



Special Module on Media Processing and Communication

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Recap



▶ Lecture 2

● Definitions

- [Compression Ratio](#), Fidelity Measures
- Data Redundancy

● Compression Techniques

- Symmetric and Asymmetric
- Loss-less and Lossy
- Loss-less

– [Variable length coding \(Huffman Coding\)](#)



Image Compression

Run Length Coding

Run: a string of the same symbol

Example

input: AAABBBCCCCCCCCCAA

output: A3B2C9A2

compression ratio = $16/8 = 2$



Image Compression

Predictive Coding

Basic premise: Current pixel is similar to the previous pixel (coherence)

Differential Coding

$$d(x,y) = I(x,y) - I(x-1,y)$$

$d(x,y)$ prediction error which is to be encoded.



Image Compression

Predictive Coding

Compression

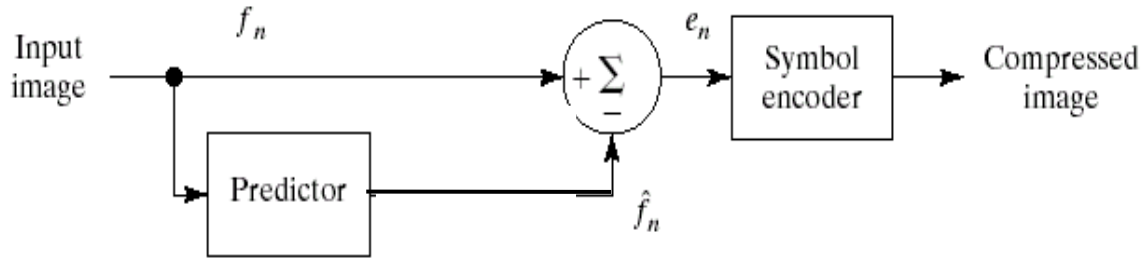




Image Compression

Predictive Coding

Decompression

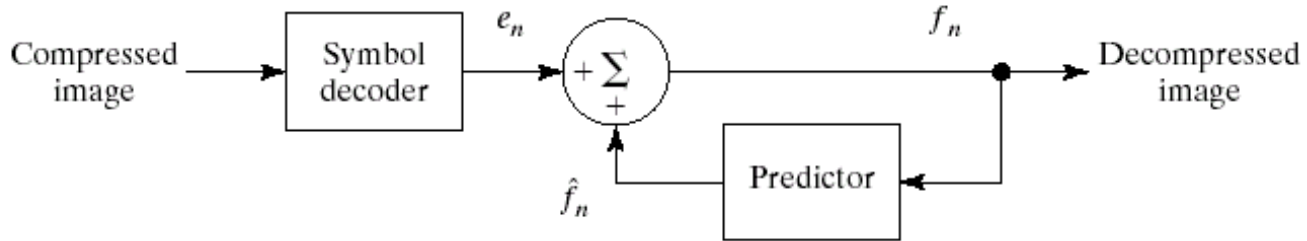


Image Compression

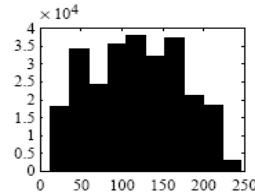
Predictive Coding



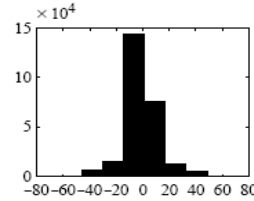
(a)



(b)



(c)



(d)

Distributions for Original versus Derivative Images. (a,b): Original gray-level image and its partial derivative image; (c,d): Histograms for original and derivative images.

Image Compression

Lossy

- Psychovisual redundancy
- Keep more important information
- Trade off between loss (degradation) and compression



Original



Compression Ratio: 7.7



Compression Ratio: 12.3



Compression Ratio: 33.9



Image Compression

Lossy



Original



Image Compression

Lossy



Compression Ratio 7.7



Image Compression

Lossy



Compression Ratio 33.9



Image Compression

Lossy

- Recall Quantization

Discrete value to represent range of values

Irreversible operation

Information loss !

