

COL783: Digital Image Processing

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Image Restoration

Linear, Position Invariant Degradation

Input-output relationship

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$



$$\eta(x, y) = 0$$

$$g(x, y) = H[f(x, y)]$$

Image Restoration

Linear, Position Invariant Degradation

- H is linear if

$$\begin{aligned} H[af_1(x, y) + bf_2(x, y)] \\ = aH[f_1(x, y)] + bH[f_2(x, y)] \end{aligned}$$

- Additivity

$$\begin{aligned} H[f_1(x, y) + f_2(x, y)] \\ = H[f_1(x, y)] + H[f_2(x, y)] \end{aligned}$$

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Linear, Position Invariant Degradation

- Homogeneity

$$H[af_1(x, y)] = aH[f_1(x, y)]$$

- Position (or space) invariant

$$H[f(x - \alpha, y - \beta)] = g(x - \alpha, y - \beta)$$

Image Restoration

Linear, Position Invariant Degradation

- In terms of a continuous impulse function

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) \delta(x - \alpha, y - \beta) d\alpha d\beta$$

$$g(x, y) = H[f(x, y)]$$

$$= H \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) \delta(x - \alpha, y - \beta) d\alpha d\beta \right]$$

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Linear, Position Invariant Degradation

$$g(x, y)$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H[f(\alpha, \beta) \delta(x - \alpha, y - \beta)] d\alpha d\beta$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) H[\delta(x - \alpha, y - \beta)] d\alpha d\beta$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x, \alpha, y, \beta) d\alpha d\beta$$

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Linear, Position Invariant Degradation

- Impulse response of H

$$h(x, \alpha, y, \beta) = H[\delta(x - \alpha, y - \beta)]$$

- In optics, the impulse becomes a point of light
- Point spread function (PSF)

$$h(x, \alpha, y, \beta)$$

- All physical optical systems blur (spread) a point of light to some degree

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Linear, Position Invariant Degradation

- If H is position invariant

$$H[\delta(x - \alpha, y - \beta)] = h(x - \alpha, y - \beta)$$

- Convolution integral

$$g(x, y) =$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x - \alpha, y - \beta) d\alpha d\beta$$

Image Restoration

Linear, Position Invariant Degradation

- If H is position invariant

$$H[\delta(x - \alpha, y - \beta)] = h(x - \alpha, y - \beta)$$

- Convolution integral

$$g(x, y) =$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x - \alpha, y - \beta) d\alpha d\beta$$

Image Restoration

Linear, Position Invariant Degradation

With added noise.

$$g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) h(x - \alpha, y - \beta) d\alpha d\beta + \eta(x, y)$$

Image Restoration

Linear, Position Invariant Degradation

- If H is position invariant

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- Restoration approach
 - Image deconvolution
 - Deconvolution filter

Image Restoration

Degradation Estimation/Modeling

- Estimation by image observation
 - In order to reduce the effect of noise in our observation, we would look for areas of strong signal content

$$H_s(u, v) = \frac{G_s(u, v)}{\hat{F}_s(u, v)}$$

Image Restoration

Degradation Estimation/Modeling

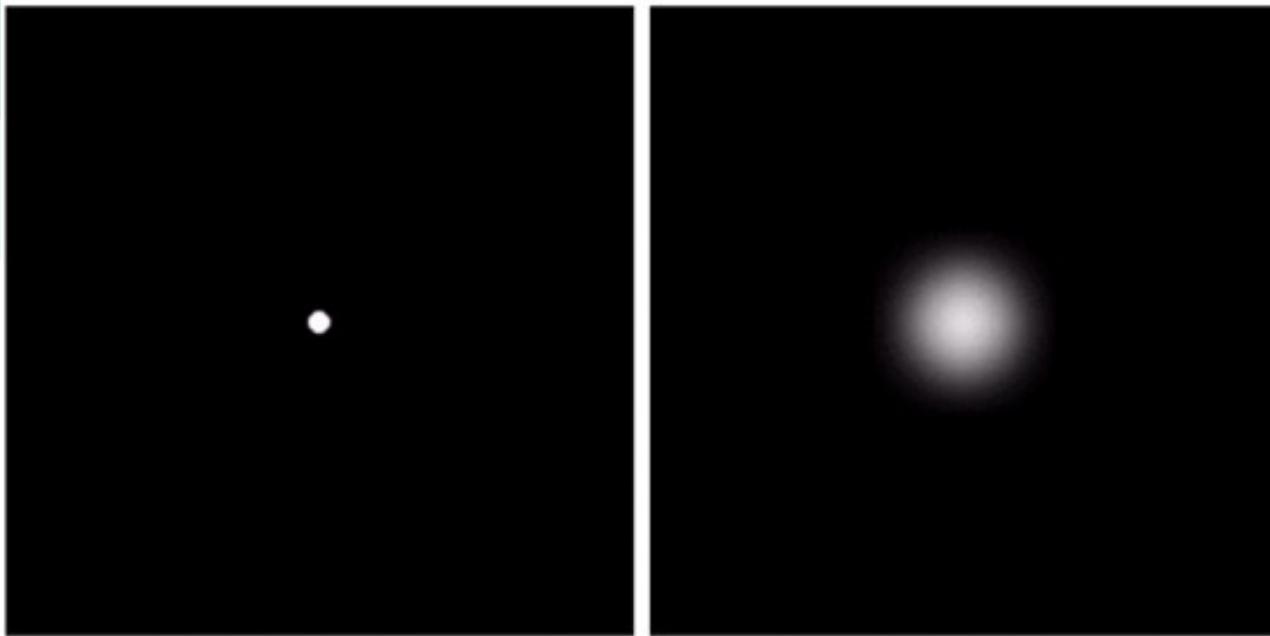
- Estimation by experimentation
 - Obtain the impulse response of the degradation by imaging an impulse (small dot of light) using the same system settings

