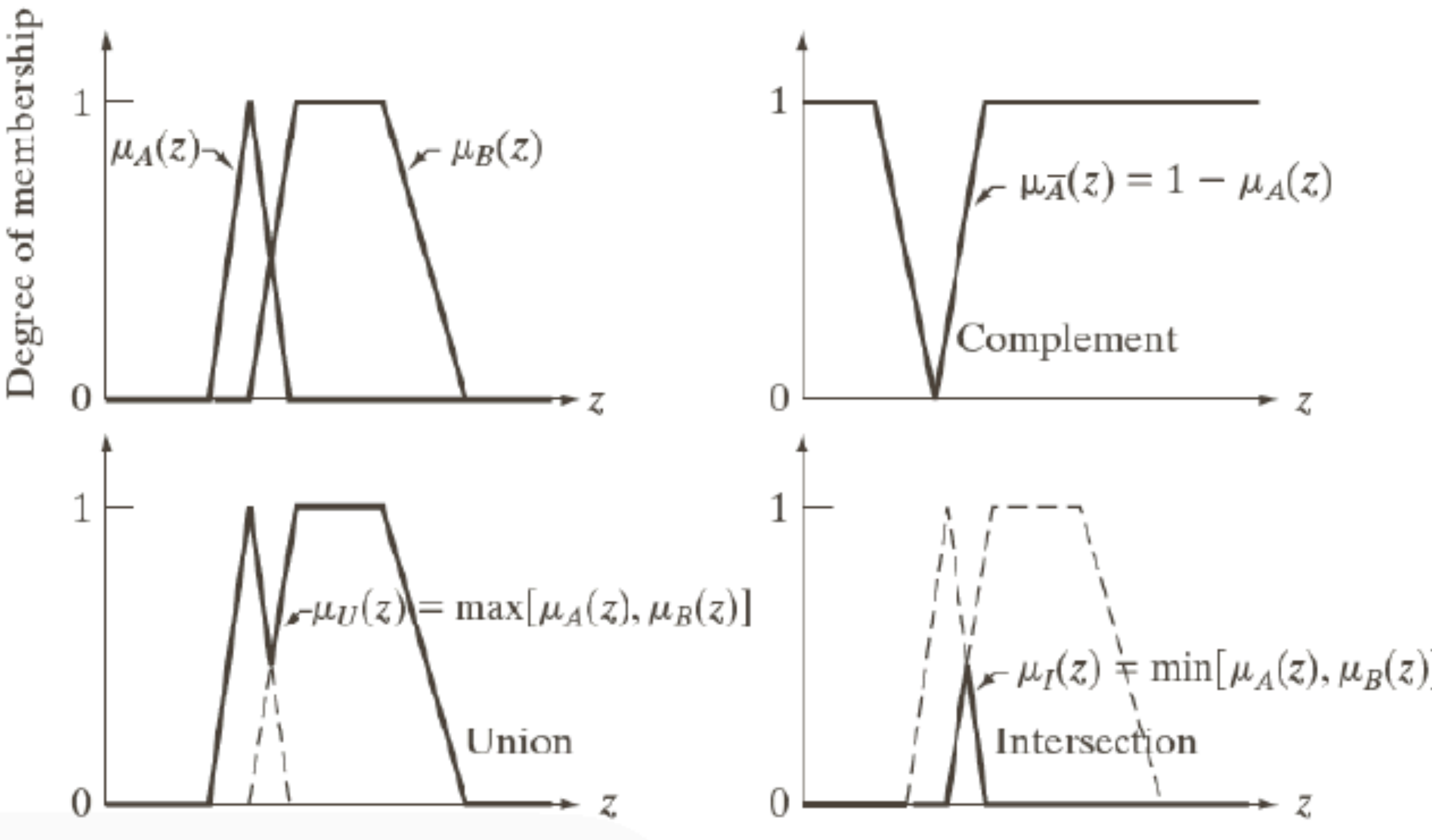
Example 3:

$A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5), (x_4, 0.6)\}$

$B(y) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$

$$A \times B = \min\{\mu_A(x), \mu_B(y)\} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0.3 \\ 0.6 & 0.6 & 0.3 \end{bmatrix} \end{array}$$

Fuzzy Operations



Fuzzy Relation

- A fuzzy relation $\tilde{R}$ is a mapping from the Cartesian space X x Y to the interval [0,1], where the strength of the mapping is expressed by the membership function of the relation $\mu_{\tilde{R}}(x,y)$

- The "strength" of the relation between ordered pairs of the two universes is measured with a membership function expressing various "degree" of strength [0,1]

Fuzzy Relation

Fuzzy Composition: Example (max-min)

$$X = \{x_1, x_2\}, \quad Y = \{y_1, y_2\}, \text{ and } \quad Z = \{z_1, z_2, z_3\}$$

Consider the following fuzzy relations:

$$\tilde{R} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{matrix} y_1 & y_2 \end{matrix} \\ \begin{bmatrix} 0.7 & 0.5 \\ 0.8 & 0.4 \end{bmatrix} \end{array} \quad \text{and} \quad \tilde{S} = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{matrix} z_1 & z_2 & z_3 \end{matrix} \\ \begin{bmatrix} 0.9 & 0.6 & 0.5 \\ 0.1 & 0.7 & 0.5 \end{bmatrix} \end{array}$$

Using max-min composition,

$$\left.\begin{aligned} \mu_{\tilde{T}}(x_1, z_1) &= \bigvee_{y \in Y} (\mu_{\tilde{R}}(x_1, y) \wedge \mu_{\tilde{S}}(y, z_1)) \\ &= \max[\min(0.7, 0.9), \min(0.5, 0.1)] \\ &= 0.7 \end{aligned}\right\} \quad \tilde{T} = \begin{array}{c} \\ x_1 \\ x_2 \end{array} \begin{array}{c} \begin{matrix} z_1 & z_2 & z_3 \end{matrix} \\ \begin{bmatrix} 0.7 & 0.6 & 0.5 \\ 0.8 & 0.6 & 0.4 \end{bmatrix} \end{array}$$

Fuzzy Relation

Problem: In computer engineering, different logic families are often compared on the basis of their power-delay product. Consider the fuzzy set $\tilde{F}$ of logic families, the fuzzy set D of delay times(ns), and the fuzzy set P of power dissipations (mw).