- Predictive Coding

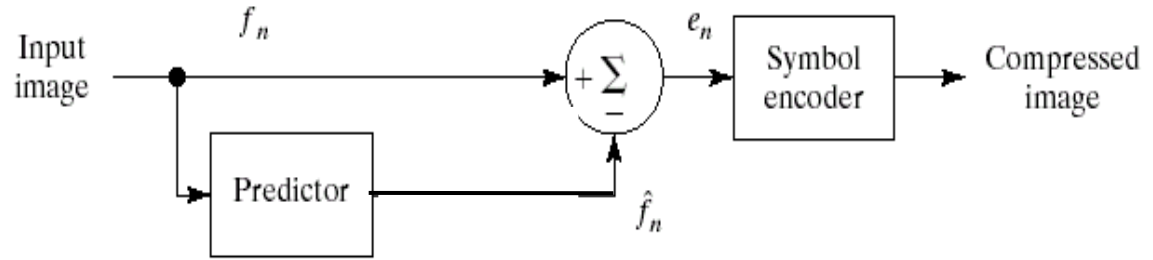
- Transform Coding



Image Compression

Predictive Coding: Loss-less (Revisit)

Compression



Decompression

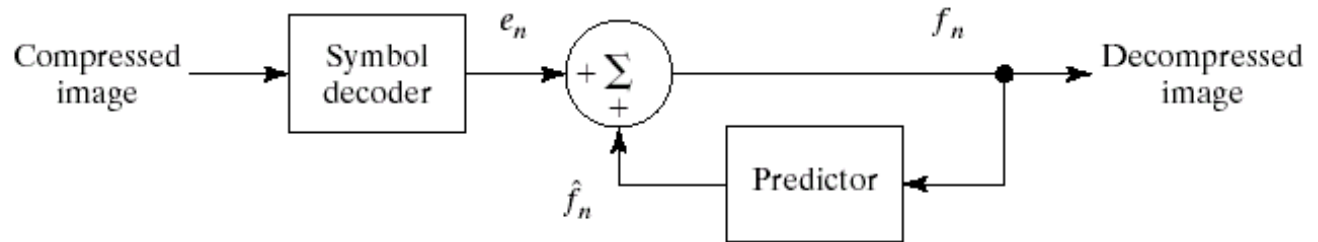
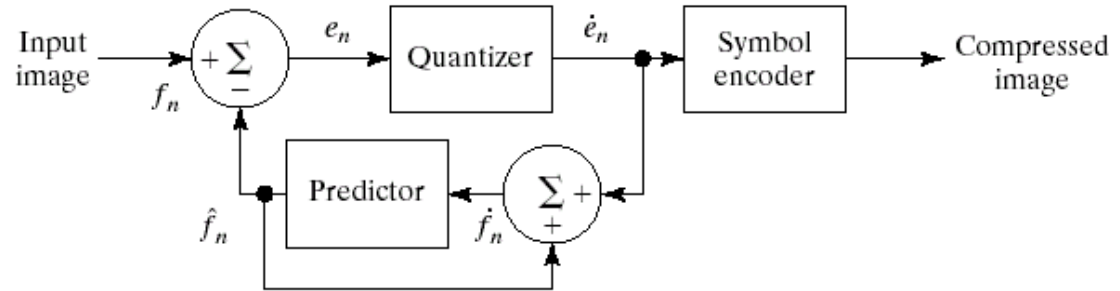




Image Compression

Predictive Coding: Lossy

Compression



Decompression

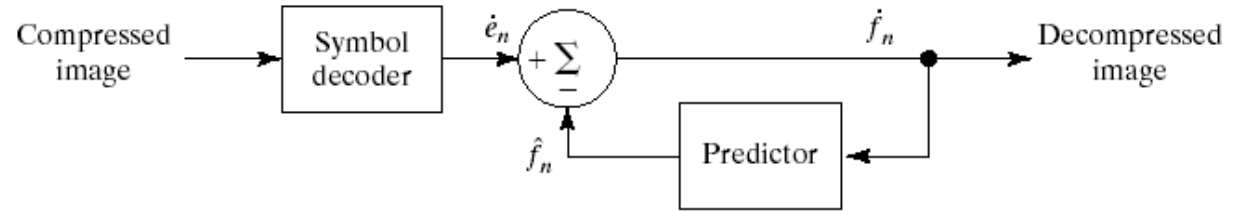




Image Compression

Predictive Coding: Lossy Delta Modulation

Example:

$$\hat{f}_n = \alpha \hat{f}_{n-1}$$

$$\text{and } \dot{e}_n = \begin{cases} +\xi & e_n > 0 \\ -\xi & e_n < 0 \end{cases}$$

