Image Compression

Compression Ratio

$$C_r = n_o/n_c$$

n_o = Number of carrying units (bits) in the original data (image)

n_r = Number of carrying units (bits) in the compressed data (image)

Also,

$$R_d = 1 - 1/\ C_r$$

R_d = Relative data redundancy

Image Compression

Fidelity Criteria

Measure of loss or degradation

- Mean Square Error (MSE)

$$MSE = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left[f(i,j) - f'(i,j)\right]^2$$

- Signal to Noise Ratio (SNR)
- Subjective Voting

Image Compression

Compression Techniques

- Loss-less Compression

  Information can be compressed and restored without any loss of information

- Lossy Compression

  Large compression, perfect recovery is not possible

Image Compression

Compression Techniques

Symmetric

- Same time for compression (coding) and decompression (decoding)
- Used for dialog (interactive) mode applications

Asymmetric

- Compression is done once so can take longer
- Decompression is done frequently so should be fast
- Used for retrieval model applications

Image Compression

Data Redundancy

- Coding

  Variable length coding with shorter codes for frequent symbols
- Interpixel

  Neighboring pixels are similar
- Psychovisual

  Human visual perception - limited

Image Compression

Coding Redundancy

Example: (from Digital Image Processing by Gonzalez and Woods)

r_k	$p_r(r_k)$	Code 1	$l_1(r_k)$	Code 2	$l_2(r_k)$
$r_0 = 0$	0.19	000	3	11	2
$r_1 = 1/7$	0.25	001	3	01	2
$r_2 = 2/7$	0.21	010	3	10	2
$r_3 = 3/7$	0.16	011	3	001	3
$r_4 = 4/7$	0.08	100	3	0001	4
$r_5 = 5/7$	0.06	101	3	00001	5
$r_6 = 6/7$	0.03	110	3	000001	6
$r_7 = 1$	0.02	111	3	000000	6

TABLE 8.1 Example of variable-length coding.

fixed length coding
Avg length=3 bits

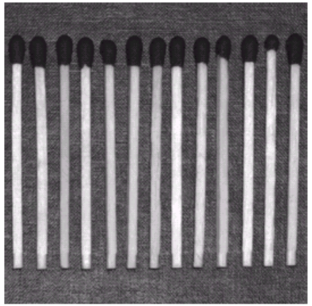
variable length coding
Avg length=2.7 bits

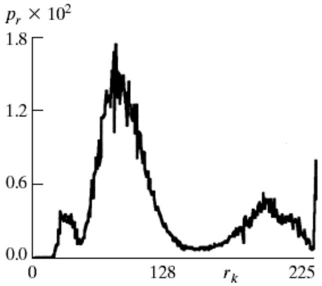
Image Compression

Interpixel Redundancy

Example: (from Digital Image Processing by Gonzalez and Woods)

Image





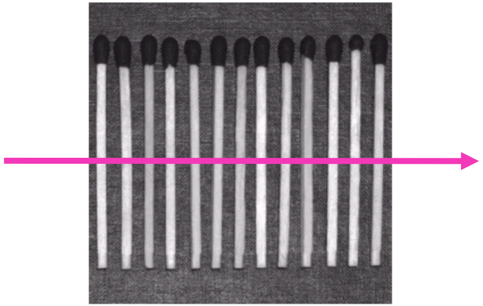
Histogram

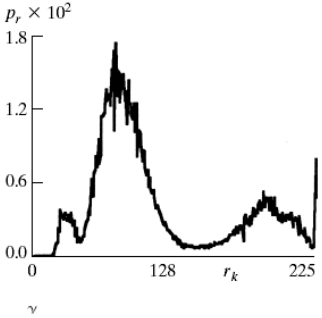
Image Compression

Interpixel Redundancy

Example: (from Digital Image Processing by Gonzalez and Woods)

Image





Histogram

High interpixel correlation

Image Compression

Psychovisual Redundancy

Example: (from Digital Image Processing by Gonzalez and Woods)

Original 256 levels **16 level quantization** **IGS quantization**

Image Compression

Loss-less Techniques

- Coding redundancy

  Variable length coding

- Interpixel redundancy

  Run length coding
  Predictive coding

Image Compression

Variable Length Coding (Huffman Coding)

Sequence of symbols (a1, a2, a3, a4, a5) with associated probabilities (p1, p2, p3, p4, p5)

- Start with two symbols of the least probability

  a1:p1

  a2:p2
- Combine (a1 or a2) with probability (p1+p2)
- Do it recursively (sort and combine)
- A binary tree construction