If $\tilde{F} = \{NMOS, CMOS, TTL, ECL, JJ\}$,

$\tilde{D} = \{0.1, 1, 10, 100\}$,

$\tilde{P} = \{0.01, 0.1, 1, 10, 100\}$

Suppose $\tilde{R}_1 = \tilde{D} \times \tilde{F}$ and $\tilde{R}_2 = \tilde{F} \times \tilde{P}$

$$\tilde{R}_1 = \begin{array}{c} \\ 0.1 \\ 1 \\ 10 \\ 100 \end{array} \begin{array}{c} \begin{array}{ccccc} N & C & T & E & J \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & .6 & 1 \\ 0 & .1 & .5 & 1 & 0 \\ .4 & 1 & 1 & 0 & 0 \\ 1 & .2 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad \text{and} \quad \tilde{R}_2 = \begin{array}{c} \\ N \\ C \\ T \\ E \\ J \end{array} \begin{array}{c} \begin{array}{ccccc} .01 & .1 & 1 & 10 & 100 \end{array} \\ \begin{bmatrix} 0 & .4 & 1 & .3 & 0 \\ .2 & 1 & 0 & 0 & 0 \\ 0 & 0 & .7 & 1 & 0 \\ 0 & 0 & 0 & 1 & .5 \\ 1 & .1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

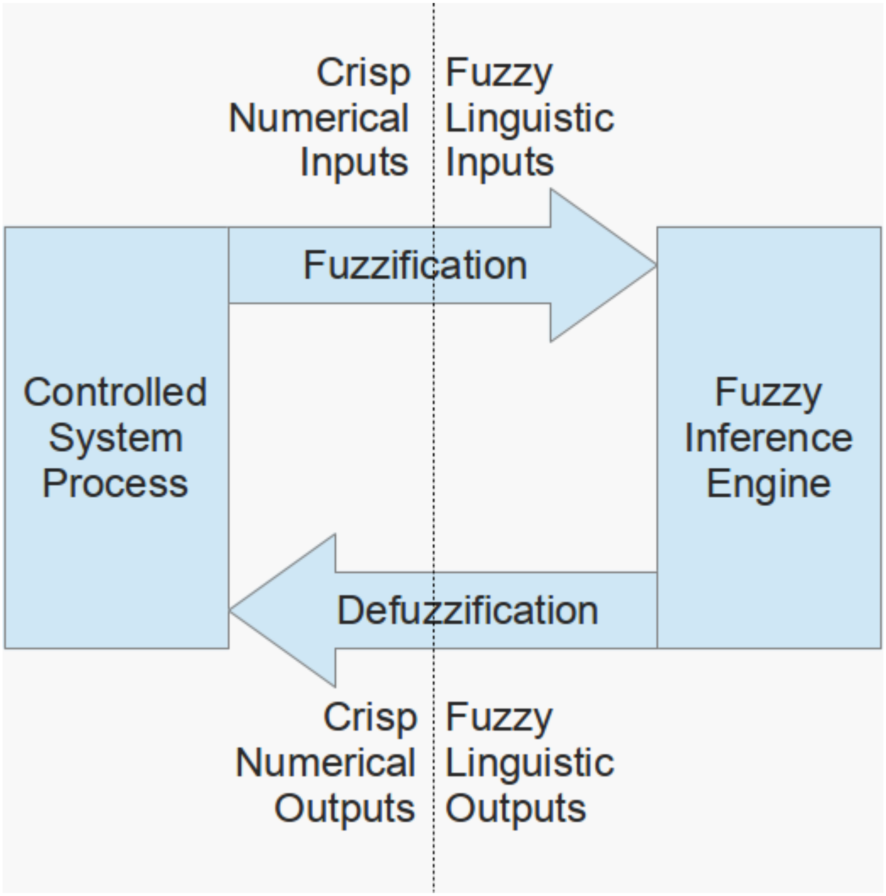
Fuzzy Relation

We can use max-min composition to obtain a relation between delay times and power dissipation: i.e., we can compute or

$$\tilde{R}_3 = \tilde{R}_1 \circ \tilde{R}_2 \qquad \mu_{R_3} = \vee(\mu_{R_1} \wedge \mu_{R_2})$$

$$\tilde{R}_3 = \begin{array}{c} \\ 0.1 \\ 1 \\ 10 \\ 100 \end{array} \begin{array}{c} \begin{array}{ccccc} .01 & .1 & 1 & 10 & 100 \end{array} \\ \begin{bmatrix} 1 & .1 & 0 & .6 & .5 \\ .1 & .1 & .5 & 1 & .5 \\ .2 & 1 & .7 & 1 & 0 \\ .2 & .4 & 1 & .3 & 0 \end{bmatrix} \end{array}$$

Fuzzy System



Fuzzy System

Fuzzification

- To apply fuzzy inference, we need our input to be in linguistic values
- These linguistic values are represented by the degree of membership in the fuzzy sets
- The process of translating the measured numerical values into fuzzy linguistic values is called fuzzification
- In other words, fuzzification is where membership functions are applied, and the degree of membership is determined