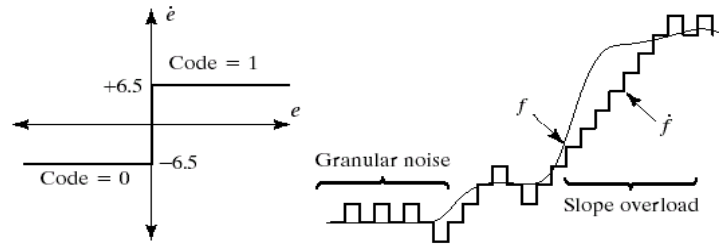
$$0 < \alpha < 1$$

prediction coefficient

$$\begin{aligned} \dot{f}_n &= \dot{e}_n + \hat{f}_n \\ &= \dot{e}_n + \alpha \dot{f}_{n-1} \end{aligned}$$

Image Compression

Predictive Coding: Lossy



a b
c

FIGURE 8.22 An example of delta modulation.

Input		Encoder			Decoder		Error	
n	f	\hat{f}	e	\hat{e}	\hat{f}	\hat{f}	\hat{f}	$[f - \hat{f}]$
0	14	—	—	—	14.0	—	14.0	0.0
1	15	14.0	1.0	6.5	20.5	14.0	20.5	-5.5
2	14	20.5	-6.5	-6.5	14.0	20.5	14.0	0.0
3	15	14.0	1.0	6.5	20.5	14.0	20.5	-5.5
·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·
14	29	20.5	8.5	6.5	27.0	20.5	27.0	2.0
15	37	27.0	10.0	6.5	33.5	27.0	33.5	3.5
16	47	33.5	13.5	6.5	40.0	33.5	40.0	7.0
17	62	40.0	22.0	6.5	46.5	40.0	46.5	15.5
18	75	46.5	28.5	6.5	53.0	46.5	53.0	22.0
19	77	53.0	24.0	6.5	59.6	53.0	59.6	17.5
·	·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·	·



Image Compression

Transform Coding

Transformation

- Represent information in another space
- Identify and remove correlation
- Quantization of transform coefficients

Information loss!

Example

time/space frequency
(Fourier Transform)

Inverse transformation

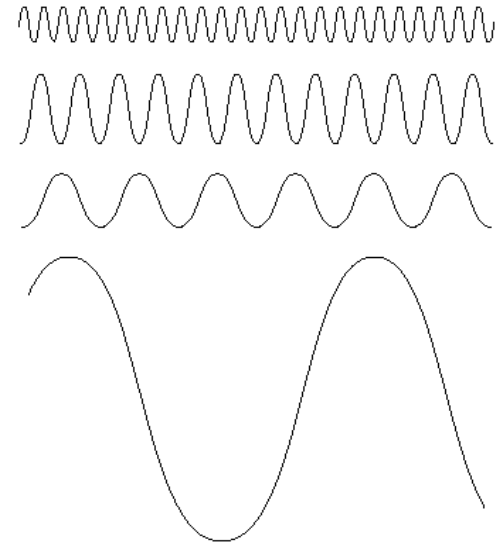
- Bring back information in the original space



Image Compression

Transform Coding

Fourier Transform (Review)



A function as sum of sines and cosines



Image Compression

Transform Coding

Fourier Transform (Review)

Mathematically

Forward

$$F(u) \equiv \mathfrak{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x)e^{-j2\pi ux} dx$$

1-D:

Inverse

$$f(x) \equiv \mathfrak{F}^{-1}\{F(u)\} = \int_{-\infty}^{\infty} F(u)e^{j2\pi ux} du$$

Discrete Fourier Transform

Fast Fourier Transform (FFT)

