COL783: Digital Image Processing

Prem Kalra
pkalra@cse.iitd.ac.in
http://www.cse.iitd.ac.in/~pkalra/col783

Department of Computer Science and Engineering
Indian Institute of Technology Delhi

Image Compression

Transform Coding

Transformation

- Represent information in anther space
- Identify and remove correlation
- Quantization of transform coefficients

Information loss!

Example

time/space frequency

(Fourier Transform)

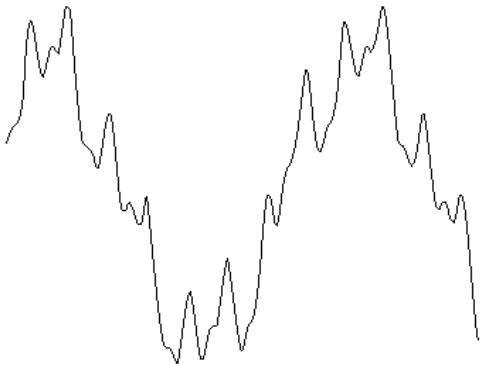
Inverse transformation

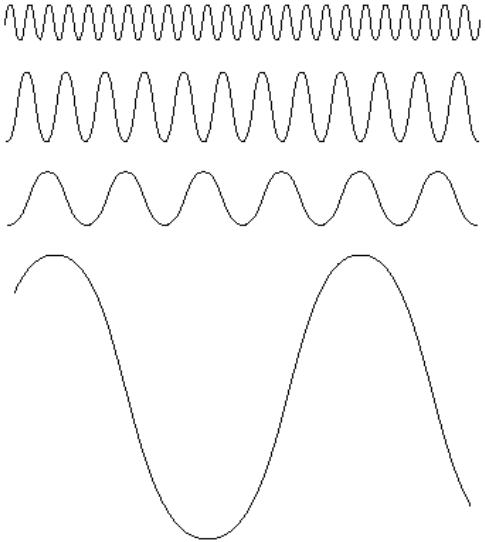
- Bring back information in the original space

Image Compression

Transform Coding

Fourier Transform (Review)





A function as sum of sines and cosines

Image Compression

Transform Coding

Fourier Transform (Review)

Mathematically

Forward

1-D:

$$F(u) \equiv \Im\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

Inverse

$$f(x) \equiv \Im^{-1}\{F(u)\} = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

2-D:

$$F(u,v) = \iint f(x,y) e^{-j2\pi(ux+vy)} dx\, dy$$

$$f(x,y) = \iint F(u,v) e^{j2\pi(ux+vy)} du\, dv$$

Discrete Fourier Transform

Fast Fourier Transform (FFT)

Image Compression

Transform Coding

Discrete Cosine Transform

Popular transform for its performance and efficiency

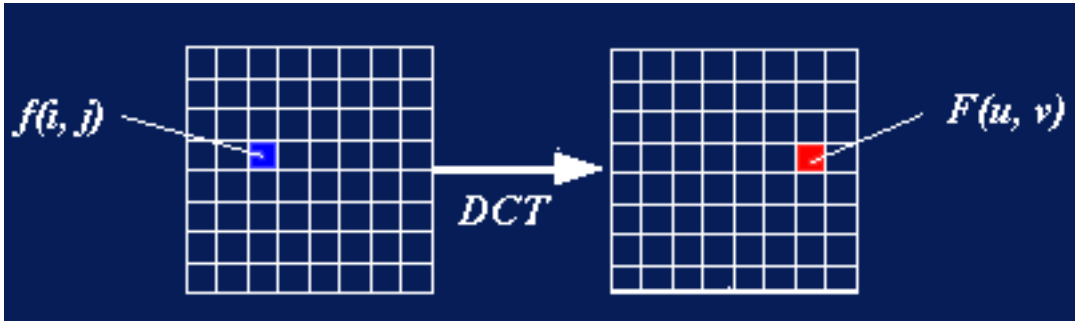


Image Compression

Transform Coding

Discrete Cosine Transform

Forward transform

$$F(u,v) = \frac{2}{N} C(u)C(v) \sum_{y=0}^{N-1} \sum_{x=0}^{N-1} f(x,y) \cos[\frac{(2x+1)u\pi}{2N}] \cos[\frac{(2y+1)v\pi}{2N}]$$

Inverse transform

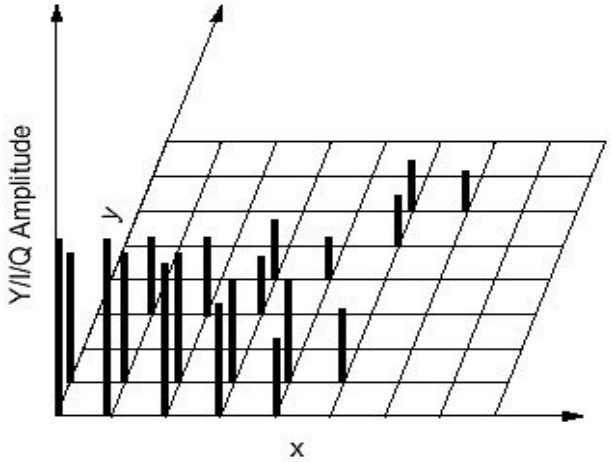
$$f(x,y) = \frac{2}{N} \sum_{v=0}^{N-1} \sum_{u=0}^{N-1} C(u)C(v)F(u,v) \cos[\frac{(2x+1)u\pi}{2N}] \cos[\frac{(2y+1)v\pi}{2N}]$$

