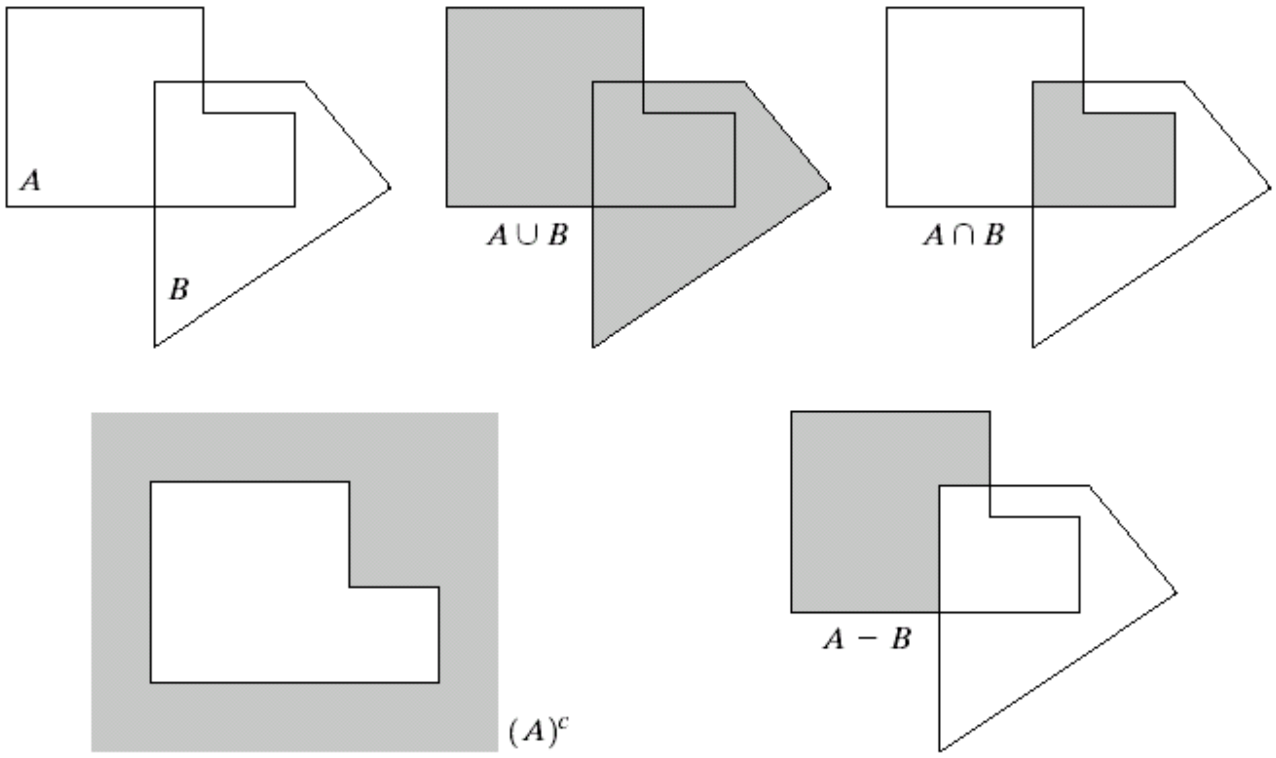
Digital Image Processing

Chapter 9: Morphological Image Processing

Mathematic Morphology

- used to extract image components that are useful in the representation and description of region shape, such as
  - boundaries extraction
  - skeletons
  - convex hull
  - morphological filtering
  - thinning
  - pruning

Basic Set Theory



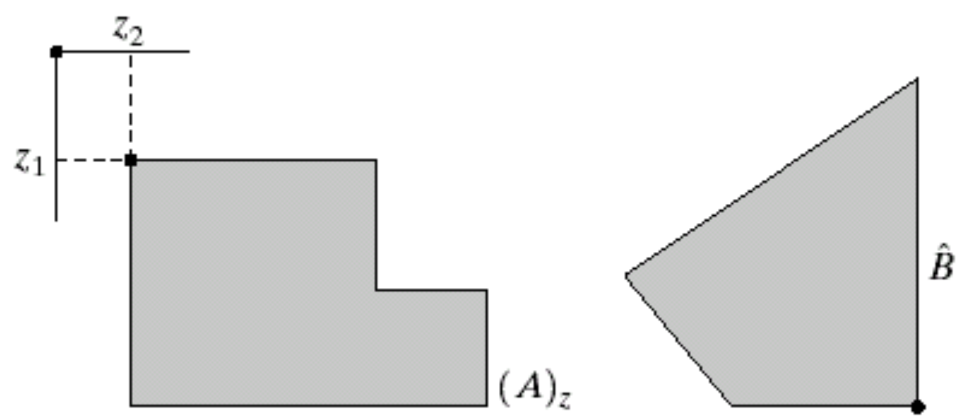
a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Reflection and Translation