2-D:

$$F(u, v) = \iint f(x, y)e^{-j2\pi(ux+vy)} dx dy$$

$$f(x, y) = \iint F(u, v)e^{j2\pi(ux+vy)} du dv$$



Image Compression

Transform Coding

Discrete Cosine Transform

Popular transform for its performance and efficiency

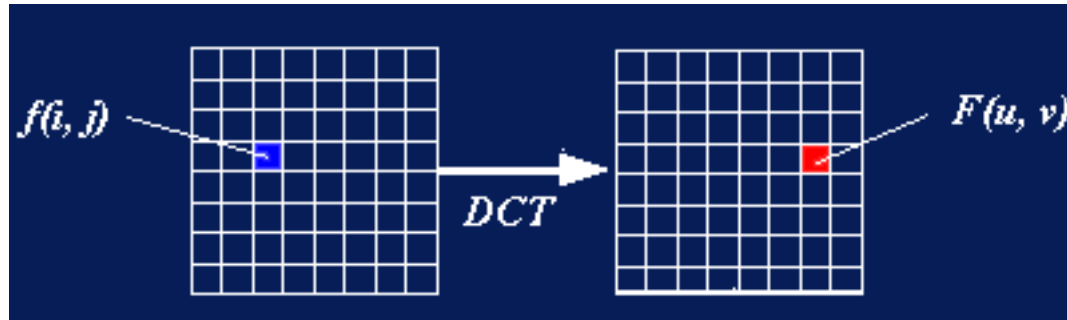




Image Compression

Transform Coding

Discrete Cosine Transform

Forward transform

$$F(u, v) = \frac{2}{N} C(u)C(v) \sum_{y=0}^{N-1} \sum_{x=0}^{N-1} f(x, y) \cos\left[\frac{(2x+1)u\pi}{2N}\right] \cos\left[\frac{(2y+1)v\pi}{2N}\right]$$

Inverse transform

$$f(x, y) = \frac{2}{N} \sum_{v=0}^{N-1} \sum_{u=0}^{N-1} C(u)C(v)F(u, v) \cos\left[\frac{(2x+1)u\pi}{2N}\right] \cos\left[\frac{(2y+1)v\pi}{2N}\right]$$



Image Compression

Transform Coding

Discrete Cosine Transform

Energy compaction

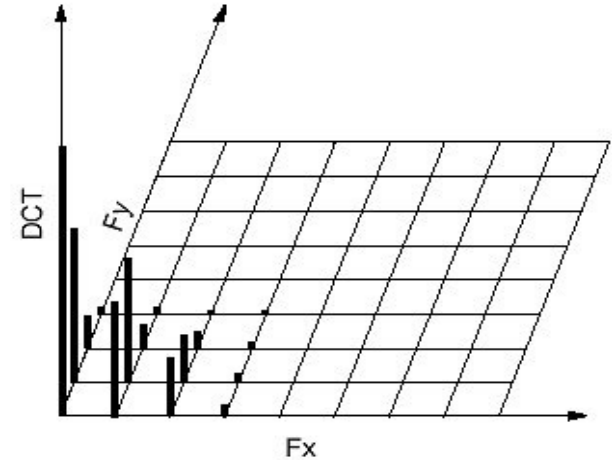
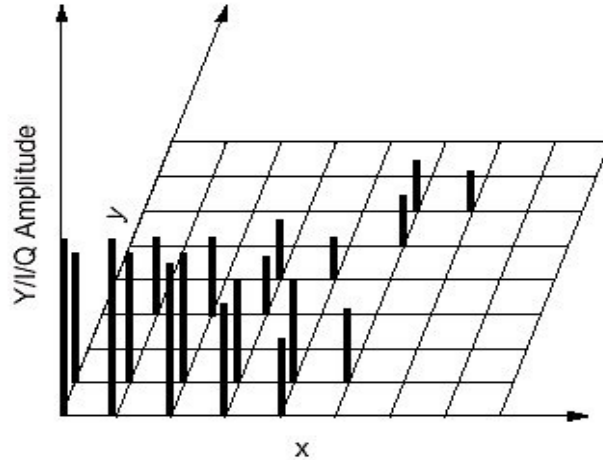




