

# COL783: Digital Image Processing

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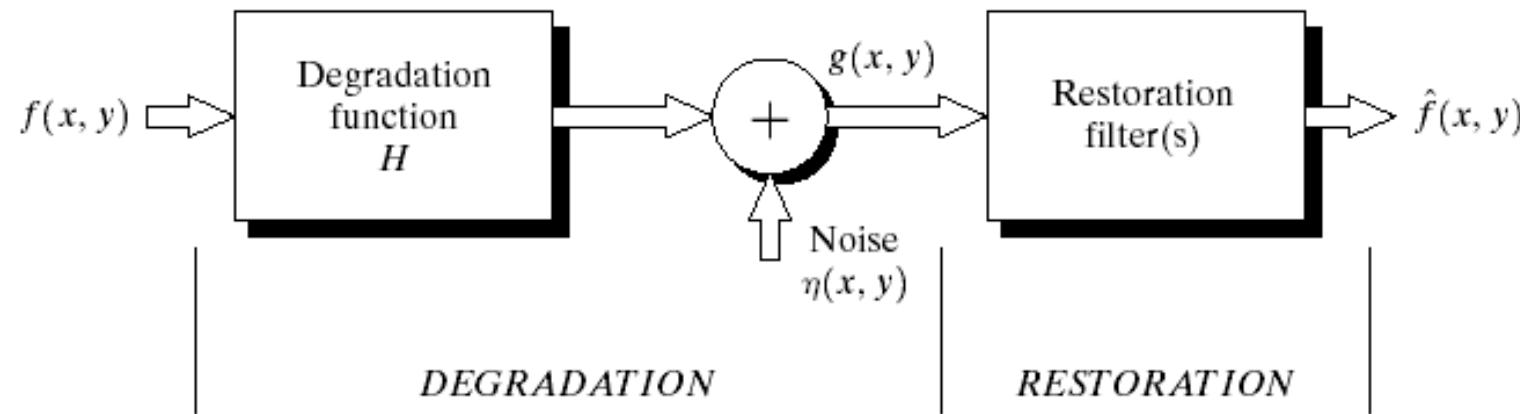
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<http://www.cse.iitd.ac.in/~pkalra/col783>

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# Image Restoration

- Use a priori knowledge of the degradation
- Modeling the degradation and apply the inverse process
- Formulate and evaluate objective criteria of goodness

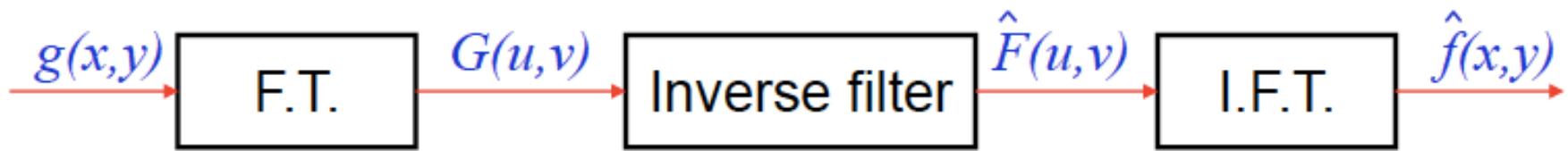


**FIGURE 5.1** A model of the image degradation/ restoration process.

# Image Restoration

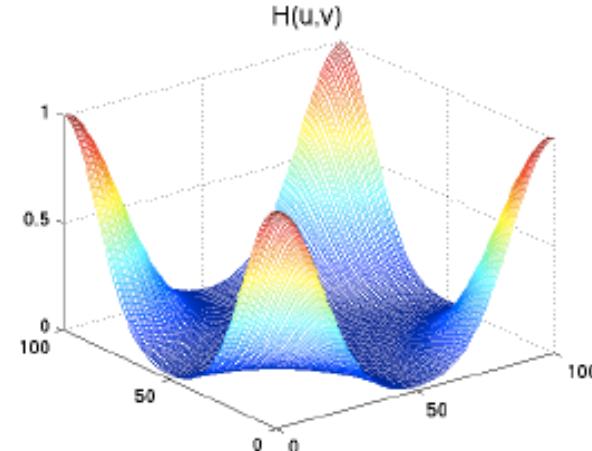
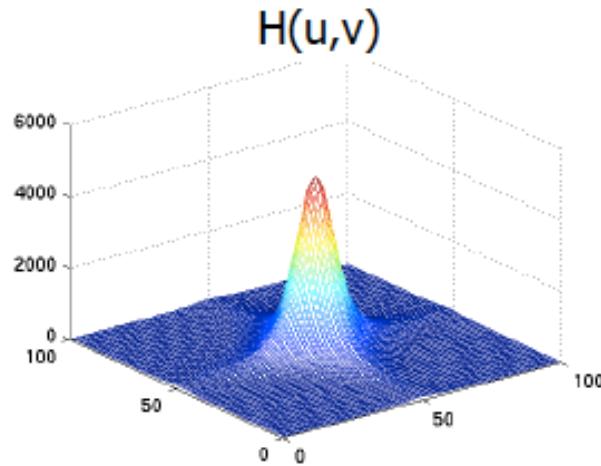
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Restoration with an inverse filter