Image Compression

Transform Coding

Discrete Cosine Transform

Energy compaction



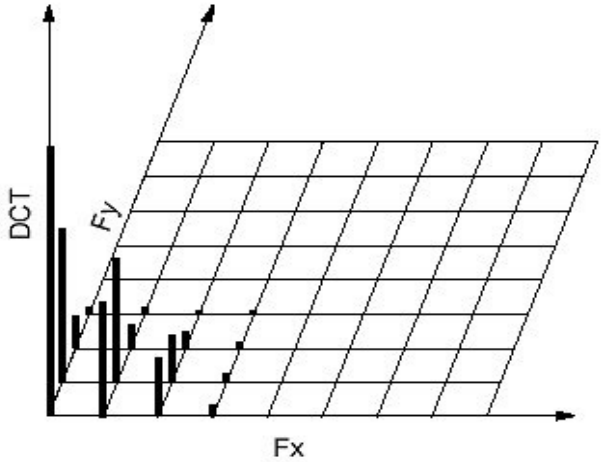
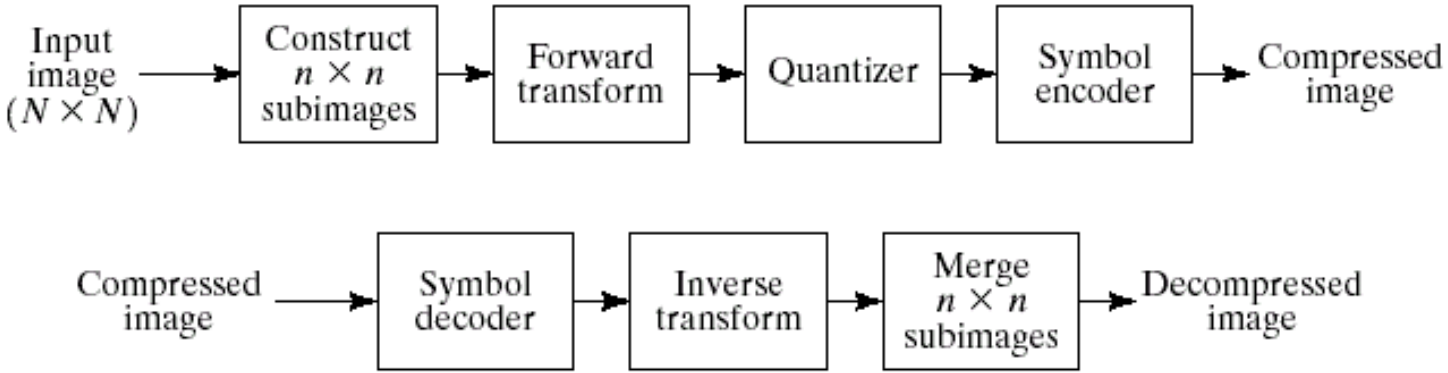


Image Compression

Transform Coding

Pipeline

Compression



Decompression

Image Compression

Transform Coding

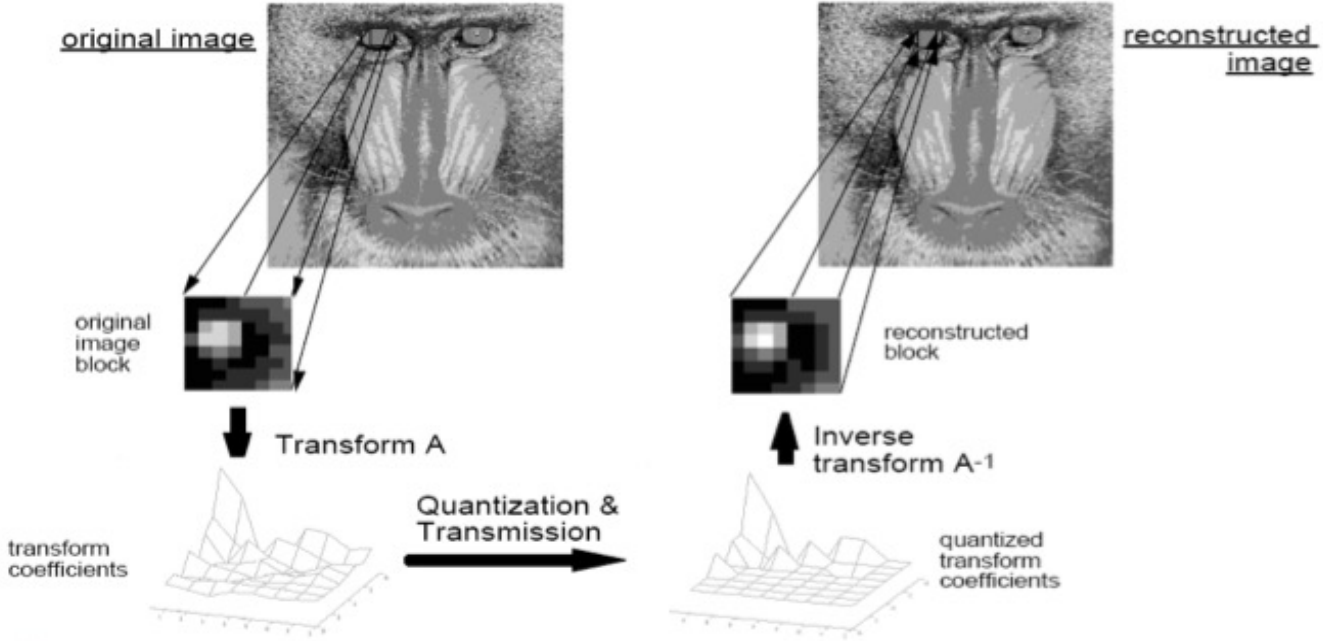


Image Compression

Transform Coding

Why sub image (nxn)?