Image Compression

Transform Coding Pipeline

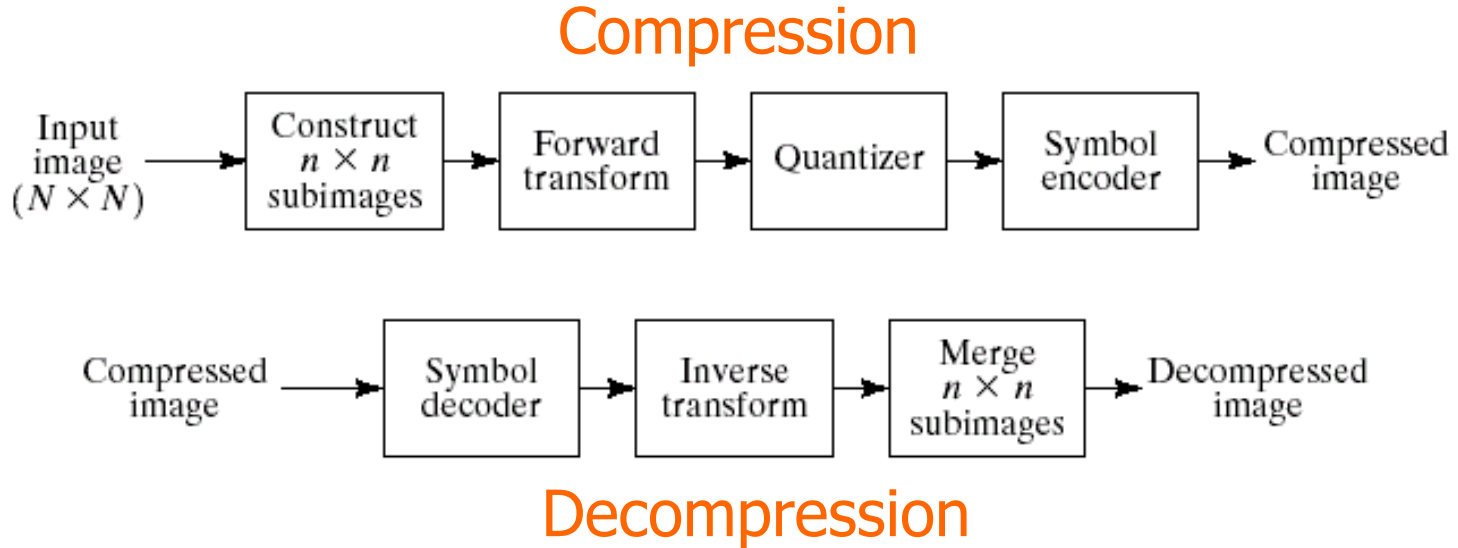


Image Compression

Transform Coding

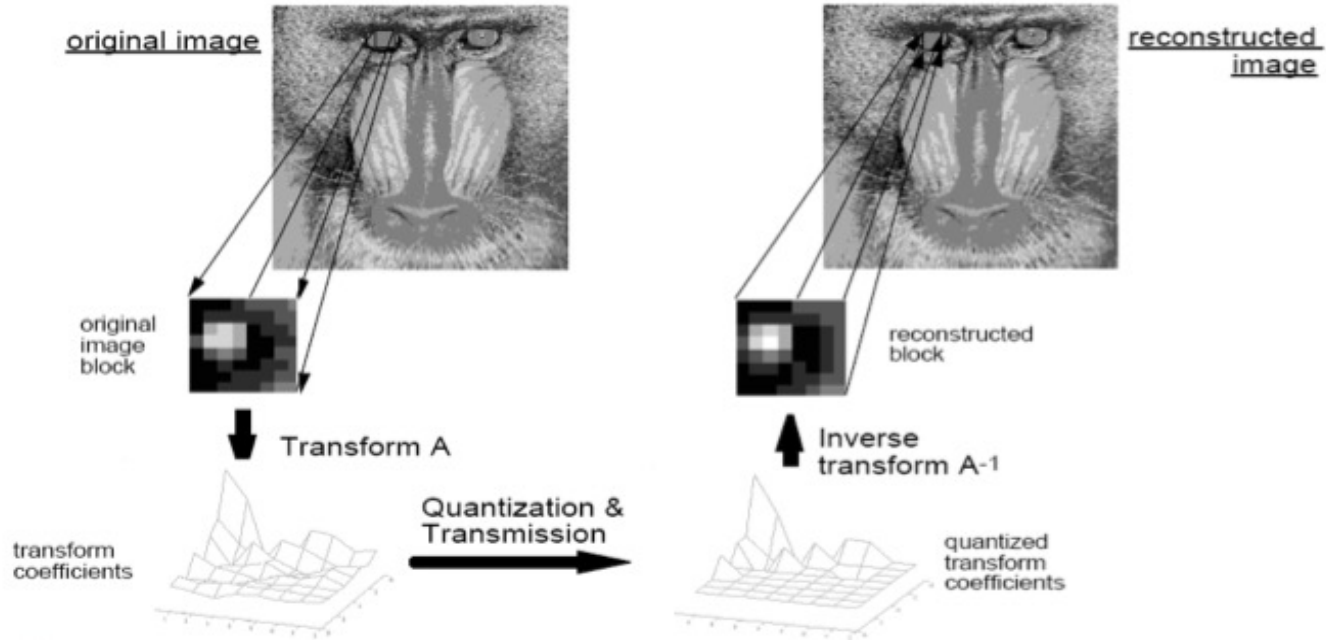




Image Compression

Transform Coding

Why sub image (nxn)?