Fuzzy System

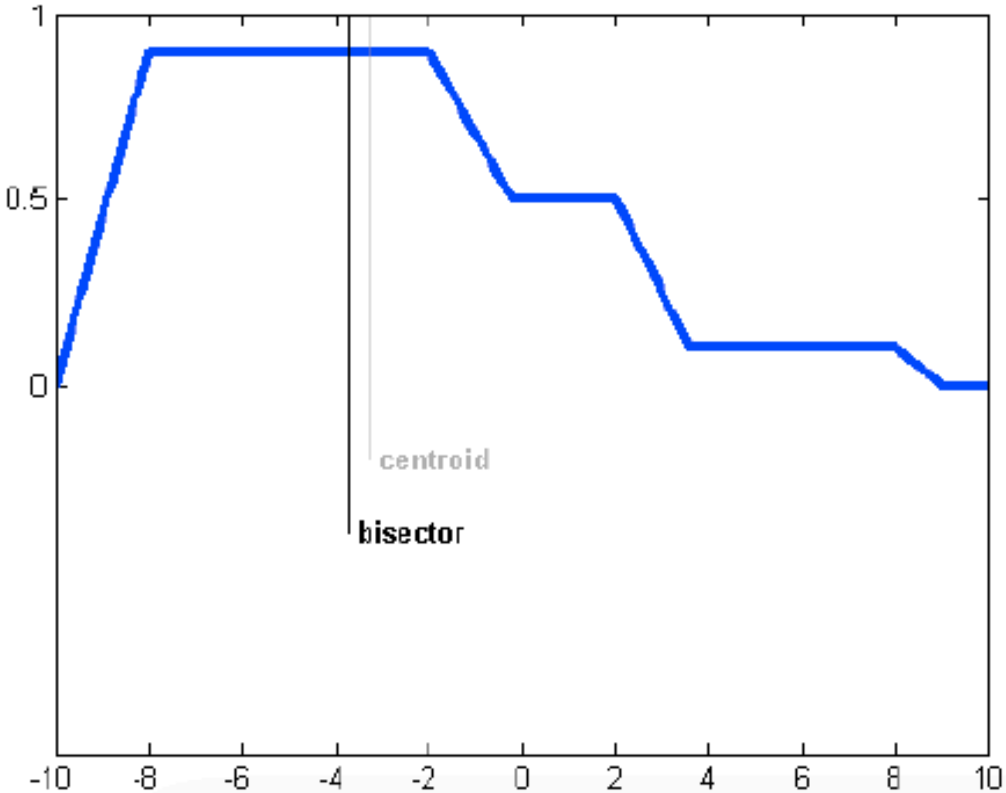
Defuzzification

- Defuzzification is the process of producing a quantifiable result in fuzzy logic
- The fuzzy inference will output a fuzzy result, described in terms of degrees of membership of the fuzzy sets
- Defuzzification interprets the membership degrees in the fuzzy sets into a specific action or real-value

Fuzzy System

Methods of Defuzzification

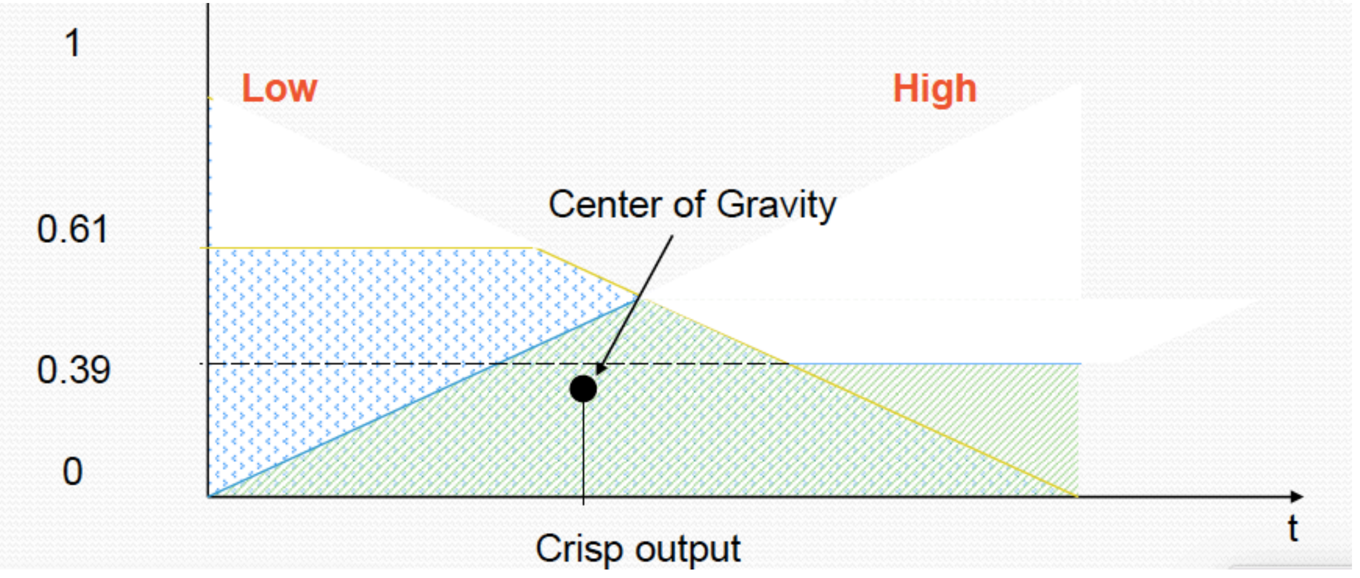
- Two other common methods are:
- Centre of gravity:
  - Calculates the centre of gravity for the area under the curve
- Bisector method:
  - Finds the value where the area on one side of that value is equal to the area on the other side