- Computational benefit
- Typically 8x8, 16x16
- Error is not very high

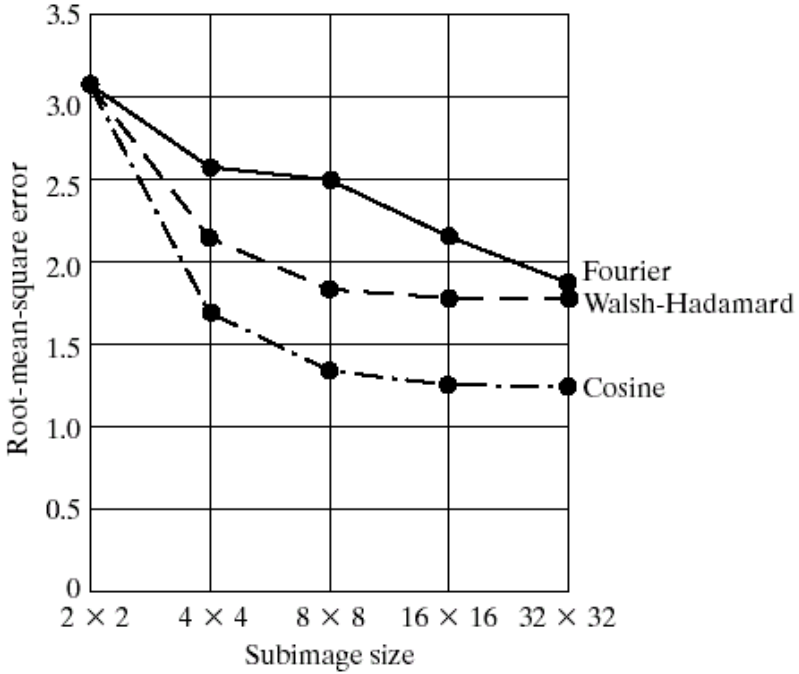


Image Compression

Transform Coding

Why sub image (nxn)?

- Computational benefit
- Typically 8x8, 16x16
- Error is not very high

Which transform?

- Low error for the same number of coefficients
- Computationally fast
- DCT is preferred

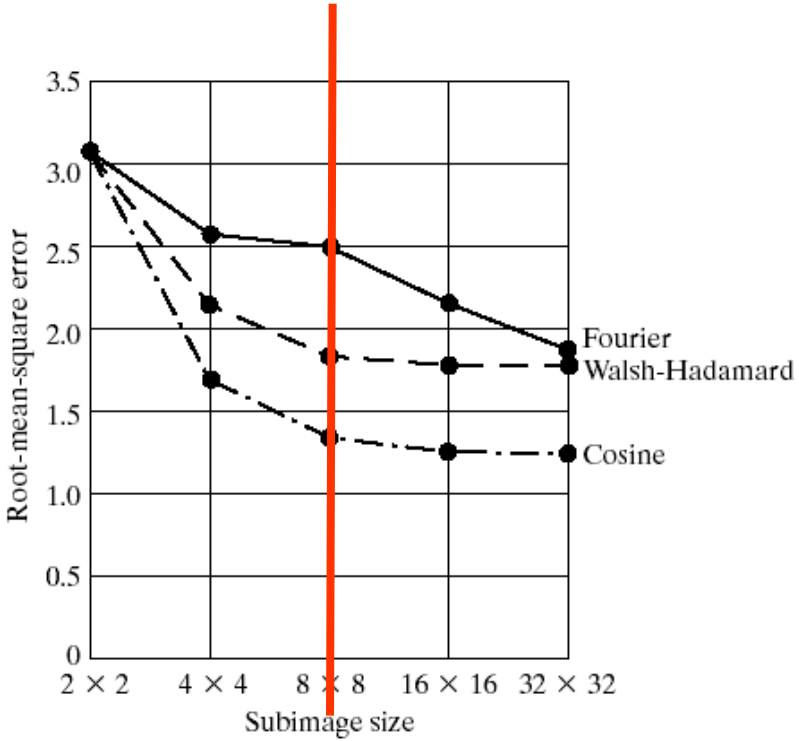


Image Compression

Transform Coding

Quantization Schemes

- Global thresholding
- Local thresholding
- For each block M out of N coefficients to retain

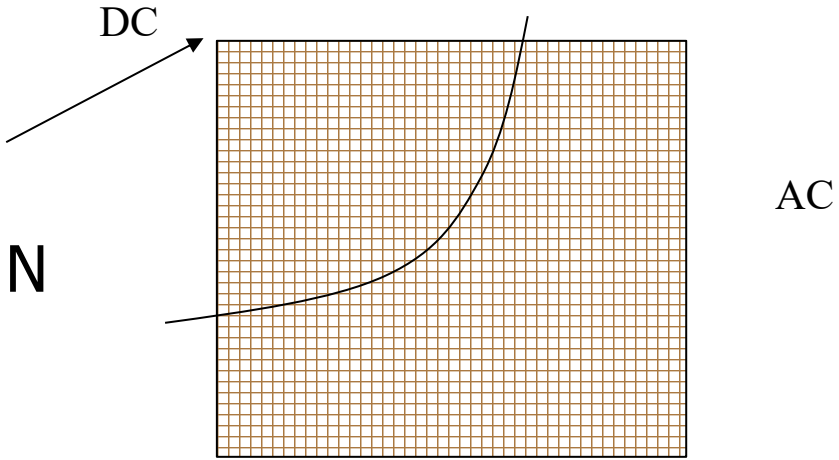


Image Compression

Transform Coding

In general 1D transform

$$\mathbf{f} = \sum_{k=0}^{N-1} t(k)\mathbf{h}_k = \mathbf{B}\mathbf{t},$$

$$where \quad \mathbf{B} = [\mathbf{h}_0, \mathbf{h}_1, \ldots, \mathbf{h}_{N-1}], \mathbf{t} = \begin{bmatrix} t(0) \\ t(1) \\ \vdots \\ t(N-1) \end{bmatrix}.$$

$$\Rightarrow \quad \mathbf{t} = \mathbf{B}^{-1}\mathbf{f} = \mathbf{A}\mathbf{f}$$

f and t form a transform pair

Image Compression

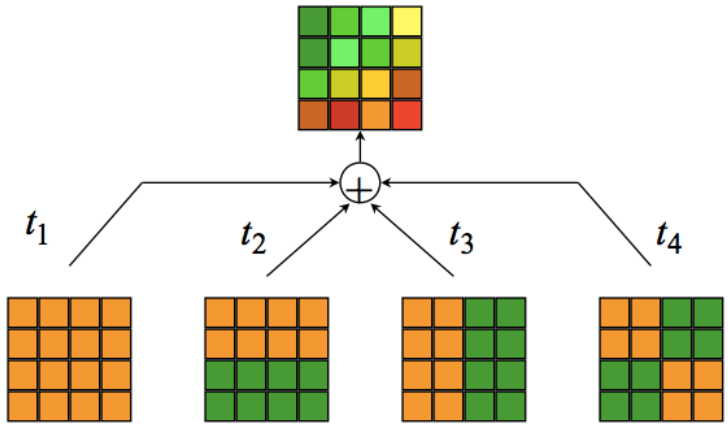
Transform Coding

In general

Represent an image (or an image block) as the linear combination of some basis images and specify the linear coefficients.