- Computational benefit
- Typically 8x8, 16x16
- Error is not very high

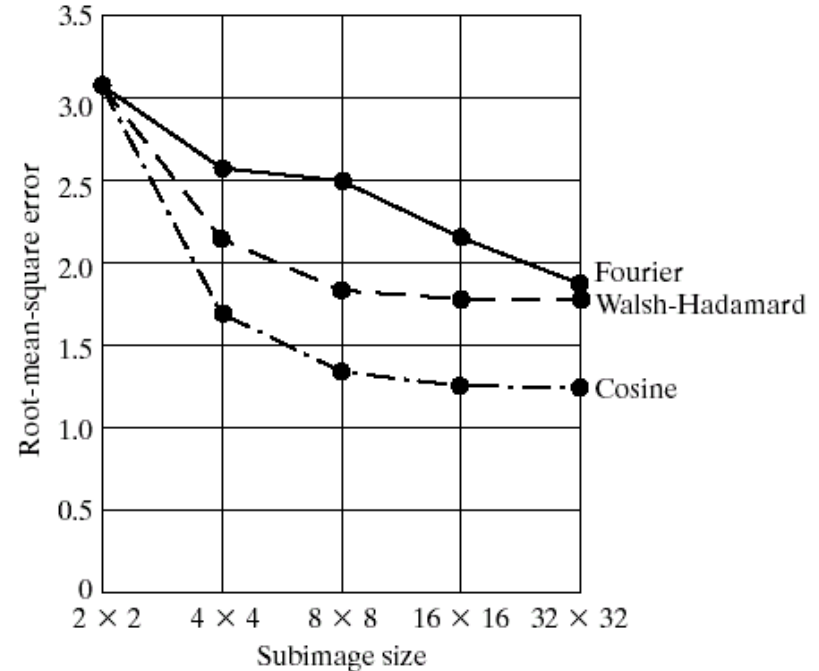




Image Compression

Transform Coding

Why sub image (nxn)?

- Computational benefit
- Typically 8x8, 16x16
- Error is not very high

Which transform?

- Low error for the same number of coefficients
- Computationally fast
- DCT is preferred

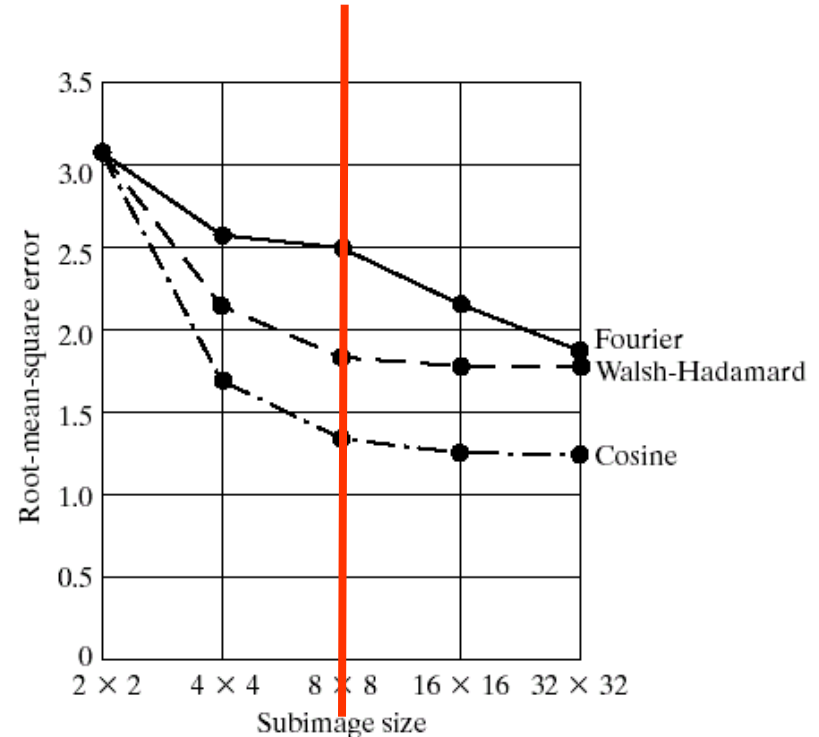




Image Compression

Transform Coding Quantization Schemes

- Global thresholding
- Local thresholding
- For each block M out of N coefficients to retain