$$\hat{B} = \{w \mid w \in -b, \quad for\ b \in B\}$$

$$(A)_z = \{c \mid c \in a + z, \quad for\ a \in A\}$$



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

Example

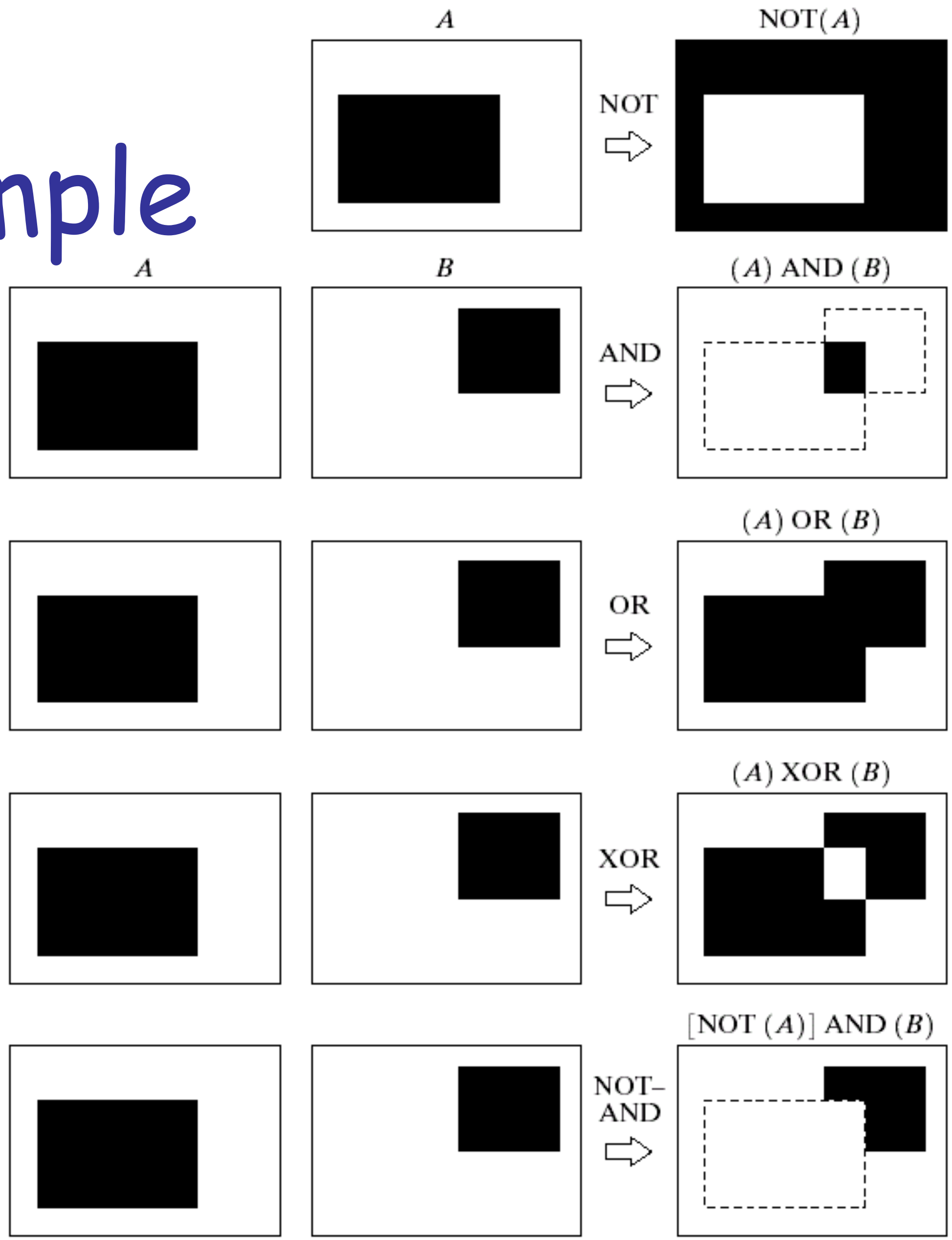
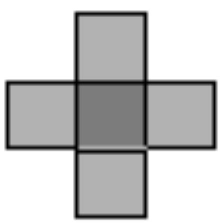
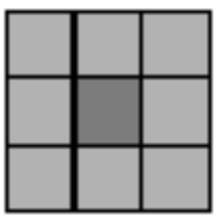