Fuzzy System

Centre of Gravity

$$C = \frac{\int_{Min}^{Max} t f(t) dt}{\int_{Min}^{Max} f(t) dt}$$

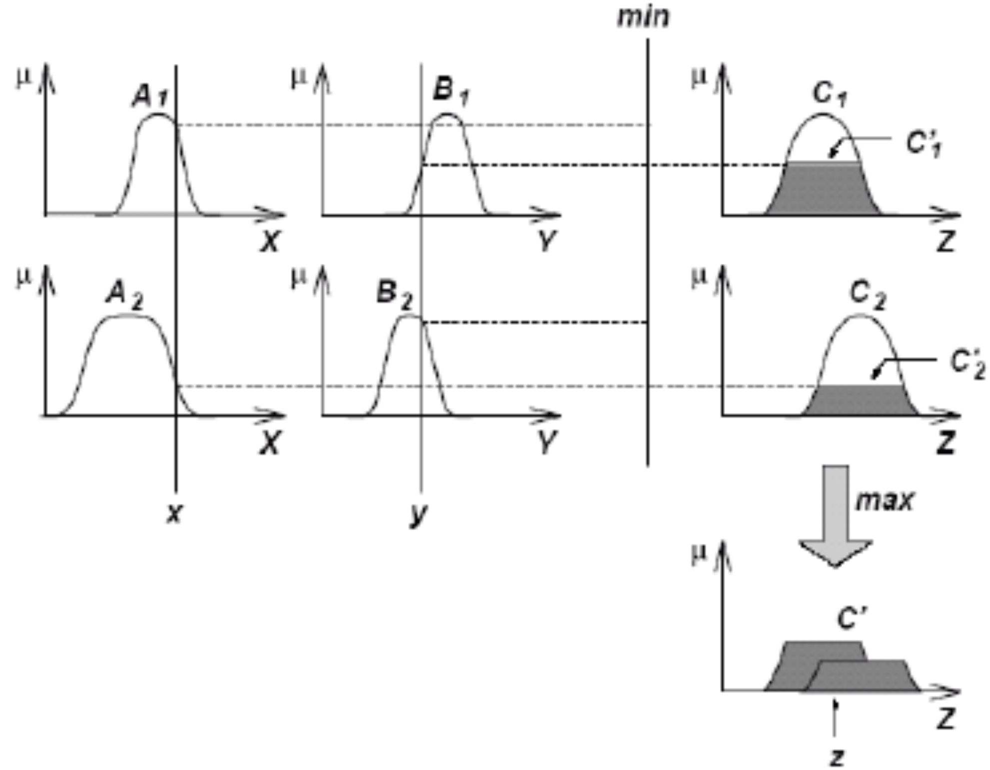


Fuzzy System

Inference Engine: Reasoning Scheme

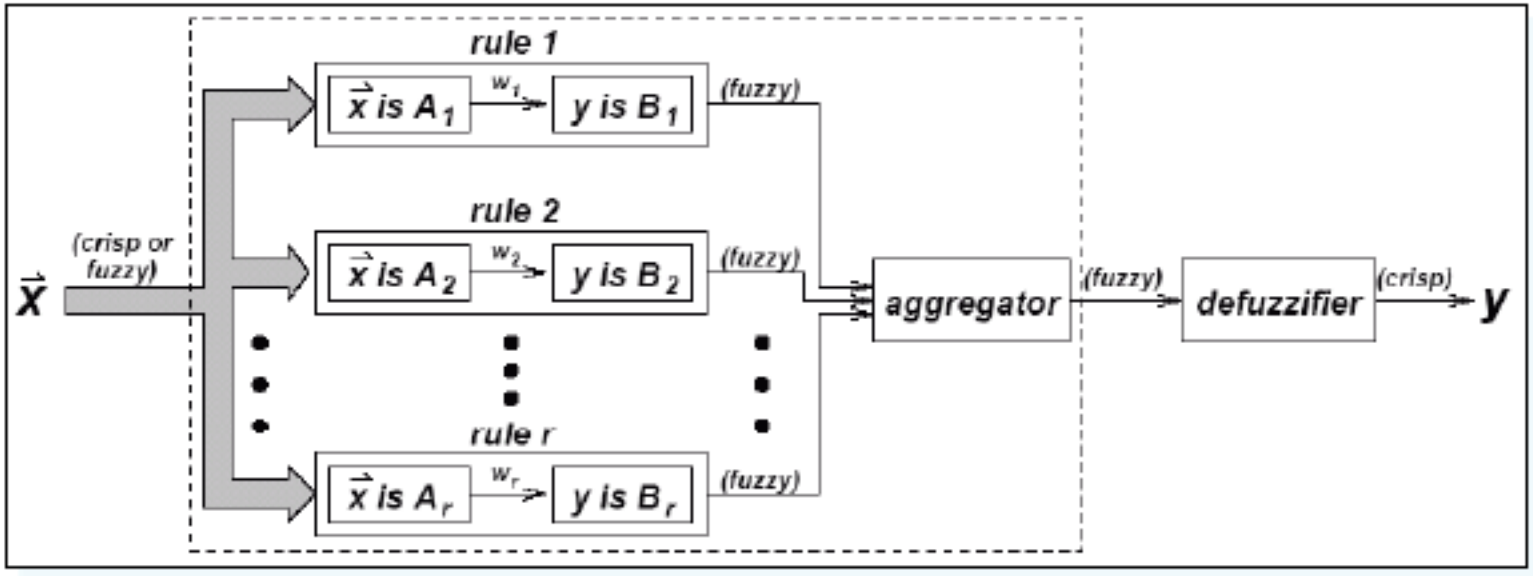
Rules

- If x is A1 and y is B1 then z is C1
- If x is A2 and y is B2 then z is C2



Fuzzy System

Inference Engine: Reasoning Scheme



Applications

Washing Machine (Control)

INPUT:

Load (Quantity)

small, medium, large

Fabric Softness:

Hard, Not so Hard, Soft, Not so soft

OUTPUT: (Wash Cycle)

Light, Normal, Strong

Applications

Washing Machine (Control)