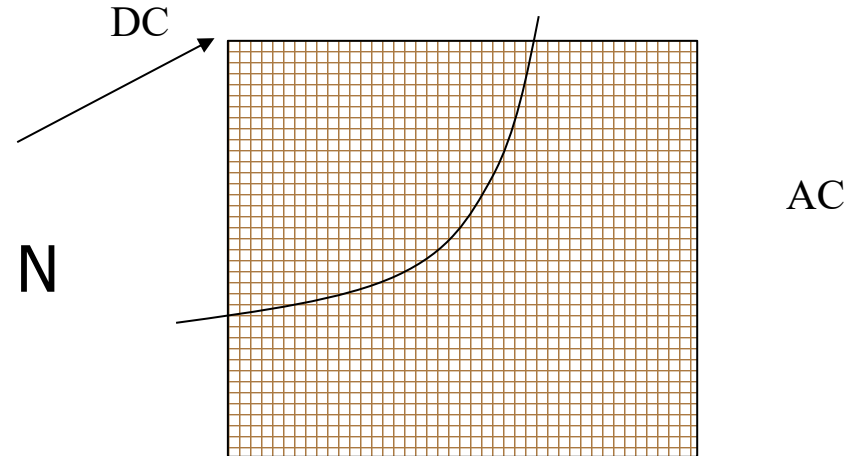




Image Compression



Recap Slides of Lecture 2



Image Compression

Compression Ratio

$$C_r = n_o/n_c$$

n_o = Number of carrying units (bits) in the **original** data (image)

n_r = Number of carrying units (bits) in the **compressed** data (image)

Also,

$$R_d = 1 - 1/ C_r$$

R_d = Relative data redundancy



Image Compression

Variable Length Coding (Huffman Coding)

Sequence of symbols (a_1, a_2, a_3, a_4, a_5) with associated probabilities (p_1, p_2, p_3, p_4, p_5)

- Start with two symbols of the least probability
 $a_1:p_1$
 $a_2:p_2$
- Combine (a_1 or a_2) with probability (p_1+p_2)
- Do it recursively (sort and combine)
- A binary tree construction



Image Compression

Variable Length Coding (Huffman Coding)