$$\mathbf{f} = \sum_{k=0}^{N-1} t(k)\mathbf{h}_k = \mathbf{B}\mathbf{t},$$

$$where \quad \mathbf{B} = [\mathbf{h}_0, \mathbf{h}_1, \ldots, \mathbf{h}_{N-1}], \mathbf{t} = \begin{bmatrix} t(0) \\ t(1) \\ \vdots \\ t(N-1) \end{bmatrix}.$$



Source: http://eeweb.poly.edu/~yao/EL5123/lecture10_TransformCoding_JPEG.pdf

Image Compression

Transform Coding

In general

Forward transform $t = Af$

Inverse transform $f = A^T t$

Considering A orthonormal $A^{-1} = A^T$

$$\begin{bmatrix} f_0 \\ \cdot \\ \cdot \\ f_{N-1} \end{bmatrix} = \begin{bmatrix} a_{0,0} & \cdot\ \cdot & a_{N-1,0} \\ \cdot & \cdot\ \cdot & \cdot \\ \cdot & \cdot\ \cdot & \cdot \\ a_{0,N-1} & \cdot\ \cdot & a_{N-1,N-1} \end{bmatrix} \begin{bmatrix} t_0 \\ \cdot \\ \cdot \\ t_{N-1} \end{bmatrix}$$

Coefficients

$$\begin{bmatrix} f_0 \\ \cdot \\ \cdot \\ f_{N-1} \end{bmatrix} = \begin{bmatrix} a_{0,0} \\ \cdot \\ \cdot \\ a_{0,N-1} \end{bmatrix} t_0 + .. + \begin{bmatrix} a_{N-1,0} \\ \cdot \\ \cdot \\ a_{N-1,N-1} \end{bmatrix} t_{N-1}$$

Basis Vectors

Image Compression

Transform Coding

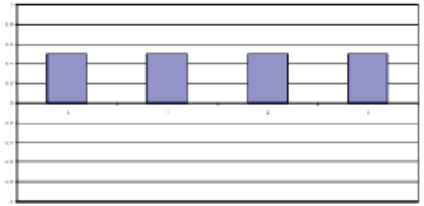
Discrete Cosine Transform

1D transform

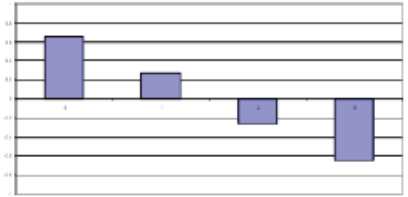
$$d_{ij} = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } i = 0 \\ \sqrt{\frac{2}{N}} \cos \frac{(2j+1)i\pi}{2N} & \text{if } i > 0 \end{cases}$$

$N = 4$

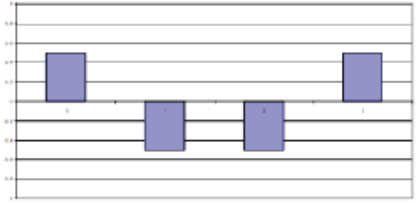
$$D = \begin{bmatrix} .5 & .5 & .5 & .5 \\ .65328 & .270598 & -.270598 & -.65328 \\ .5 & -.5 & -.5 & .5 \\ .270598 & -.65328 & .65328 & -.270598 \end{bmatrix}$$



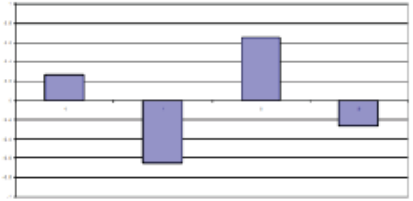
row 0



row 1



row 2



row 3

Image Compression

Transform Coding

$$d_{ij} = \begin{cases} \sqrt{\frac{1}{N}} & \text{if } i = 0 \\ \sqrt{\frac{2}{N}} \cos \frac{(2j+1)i\pi}{2N} & \text{if } i > 0 \end{cases}$$

$N = 4$

$$D = \begin{bmatrix} .5 & .5 & .5 & .5 \\ .65328 & .270598 & -.270598 & -.65328 \\ .5 & -.5 & -.5 & .5 \\ .270598 & -.65328 & .65328 & -.270598 \end{bmatrix}$$

Discrete Cosine Transform

1D transform

$$\begin{bmatrix} .5 \\ .5 \\ .5 \\ .5 \end{bmatrix} c_0 + \begin{bmatrix} .653281 \\ .270598 \\ -.270598 \\ -.653281 \end{bmatrix} c_1 + \begin{bmatrix} .5 \\ -.5 \\ -.5 \\ .5 \end{bmatrix} c_2 + \begin{bmatrix} .270598 \\ -.653281 \\ .653281 \\ -.270598 \end{bmatrix} c_3 = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

DC coefficient

AC coefficients

Image Compression

Transform Coding

In general 2D transform

$$\mathbf{F} = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} T(k,l)\mathbf{H}_{k,l},$$

Input

$$F(m,n) = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} T(k,l)H_{k,l}(m,n)$$