Image Compression

Variable Length Coding (Huffman Coding)

Example:

Symbols and their probabilities of occurrence
a1 (0.2), a2 (0.4), a3 (0.2), a4 (0.1), a5 (0.1)

Sort in probability

a2 (0.4)

a1(0.2)

a3(0.2)

a4(0.1)

a5(0.1)

Image Compression

Variable Length Coding (Huffman Coding)

Sort

a2 (0.4)

a1(0.2)

a3(0.2)

a4(0.1)

a5(0.1)

Image Compression

Variable Length Coding (Huffman Coding)

Sort combine

a2 (0.4)

a1(0.2)

a3(0.2)

a4(0.1) 0.2

a5(0.1)

Image Compression

Variable Length Coding (Huffman Coding)

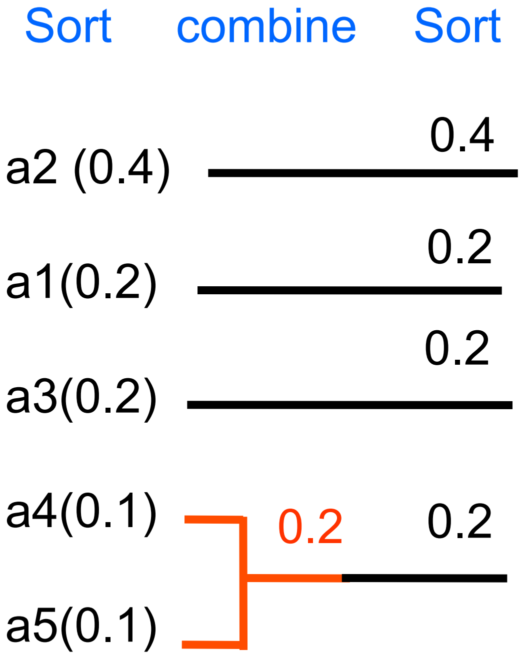


Image Compression

Variable Length Coding (Huffman Coding)