If

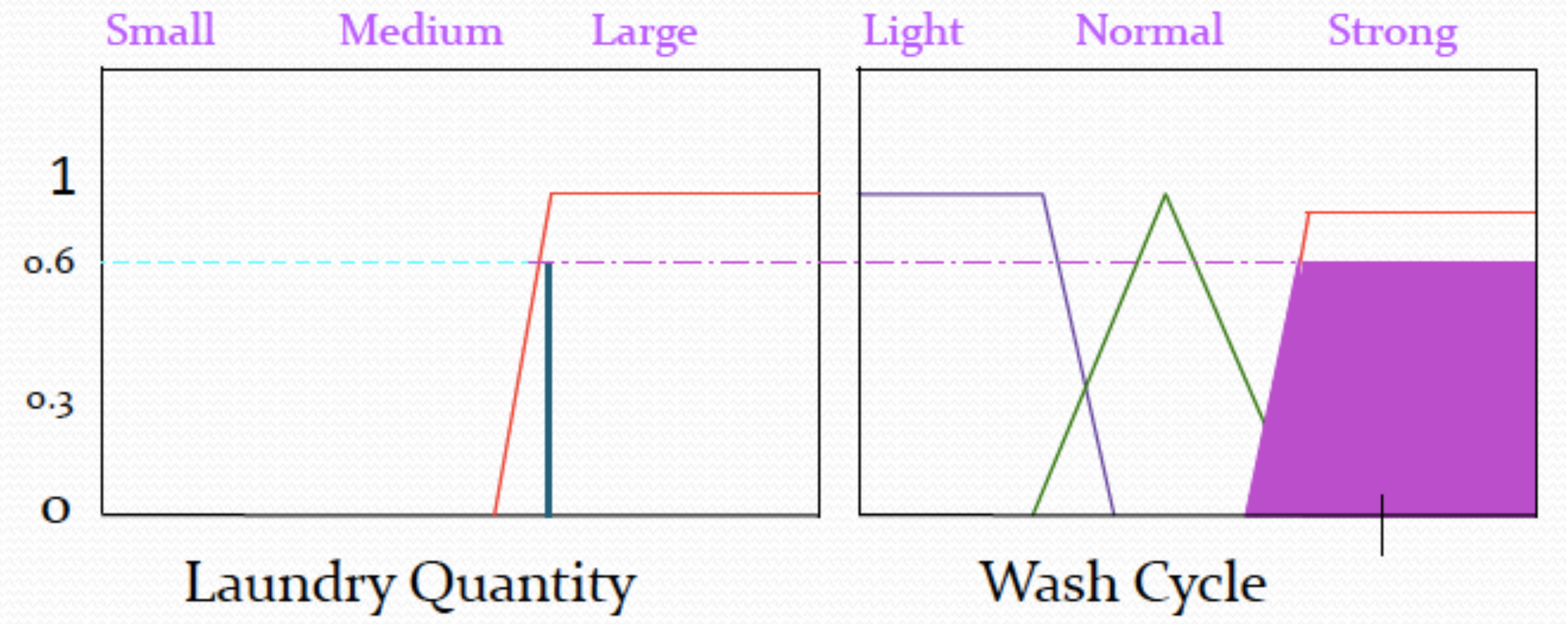
Laundry quantity is large *(Fuzzy)*

then

wash cycle is strong *(Fuzzy)*

Applications

Washing Machine (Control)



If Laundry quantity is large *(Fuzzy)* then wash cycle is strong *(Fuzzy)*

Washing machine needs a NON-fuzzy information.