Inverse Transform

Transform Coefficients

Image Compression

Transform Coding

2D transform

$$\mathbf{F} = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} T(k,l)\mathbf{H}_{k,l},$$

$$F(m,n) = \sum_{k=0}^{M-1}\sum_{l=0}^{N-1} T(k,l)H_{k,l}(m,n)$$

Discrete Cosine Transform

$$H_{k,l}(m,n) = \alpha(k)\alpha(l)\cos\left[\frac{(2m+1)k\pi}{2N}\right]\cos\left[\frac{(2n+1)l\pi}{2N}\right]$$

$$where\ \alpha(k) = \begin{cases} \sqrt{1/N} & k=0 \\ \sqrt{2/N} & k=1,\ldots,N-1 \end{cases}$$

Image Compression

Transform Coding

2D transform

Discrete Cosine Transform

8x8 DCT Basis Images

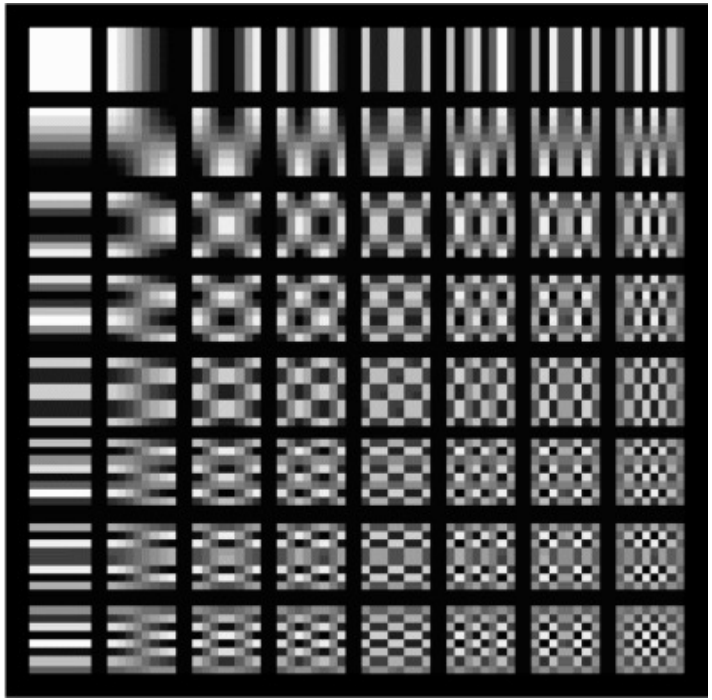


Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

Covariance Matrix

$$C_x = E\{(x - m_x)(x - m_x)^T\}$$

$$m_x = E\{x\}$$

$$m_x = \frac{1}{M}\sum_{k=0}^{M-1} x_k$$

$$C_x = \frac{1}{M}\sum_{k=0}^{M-1}(x_k - m_x)(x_k - m_x)^T$$

$$C_x = \frac{1}{M}\sum_{k=0}^{M-1} x_k x_k - m_x m_x^T$$

Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

$$Y = A(X - m_x)$$

Inverse Transform

$$X = A^T Y + m_x$$

$$C_x T_i = \lambda_i T_i$$

$A =$ Matrix with rows of eigen vectors $T_i s$ with $\lambda_0 > \lambda_1 > \lambda_2 \ldots$

Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

$$m_y = E\{Y\} = E\{A(X - m_x)\} = AE\{X\} - Am_x = 0$$

$$\begin{aligned} C_y &= E\{(Y - m_y)(Y - m_y)^T\} = E\{(Y)(Y)^T\} \\ &= E\{(AX - Am_x)(AX - Am_x)^T\} \\ &= E\{A(X - m_x)(X - m_x)^T A^T\} \\ &= AE\{(X - m_x)(X - m_x)^T\}A^T \\ &= AC_xA^T = AA^TD = D \end{aligned}$$

Covariance Matrix is a Diagonal Matrix

$$D = \begin{bmatrix} \lambda_1 & . & . & 0 \\ . & \lambda_2 & . & . \\ . & . & .. & . \\ 0 & . & . & \lambda_N \end{bmatrix}$$

Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

Inverse Transform $$X = A^T Y + m_x$$

$$X' = A_k^T Y + m_x$$

$$e_{ms} = \sum_{j=0}^{N-1} \lambda_j - \sum_{j=0}^{k-1} \lambda_j = \sum_{j=K}^{N-1} \lambda_j$$

Image Compression

Transform Coding

Hotelling Transform (Karhunen-Loeve Transform KLT)

Geometrically

Principal Component Analysis

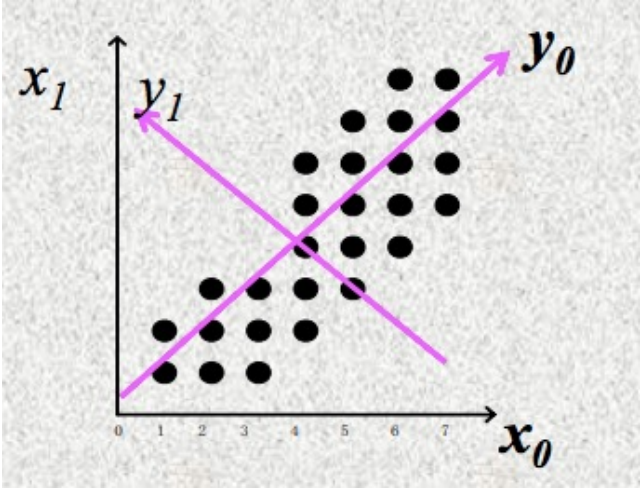
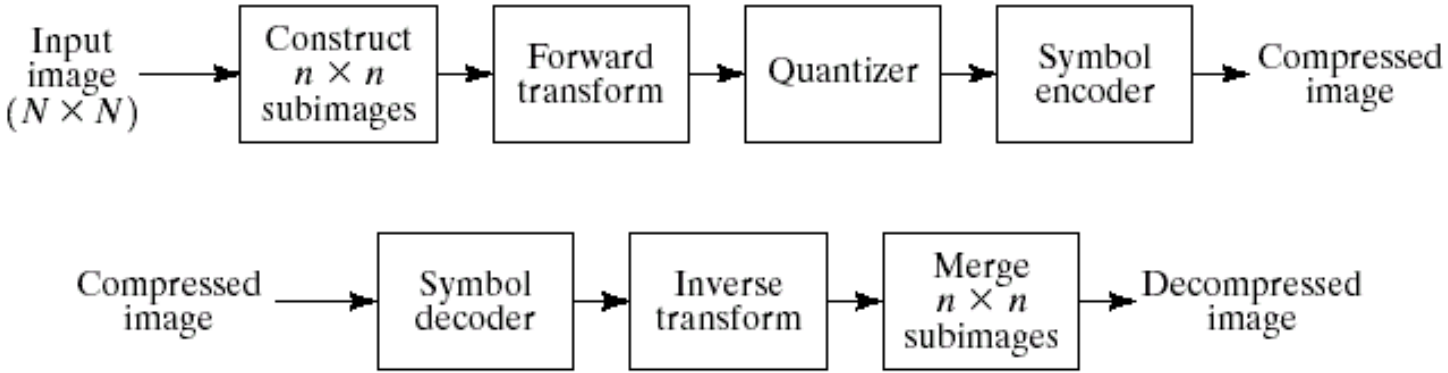