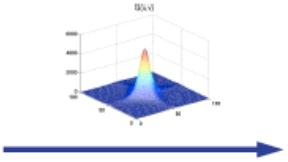


# Image Restoration

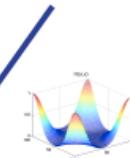
- inverse filter  $\hat{H}(u, v) = 1/H(u, v)$
- recovered image  $\hat{F}(u, v) = G(u, v)\hat{H}(u, v)$



# Image Restoration



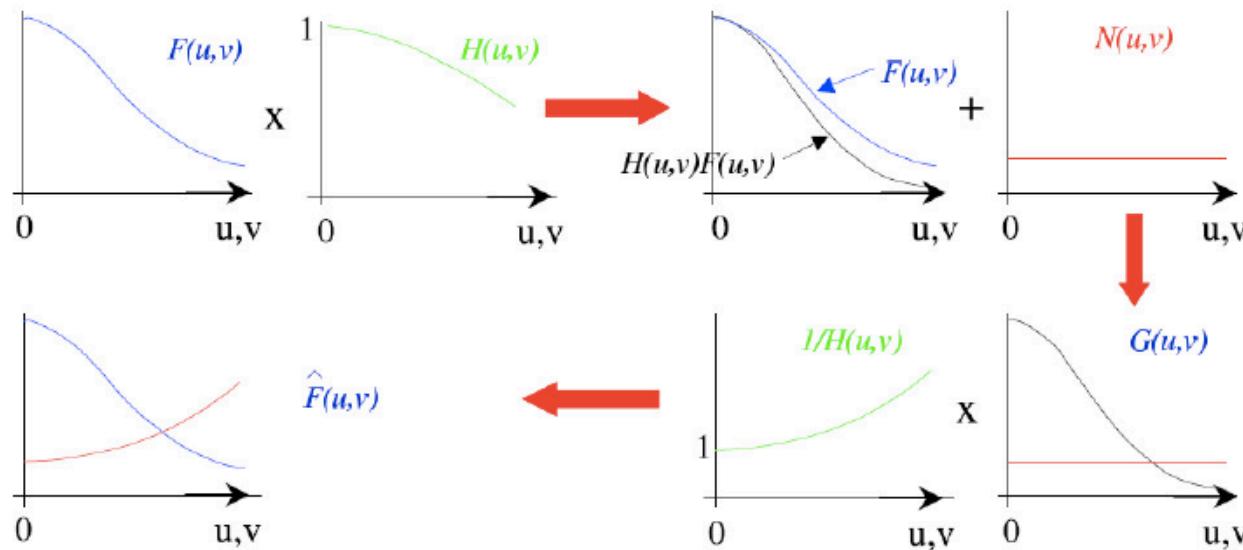
loss of information



# Image Restoration

$$G(u, v) = F(u, v)H(u, v) + N(u, v) \quad \hat{H}(u, v) = 1/H(u, v)$$

$$\hat{F}(u, v) = G(u, v)\hat{H}(u, v) = F(u, v) + \frac{N(u, v)}{\hat{H}(u, v)}$$



# Image Restoration

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Inverse filter with  
cut-off:

$$\hat{H}(u, v) = \begin{cases} 1/H(u, v), & |u^2 + v^2| \leq \eta \\ 0, & |u^2 + v^2| > \eta \end{cases}$$

Pseudo-inverse filter:

$$\hat{H}(u, v) = \begin{cases} 1/H(u, v), & |H(u, v)| \geq \epsilon \\ 0, & |H(u, v)| < \epsilon \end{cases}$$

# Image Restoration

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## Algebraic Approach

$$g = Hf + n$$

# Image Restoration

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## Wiener Filter

$$W(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + S_{\eta\eta}(u, v)/S_{ff}(u, v)}$$

$$W(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + K(u, v)}$$

# Image Restoration

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## Wiener Filter

Aim is to find filter which minimizes

$$\mathcal{E} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (f(x, y) - \hat{f}(x, y))^2 dx dy$$

$$\begin{aligned}\mathcal{E} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |f(x, y) - \hat{f}(x, y)|^2 dx dy \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |F(u, v) - \hat{F}(u, v)|^2 du dv\end{aligned}\quad \text{Parseval's Theorem}$$

$$\hat{F} = WG = WHF + WN$$

$$F - \hat{F} = (1 - WH)F - WN$$

$$\begin{aligned}\mathcal{E} &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |(1 - WH)F - WN|^2 du dv \\ &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \{|(1 - WH)F|^2 + |WN|^2\} du dv\end{aligned}\quad \text{since } f(x, y) \text{ and } \eta(x, y) \text{ uncorrelated}$$

- Note, integrand is sum of two squares

# Image Restoration

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## Wiener Filter

Minimize integral if integrand minimum for all  $(u,v)$

NB  $\frac{\partial}{\partial z}(zz^*) = 2z^*$

$$\frac{\partial}{\partial z} \rightarrow 2(-(1 - W^*H^*)H|F|^2 + W^*|N|^2) = 0$$

$$W^* = \frac{H|F|^2}{|H|^2|F|^2 + |N|^2}$$

$$W = \frac{H^*}{|H|^2 + |N|^2/|F|^2}$$

Note: filter is defined in the Fourier domain

# Image Restoration

## Wiener Filter