$$H(u, v) = \frac{G(u, v)}{A}$$

- Observed image $G(u, v)$
- The strength of the impulse A

Image Restoration

Degradation Estimation/Modeling



a b

FIGURE 5.24
Degradation estimation by impulse characterization.
(a) An impulse of light (shown magnified).
(b) Imaged (degraded) impulse.

Image Restoration

Degradation Estimation/Modeling

- Estimation by modeling
 - Hufnagel and Stanley
 - Physical characteristic of atmospheric turbulence

$$H(u, v) = e^{-k(u^2 + v^2)^{\frac{5}{6}}}$$

Image Restoration

Degradation Estimation/Modeling

a
b
c
d

FIGURE 5.25
Illustration of the atmospheric turbulence model.
(a) Negligible turbulence.
(b) Severe turbulence,
 $k = 0.0025$.
(c) Mild turbulence,
 $k = 0.001$.
(d) Low turbulence,
 $k = 0.00025$.
(Original image courtesy of NASA.)



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Degradation Estimation/Modeling

- Image motion

$$g(x, y) = \int_0^T f[x - x_0(t), y - y_0(t)] dt$$

Image Restoration

Degradation Estimation/Modeling

$$\begin{aligned} G(u, v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) e^{-j2\pi(u x + v y)} dx dy \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_0^T f[x - x_0(t), y - y_0(t)] dt \right] \\ &\quad e^{-j2\pi(u x + v y)} dx dy \\ &= \int_0^T \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f[x - x_0(t), y - y_0(t)] \right. \\ &\quad \left. e^{-j2\pi(u x + v y)} dx dy \right] dt \end{aligned}$$

Image Restoration

Degradation Estimation/Modeling

$$\begin{aligned} G(u, v) &= \int_0^T F(u, v) e^{-j2\pi[u x_0(t) + v y_0(t)]} dt \\ &= F(u, v) \int_0^T e^{-j2\pi[u x_0(t) + v y_0(t)]} dt \\ &= F(u, v) H(u, v) \end{aligned}$$

- Where

$$H(u, v) = \int_0^T e^{-j2\pi[u x_0(t) + v y_0(t)]} dt$$

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Degradation Estimation/Modeling

- If $x_0(t) = at/T$ and $y_0(t) = 0$

$$\begin{aligned} H(u, v) &= \int_0^T e^{-j2\pi[u x_0(t)]} dt \\ &= \int_0^T e^{-j2\pi[u at/T]} dt \\ &= \frac{T}{\pi ua} \sin(\pi ua) e^{-j\pi ua} \end{aligned}$$

Image Restoration

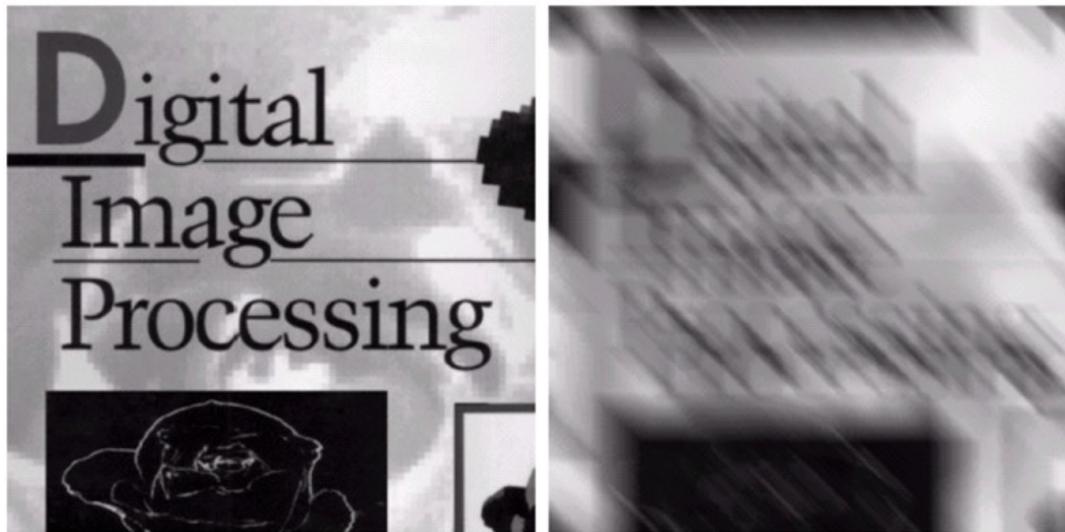
Degradation Estimation/Modeling

- If $x_0(t) = at / T$ and $y_0(t) = bt / T$

$$H(u, v) = \frac{T}{\pi(ua + vb)} \sin[\pi(ua + vb)] e^{-j\pi(ua + vb)}$$

Image Restoration

Degradation Estimation/Modeling



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.