Image Compression

Transform Coding

Pipeline

Compression



Decompression

Image Compression

Transform Coding

Pipeline

Compression

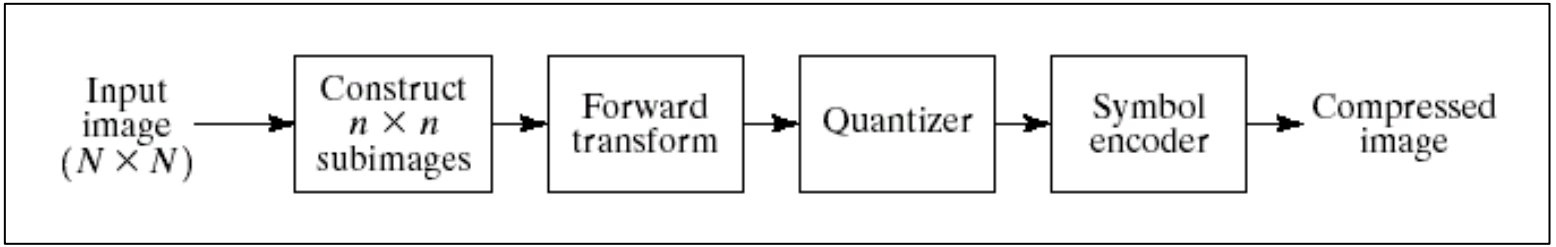
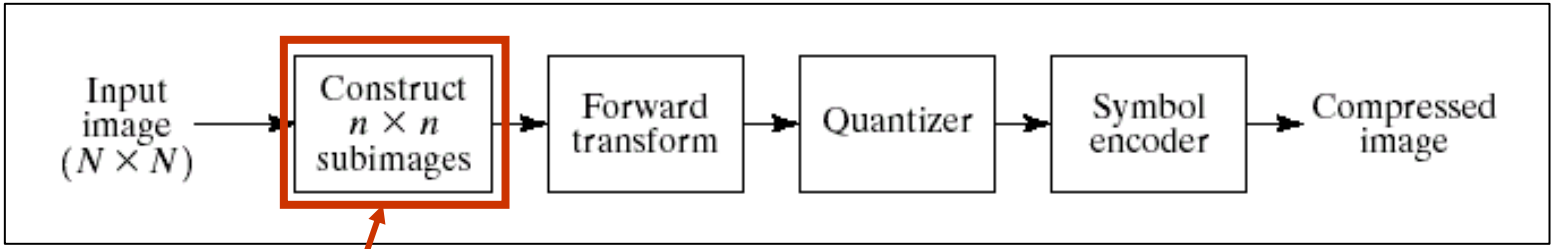


Image Compression

Transform Coding

Pipeline

Compression



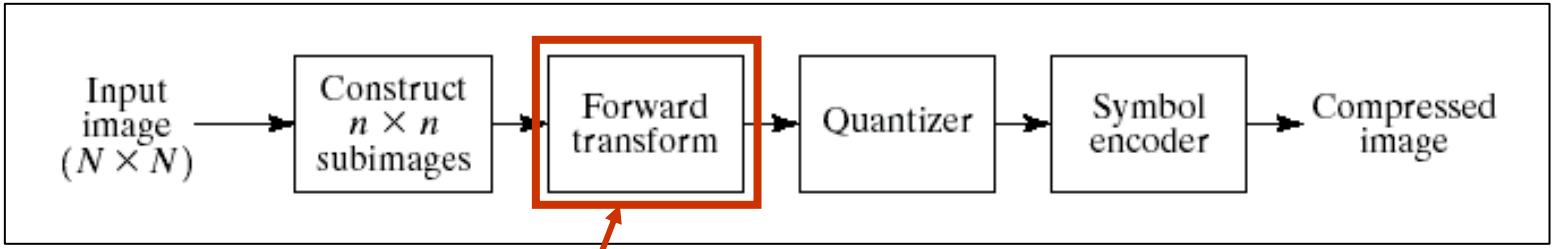
Typically
8x8 or 16x16

Image Compression

Transform Coding

Pipeline

Compression



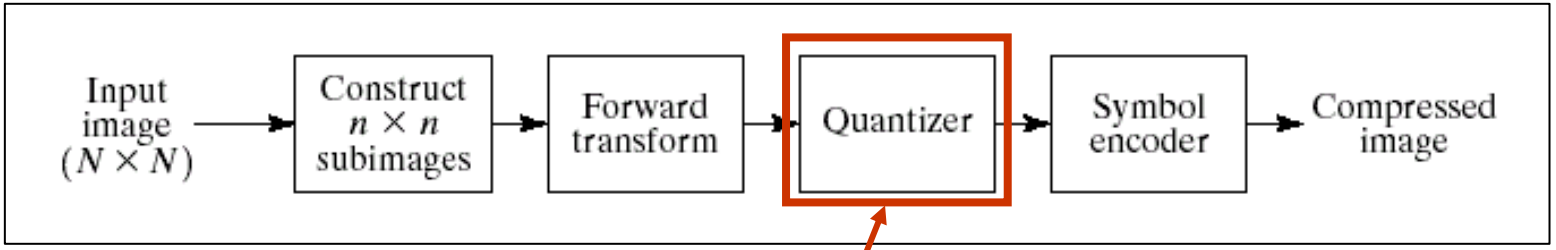
Discrete Fourier Transform
Discrete Cosine Transform
Karhunen-Loeve Transform
.....