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

Structuring element (SE)

- small set to probe the image under study
- for each SE, define origo
- shape and size must be adapted to geometric properties for the objects

Basic morphological operations

- Erosion shrink
- Dilation grow
- combine to
  - Opening → object
  - Closening → background

keep general shape but smooth with respect to

Erosion

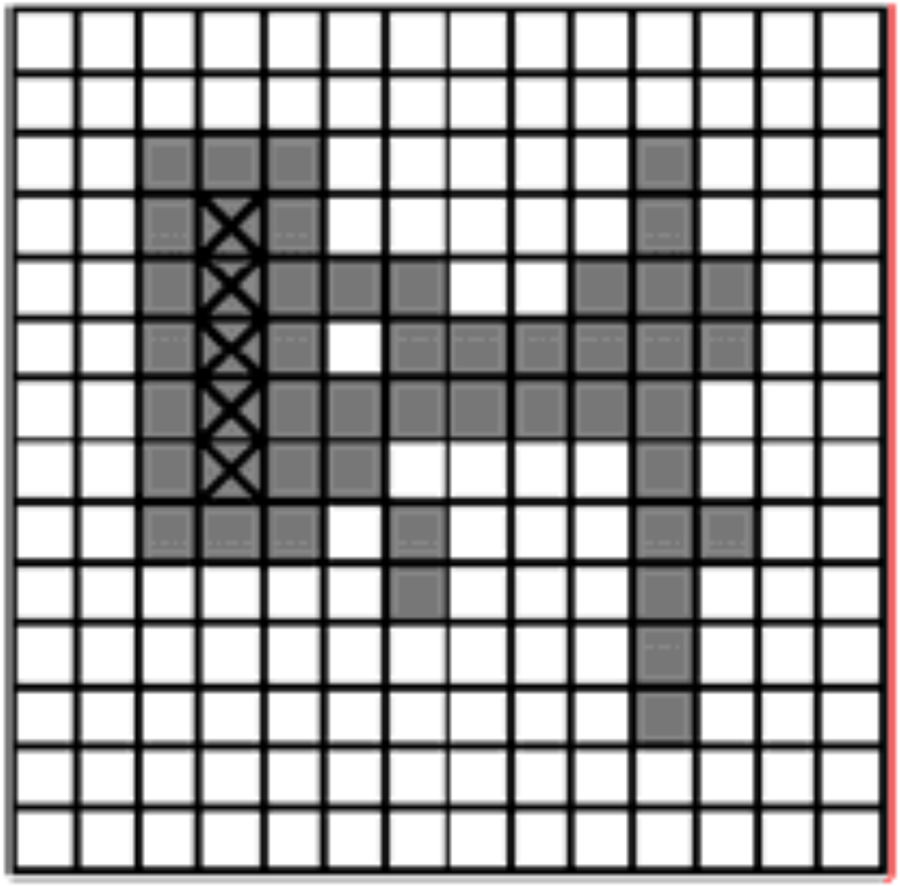
- Does the structuring element **fit the set?**

erosion of a set A by structuring element B: all z in A such that B is in A when origin of B=z