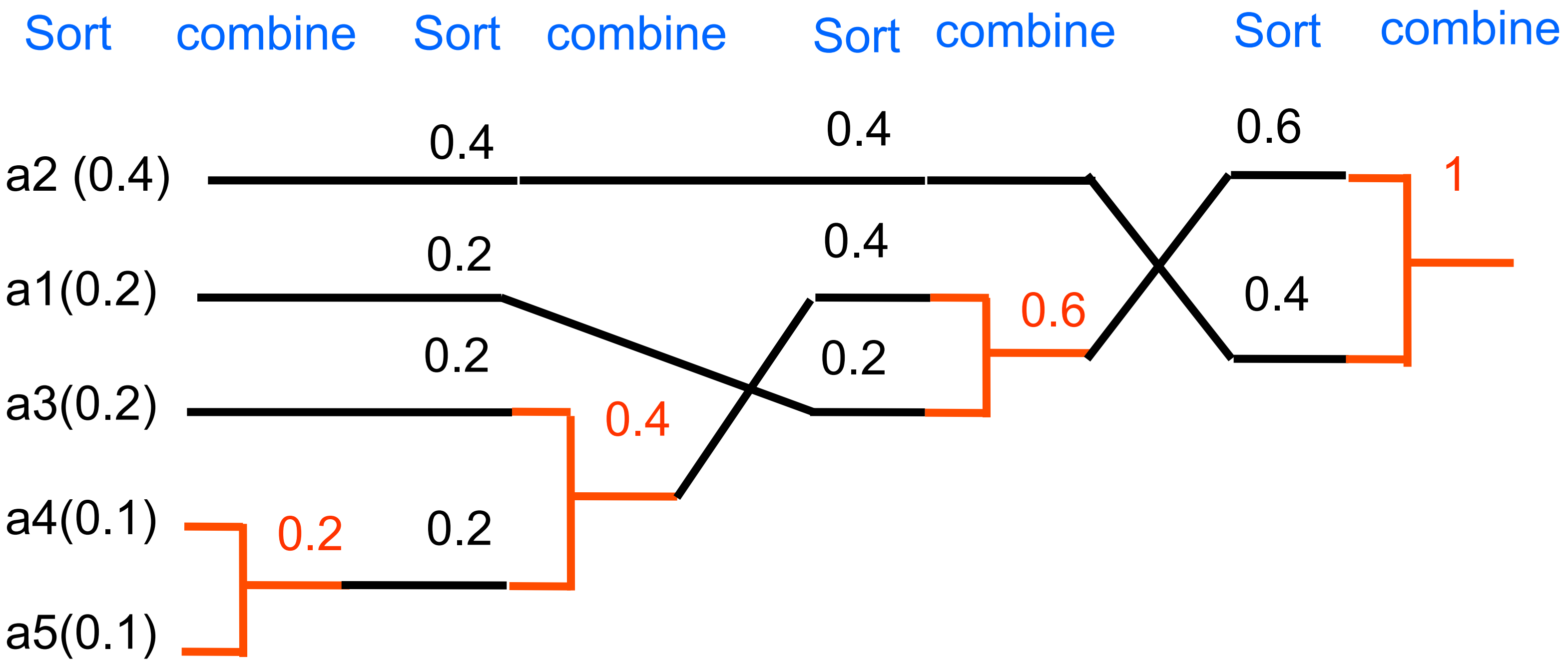


Image Compression

Variable Length Coding (Huffman Coding)

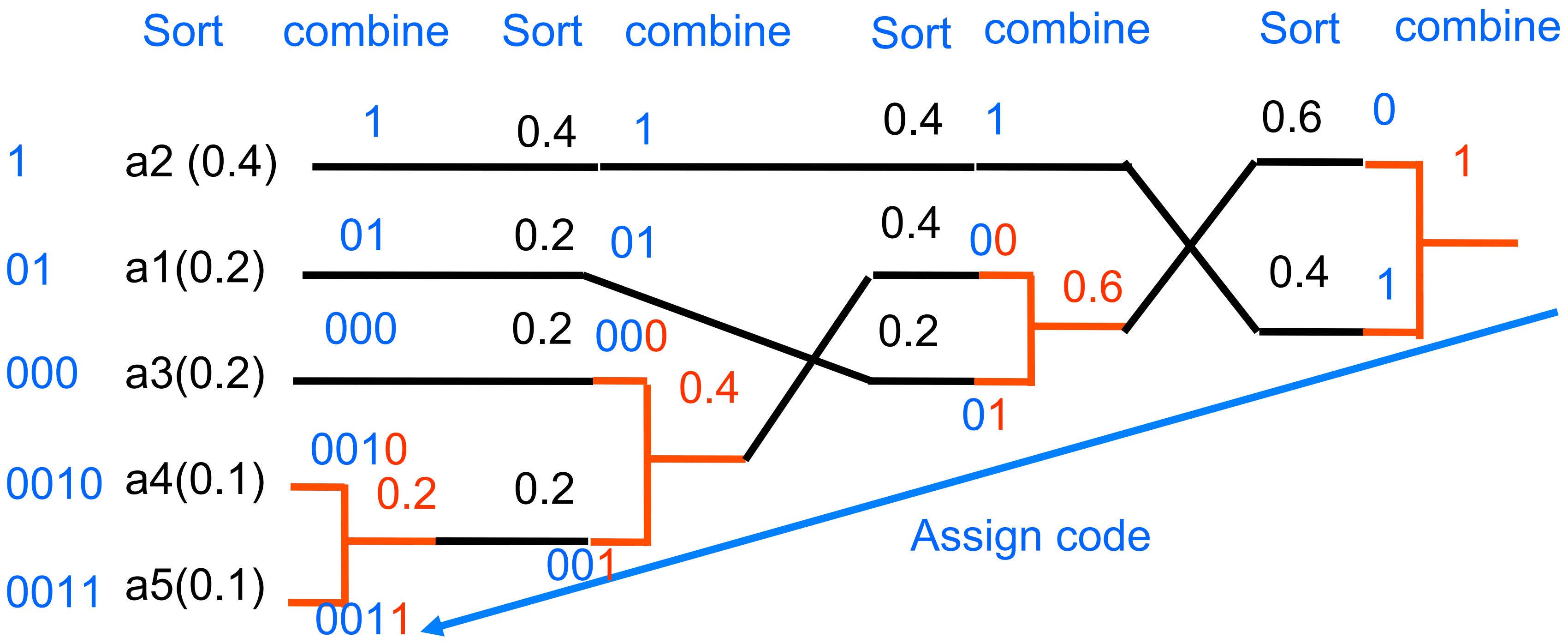


Image Compression

Variable Length Coding (Huffman Coding)

Example:

Avg length code:

$$0.4x1 + 0.2x2 + 0.2x3 + 0.1x4 + 0.1x4$$

$$= 2.2 \text{ bits}$$

Image Compression

Variable Length Coding (Huffman Coding)