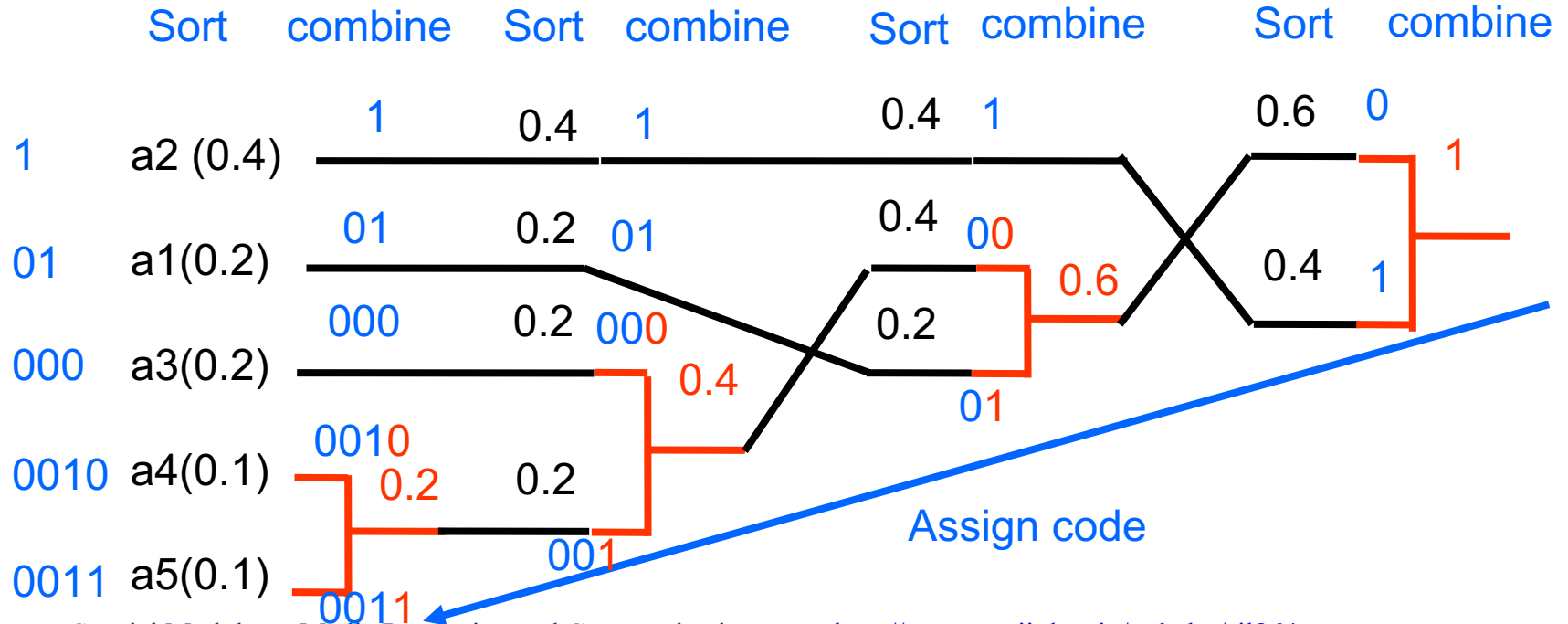




Image Compression

Variable Length Coding (Huffman Coding)

Example:

Avg length code:

$$0.4 \times 1 + 0.2 \times 2 + 0.2 \times 3 + 0.1 \times 4 + 0.1 \times 4 = 2.2 \text{ bits}$$

Entropy

A measure of information that captures uncertainty
[$I(e) = \log (1/P(e))$]

$$H = - \sum_{i=0}^{L-1} p(a_i) \log_2 p(a_i) \quad \text{bits / symbol}$$



Image Compression

Variable Length Coding (Huffman Coding)

Example: Decoding

00111010001

?



Image Compression

Variable Length Coding (Huffman Coding)

Example:

00111010001

Decoding

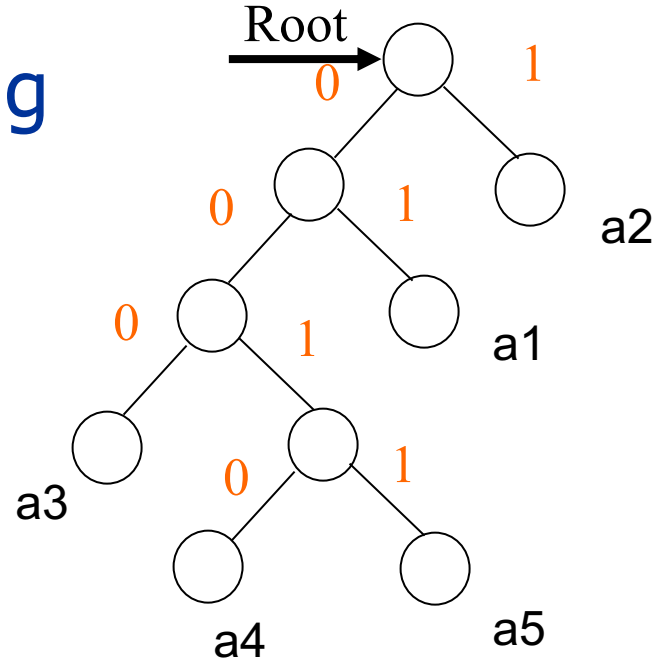
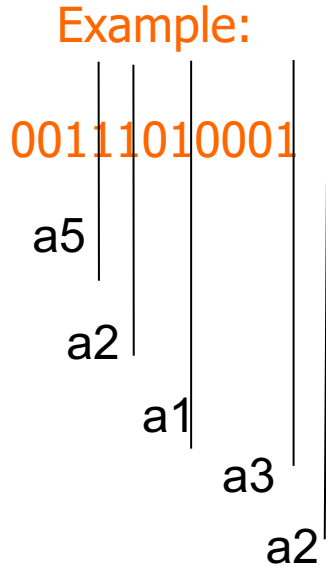


Image Compression

Variable Length Coding (Huffman Coding)



Decoding