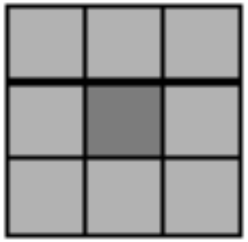
$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

shrink the object

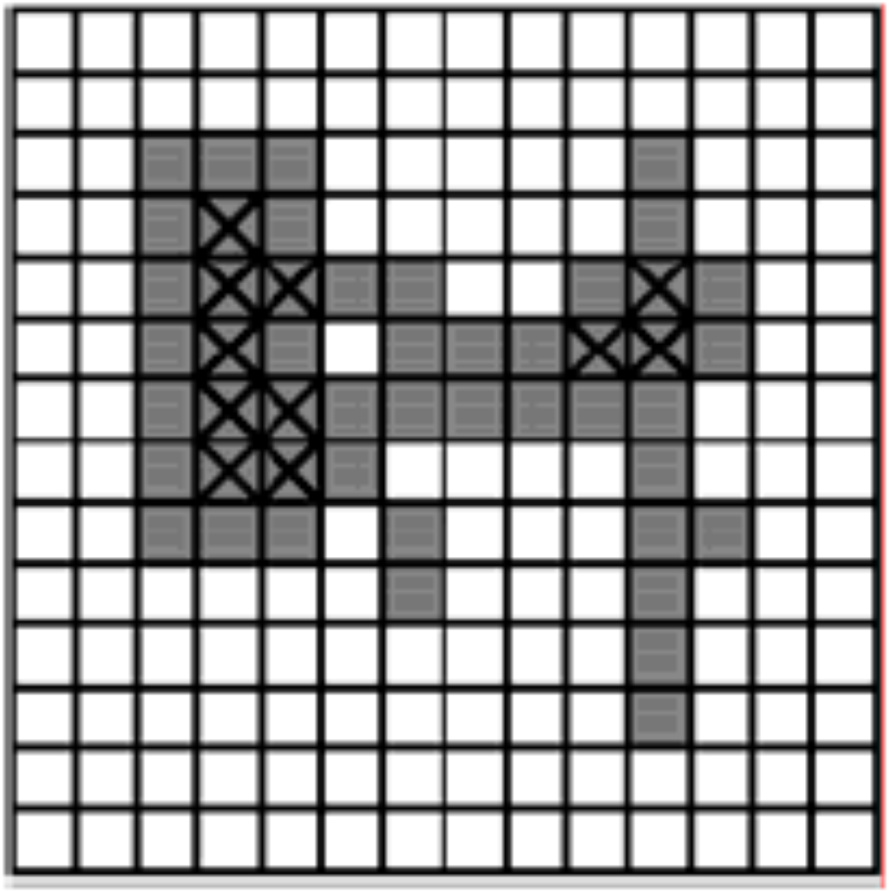
Erosion



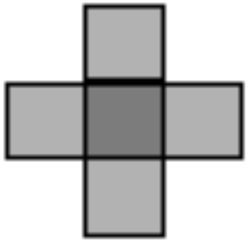
SE=



Erosion



SE=



Erosion

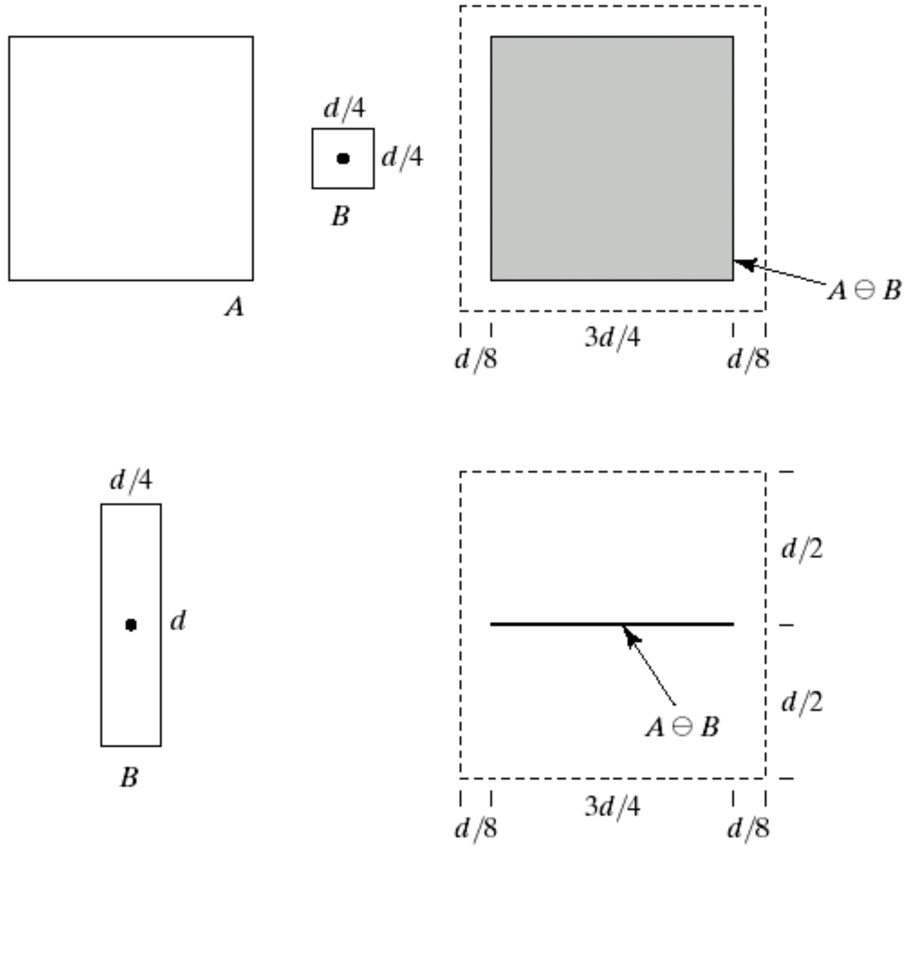


FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