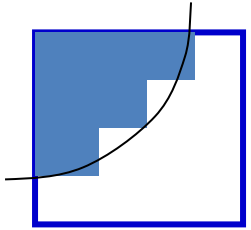
Image Compression

Transform Coding

Pipeline

Compression





Threshold Coding
Zonal Coding

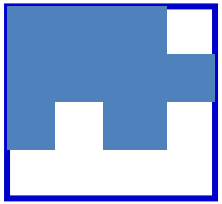
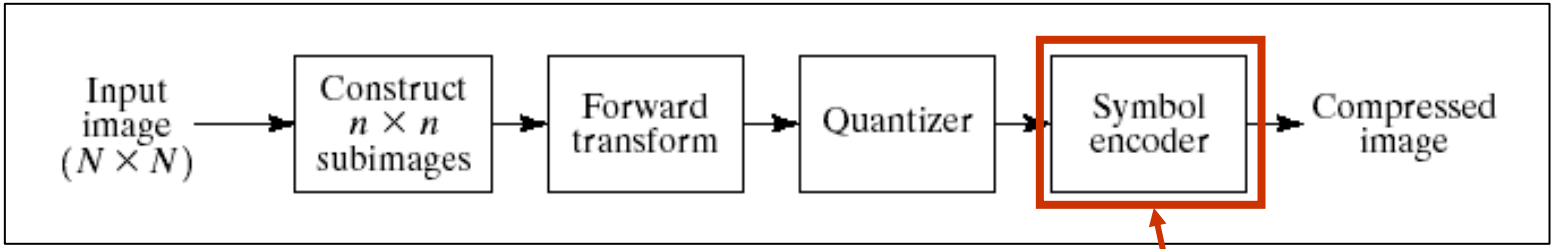


Image Compression

Transform Coding

Pipeline

Compression



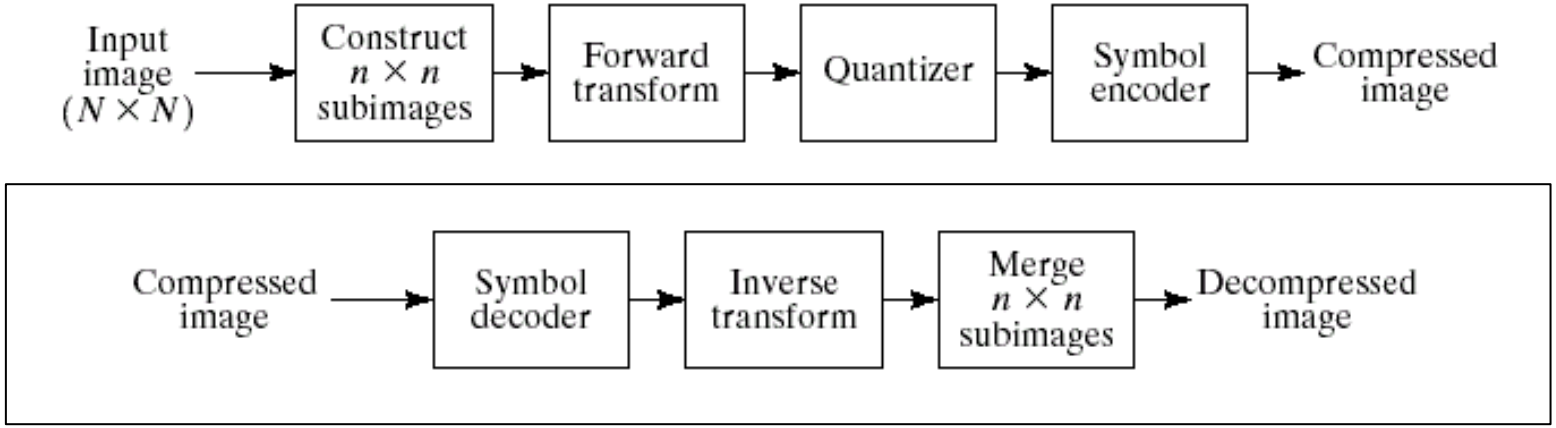
Variable Length Coding (Huffman)