Entropy

A measure of information that captures uncertainity
[I(e) = log (1/P(e))]

$$H = -\sum_{i=0}^{L-1} p(a_i) \log_2 p(a_i) \quad \text{bits / symbol}$$

Image Compression

Entropy

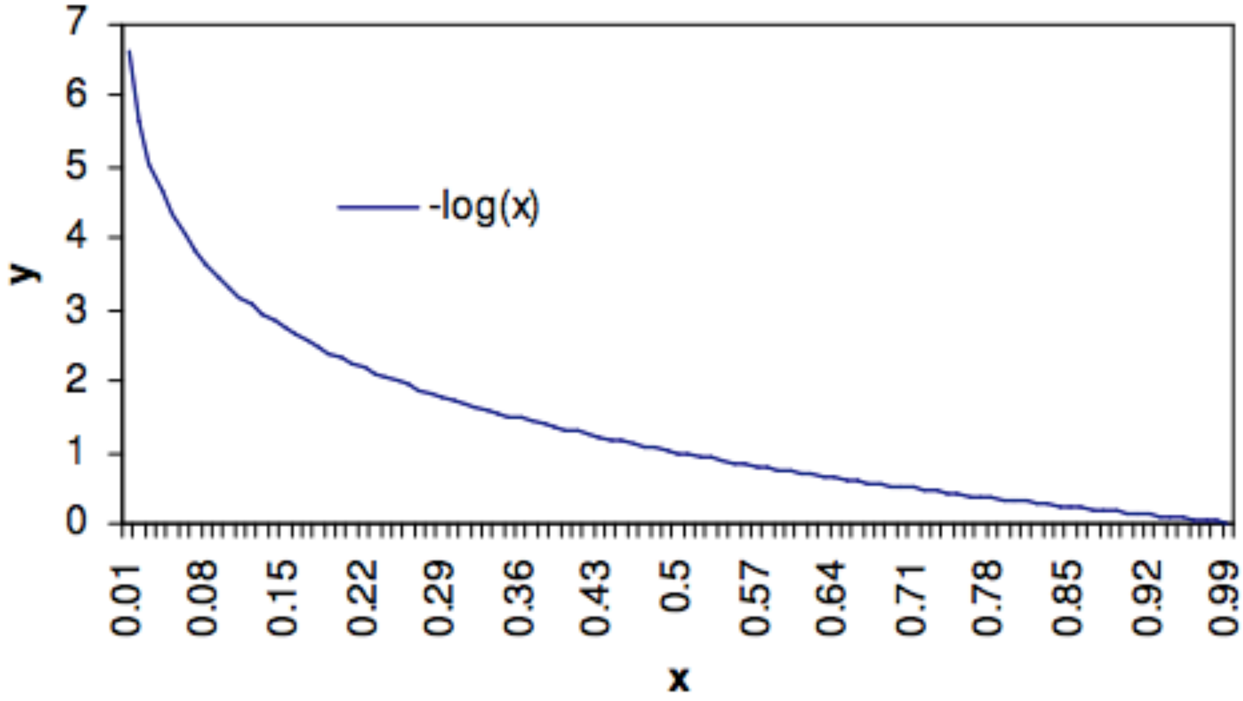


Image Compression

Entropy

Example:

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

$$P_0 = \frac{63}{64}$$

$$P_1 = \frac{1}{64}$$

$$H = -\frac{63}{64}\log_2\frac{63}{64} - \frac{1}{64}\log_2\frac{1}{64}$$

$$= 0.116 \text{ bits/pixel}$$

0	1	1	0	1	0	1	0
1	0	1	0	1	0	0	1
1	1	0	1	0	1	1	0
0	1	0	0	1	1	0	0
1	0	0	0	1	0	1	1
0	0	1	0	1	1	1	1
0	1	0	1	1	1	0	1
0	1	0	0	0	1	0	0

$$P_0 = \frac{32}{64} = \frac{1}{2}$$

$$P_1 = \frac{32}{64} = \frac{1}{2}$$

$$H = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2}$$

$$= 1.0 \text{ bits/pixel}$$

Image Compression

Entropy

Example:

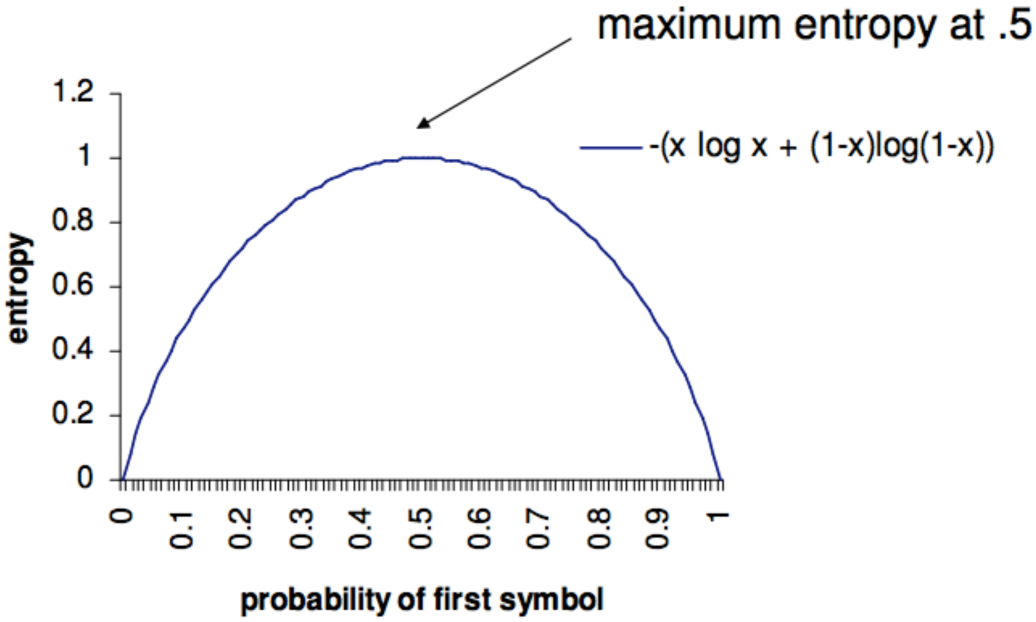
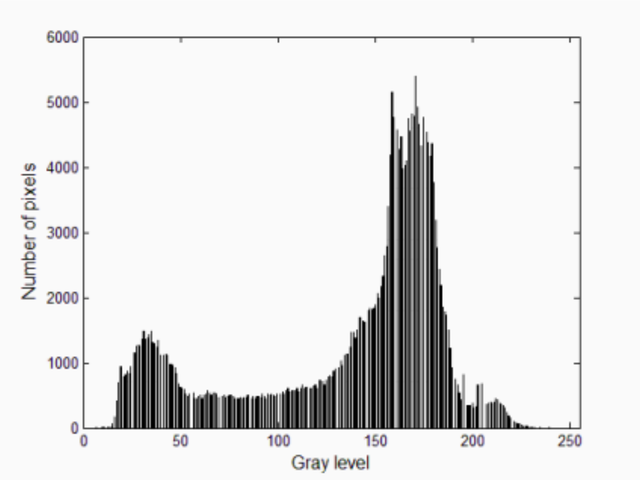


Image Compression

Entropy

Example:

- Image Histogram



Entropy = 7.63 bits/pixel

Image Compression

Variable Length Coding (Huffman Coding)

Example:

Decoding

00111010001

?

Image Compression

Variable Length Coding (Huffman Coding)

Example:

Decoding

00111010001

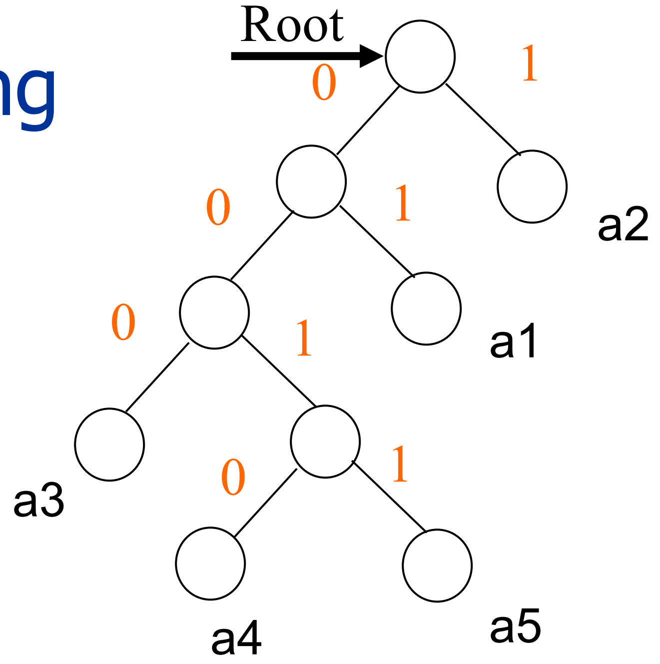


Image Compression

Variable Length Coding (Huffman Coding)