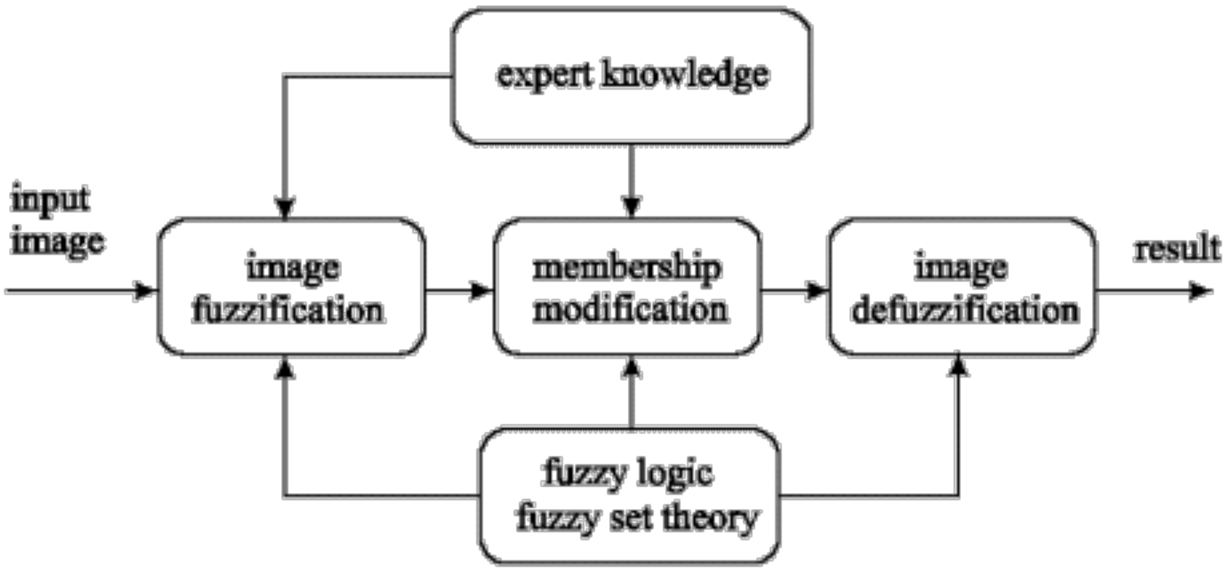
Applications

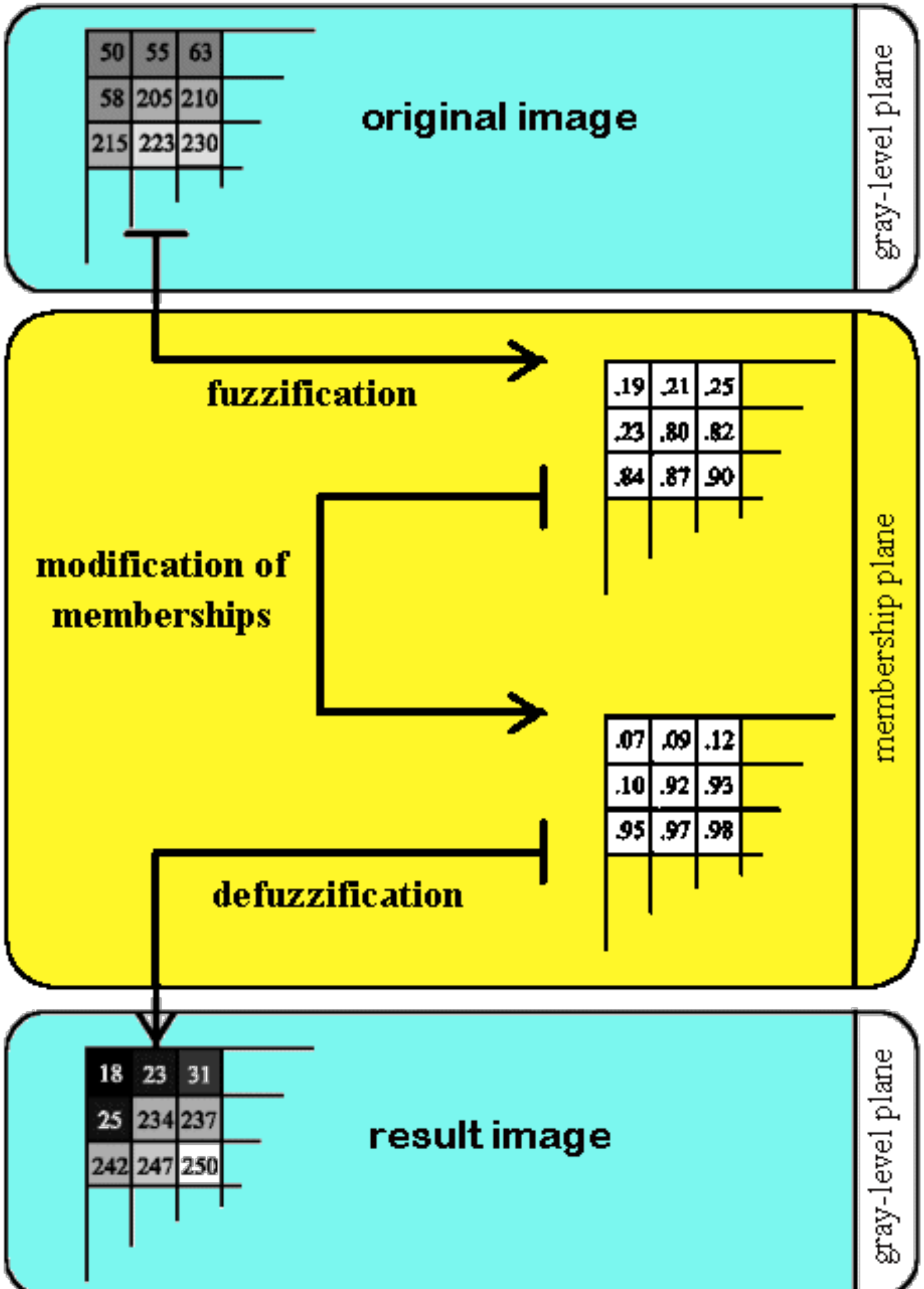
Washing Machine (Control)

Rule 1: If Laundry quantity is LARGE and Laundry softness is HARD then wash cycle is strong.

Rule 2: If Laundry quantity is MEDIUM and Laundry softness is NOT SO HARD then wash cycle is normal.