Image Compression

Transform Coding

Pipeline

Compression



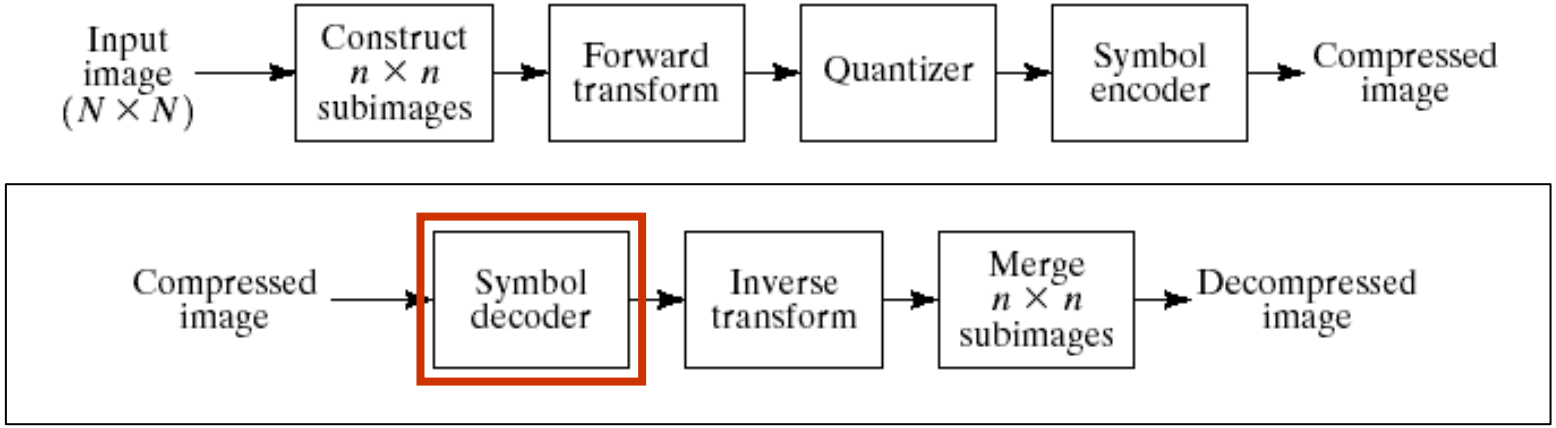
Decompression

Image Compression

Transform Coding

Pipeline

Compression



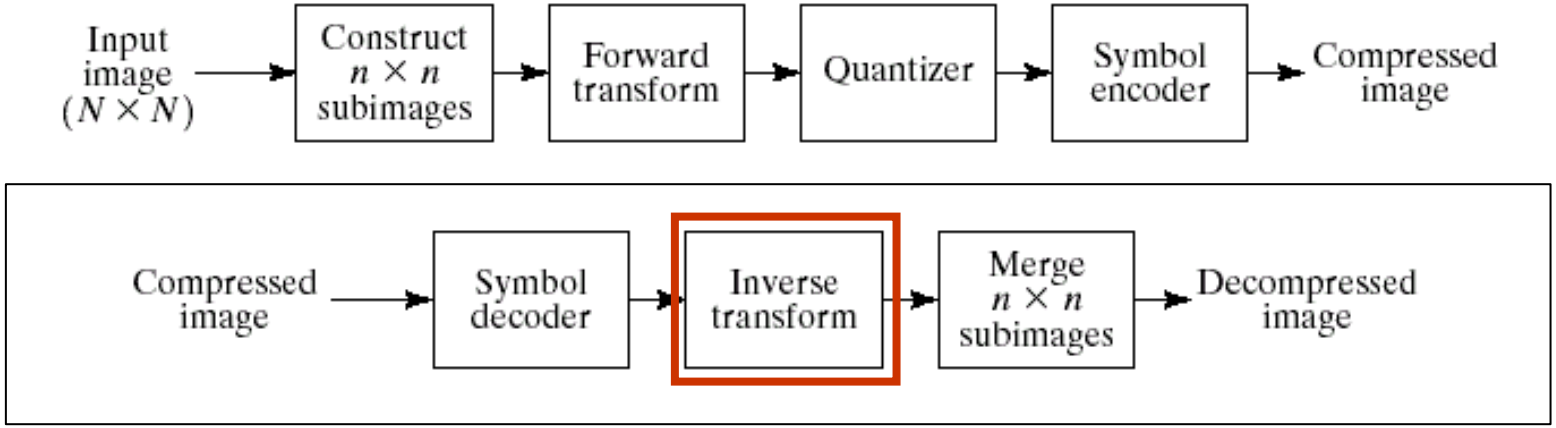
Decompression

Image Compression

Transform Coding

Pipeline

Compression



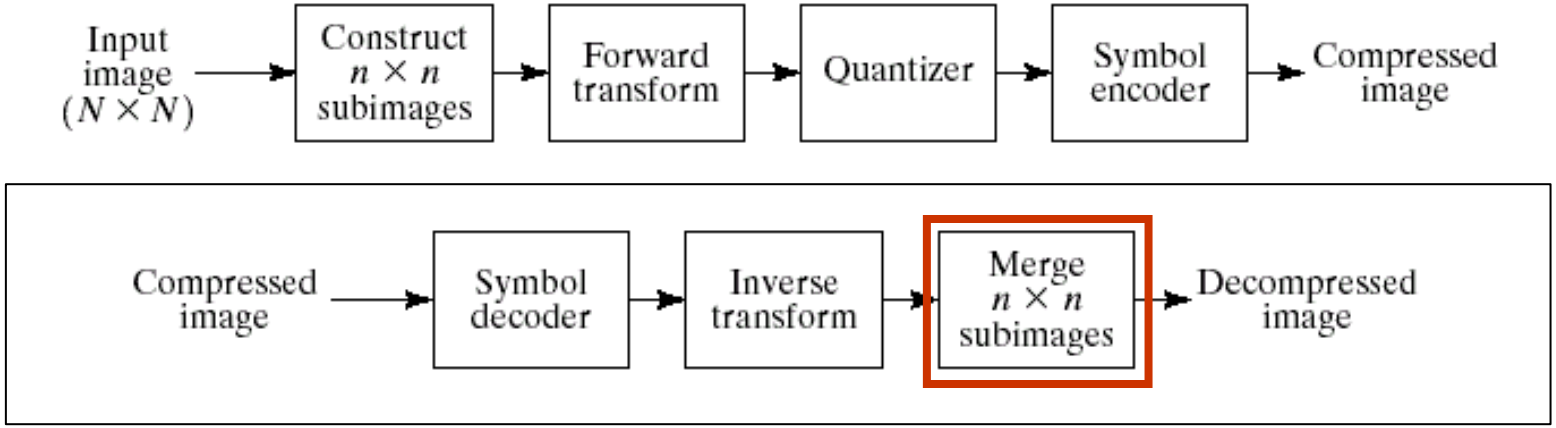
Decompression

Image Compression

Transform Coding

Pipeline

Compression



Decompression