Decoding

Example:

0011101 0001

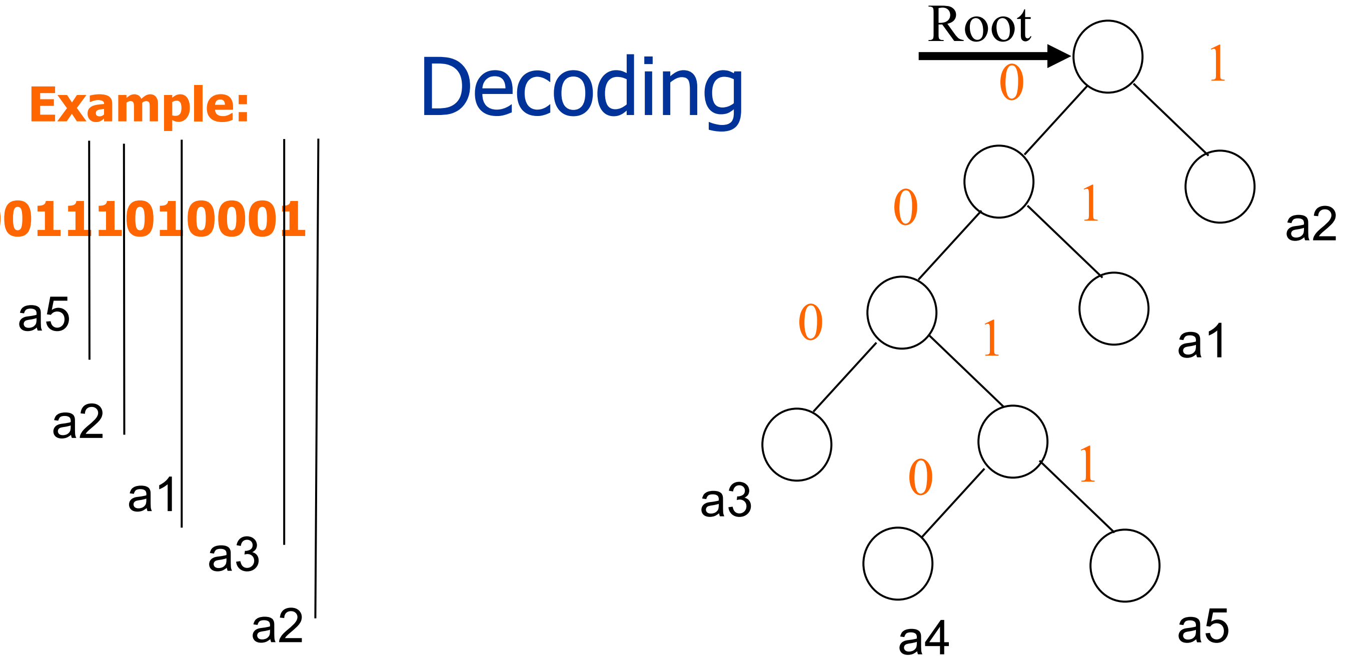


Image Compression

A Method for the Construction of Minimum-Redundancy Codes*

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Summary—An optimum method of coding an ensemble of messages consisting of a finite number of members is developed. A minimum-redundancy code is one constructed in such a way that the average number of coding digits per message is minimized.

Introduction

ONE IMPORTANT METHOD of transmitting messages is to transmit in their place sequences of symbols. If there are more messages which might be sent than there are kinds of symbols available, then some of the messages must use more than one symbol. If it is assumed that each symbol requires the same time for transmission, then the time for transmission (length) of a message is directly proportional to the number of symbols associated with it. In this paper, the symbol or sequence of symbols associated with a given message will be called the "message code." The entire will be defined here as an ensemble code which, for a message ensemble consisting of a finite number of members, N, and for a given number of coding digits, D, yields the lowest possible average message length. In order to avoid the use of the lengthy term "minimum-redundancy," this term will be replaced here by "optimum." It will be understood then that, in this paper, "optimum code" means "minimum-redundancy code."

The following basic restrictions will be imposed on an ensemble code:

(a) No two messages will consist of identical arrangements of coding digits.

(b) The message codes will be constructed in such a way that no additional indication is necessary to specify where a message code begins and ends once the starting point of a sequence of messages