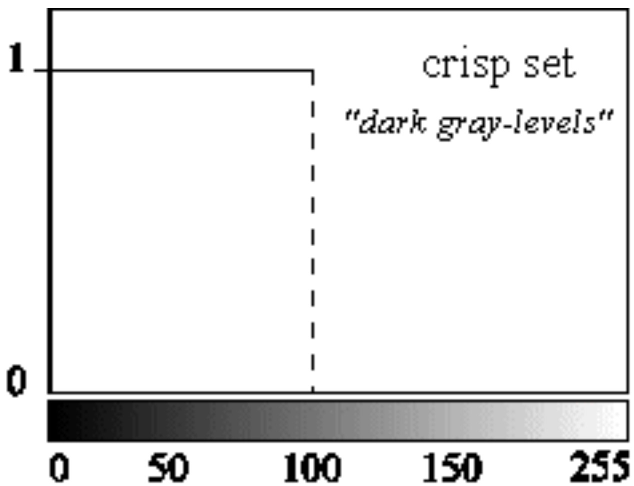
Fuzzy Image Processing

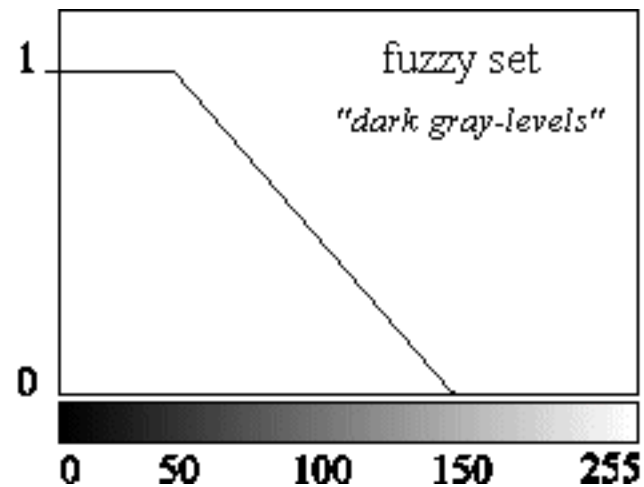




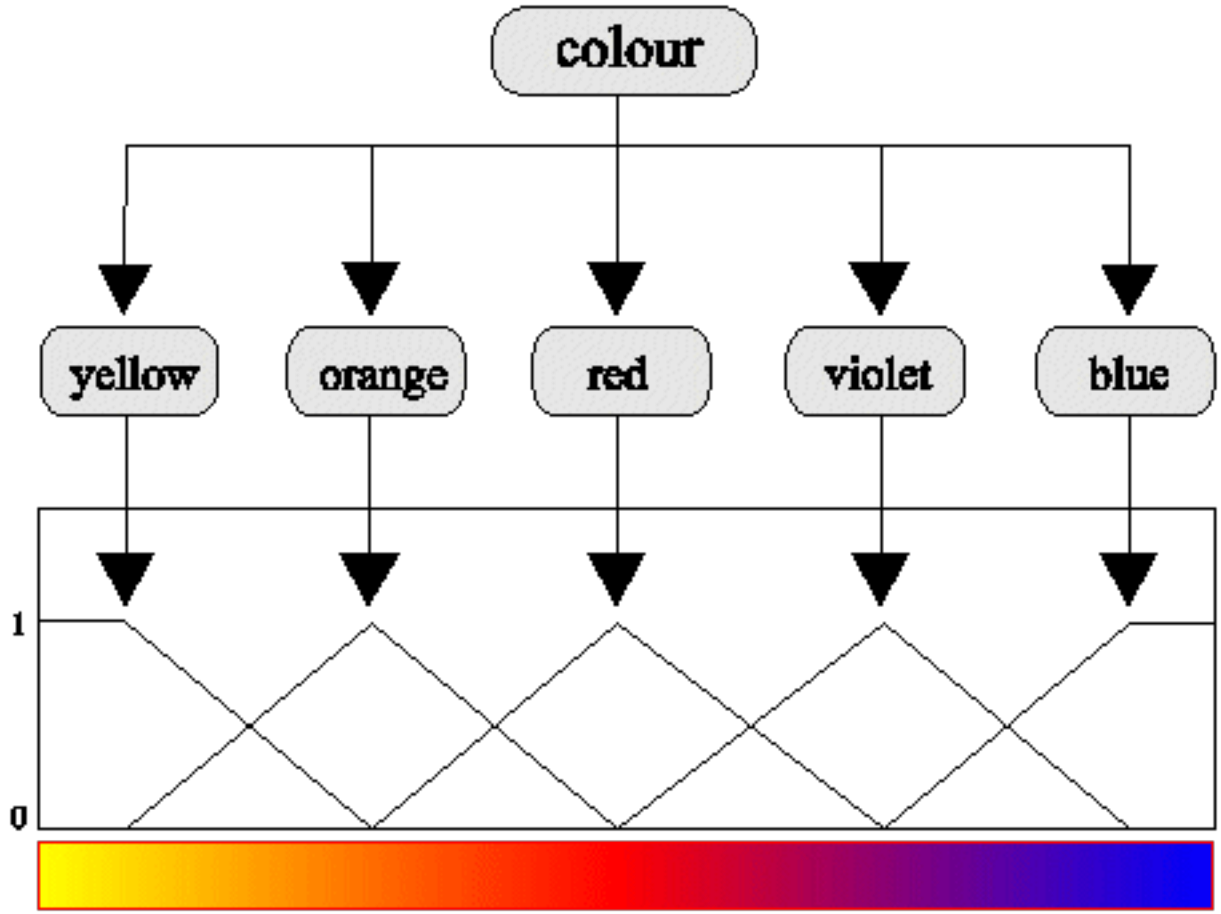
50 55 63
58 205 210
215 223 230
original image
gray-level plane
fuzzification
.19 .21 .25
.23 .80 .82
.84 .87 .90
modification of memberships
.07 .09 .12
.10 .92 .93
.95 .97 .98
defuzzification
membership plane
18 23 31
25 234 237
242 247 250
result image
gray-level plane

Fuzzy Image Processing





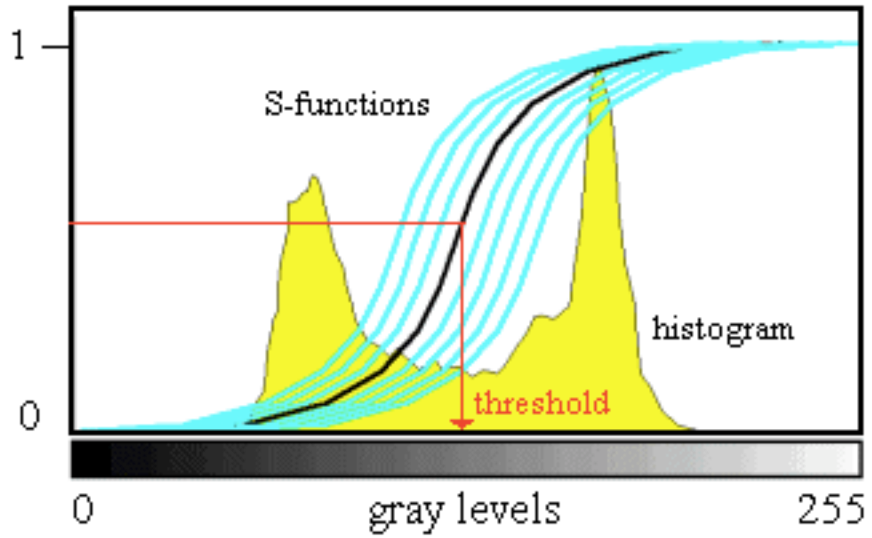
Fuzzy Image Processing



Fuzzy Image Processing

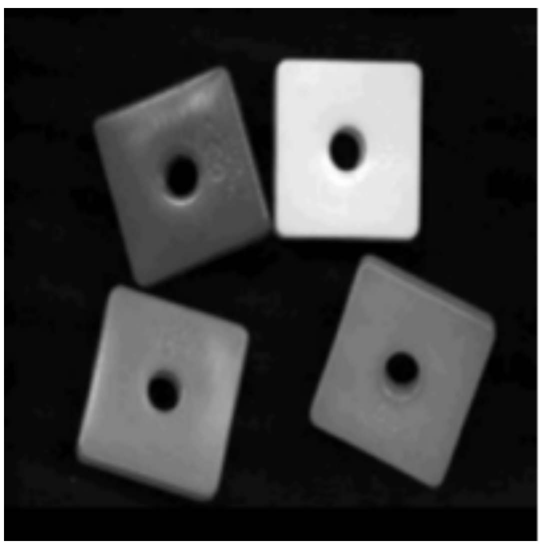
Image Segmentation

Thresholding by minimizaion of fuzziness



Fuzzy Image Processing

Image Segmentation



Fuzzy Image Processing

Image Enhancement (INT operator)

1. Define membership function

$$\mu_{mn} = G(g_{mn}) = \left[1 + \frac{g_{max} - g_{mn}}{F_d}\right]^{-F_e}$$

2. Modify membership values

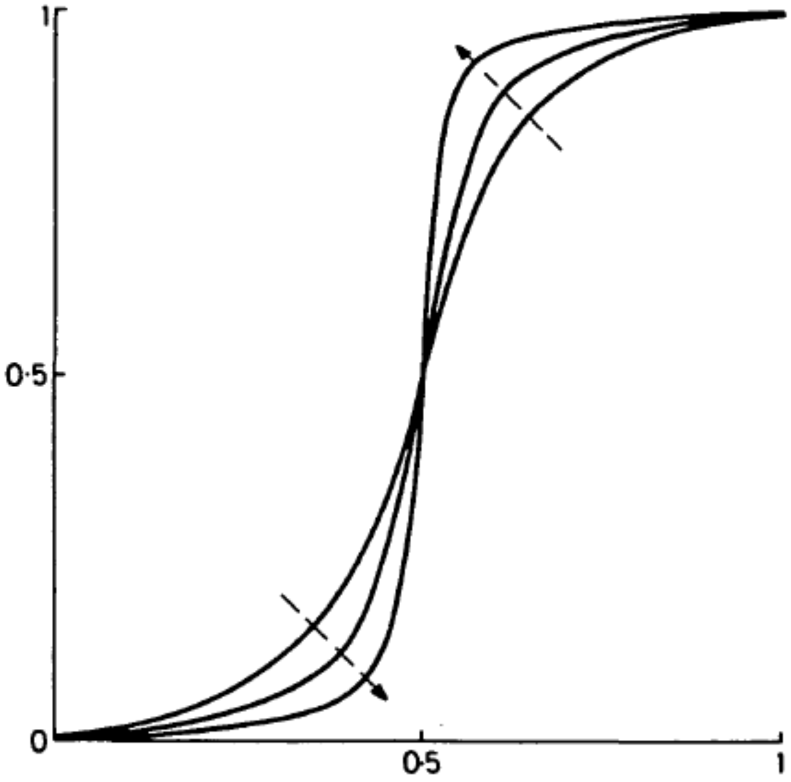
$$\mu'_{mn} = \begin{cases} 2 \cdot [\mu_{mn}]^2 & 0 \le \mu_{mn} \le 0.5 \\ 1 - 2 \cdot [1 - \mu_{mn}]^2 & 0.5 \le \mu_{mn} \le 1 \end{cases}$$

Fuzzy Image Processing

Image Enhancement (INT operator)

3. Generate new values

$$g'_{mn} = G^{-1}(\mu'_{mn}) = g_{max} - F_d\left((\mu'_{mn})^{\frac{-1}{F_e}} - 1\right.$$



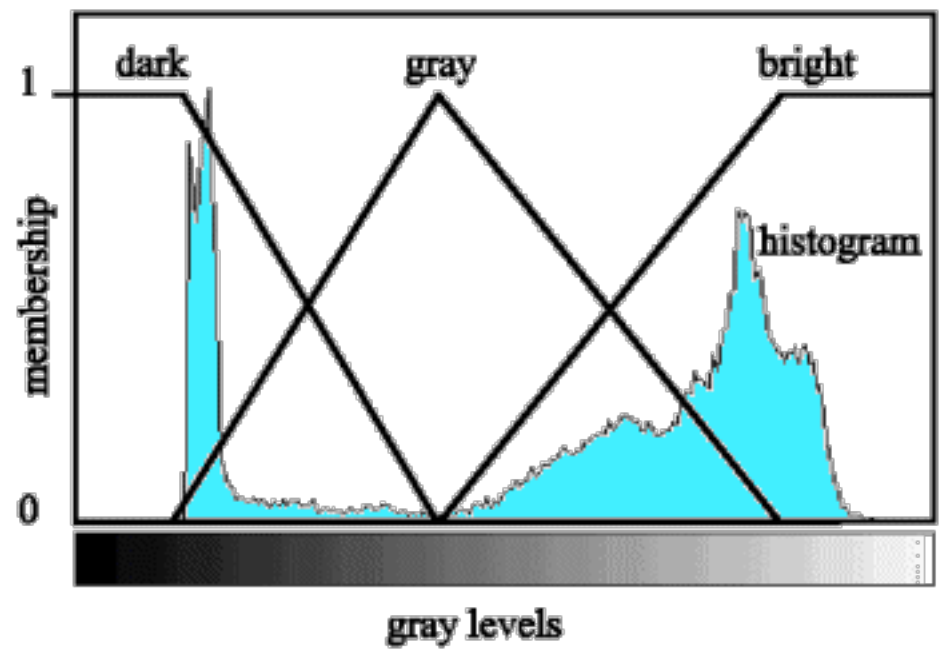
Fuzzy Image Processing

Fuzzy Image Processing

Image Enhancement (If Then Rules)

Step 1: Setting the parameter of inference system (input features, membership functions,..)

Step 2: Fuzzification of the actual pixel (memberships to the dark, gray and bright sets of pixels)

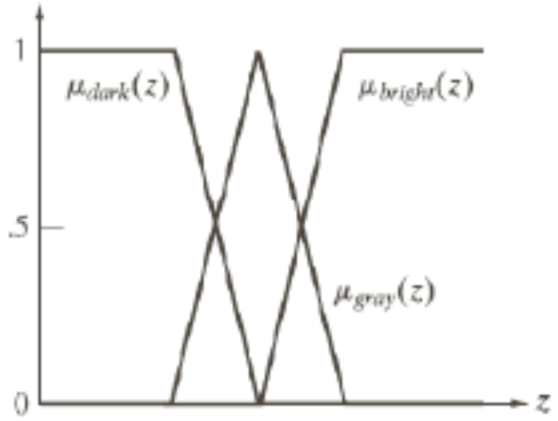


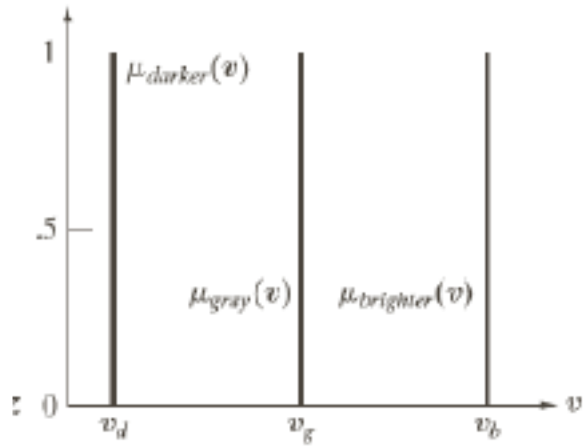
Fuzzy Image Processing

Image Enhancement (If Then Rules)

Step 3: Inference (e.g. if dark then darker, if gray then gray, if bright then brighter)

Step 4: Defuzzification of the inference result by the use of three singletons





Fuzzy Image Processing