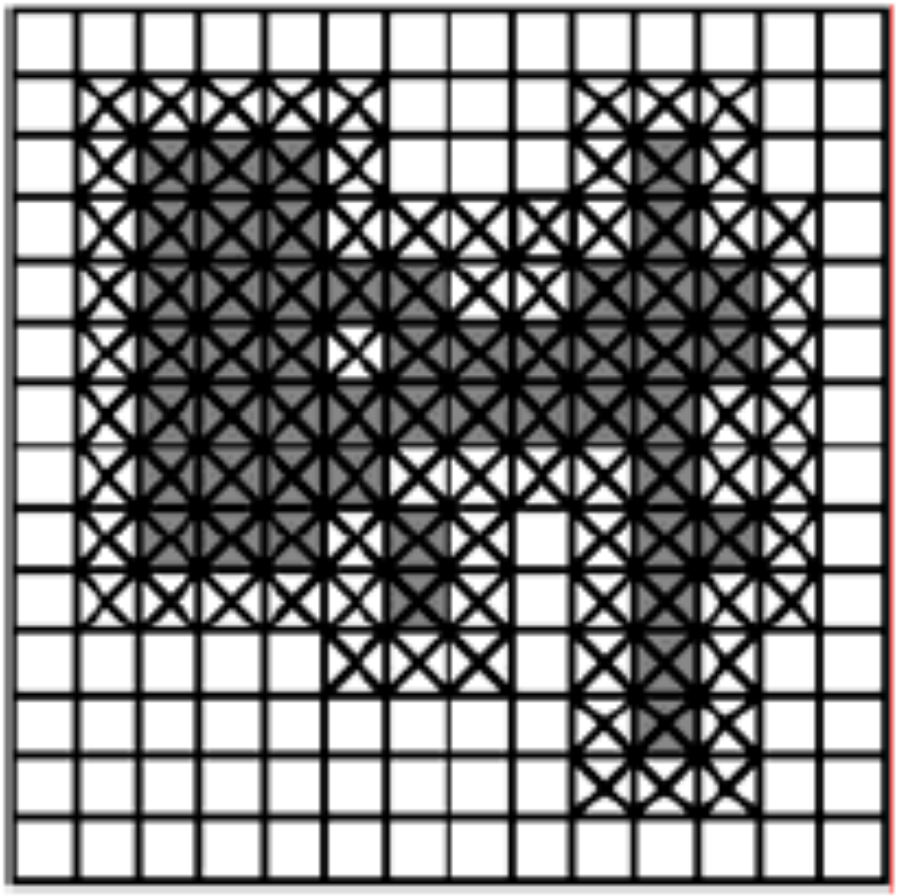
Dilation

- Does the structuring element **hit the set?**
- dilation of a set A by structuring element B: all z in A such that B hits A when origin of B=z

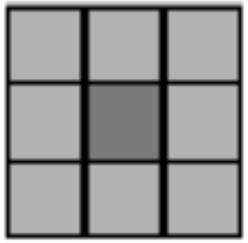
$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \Phi\}$$

- **grow the object**

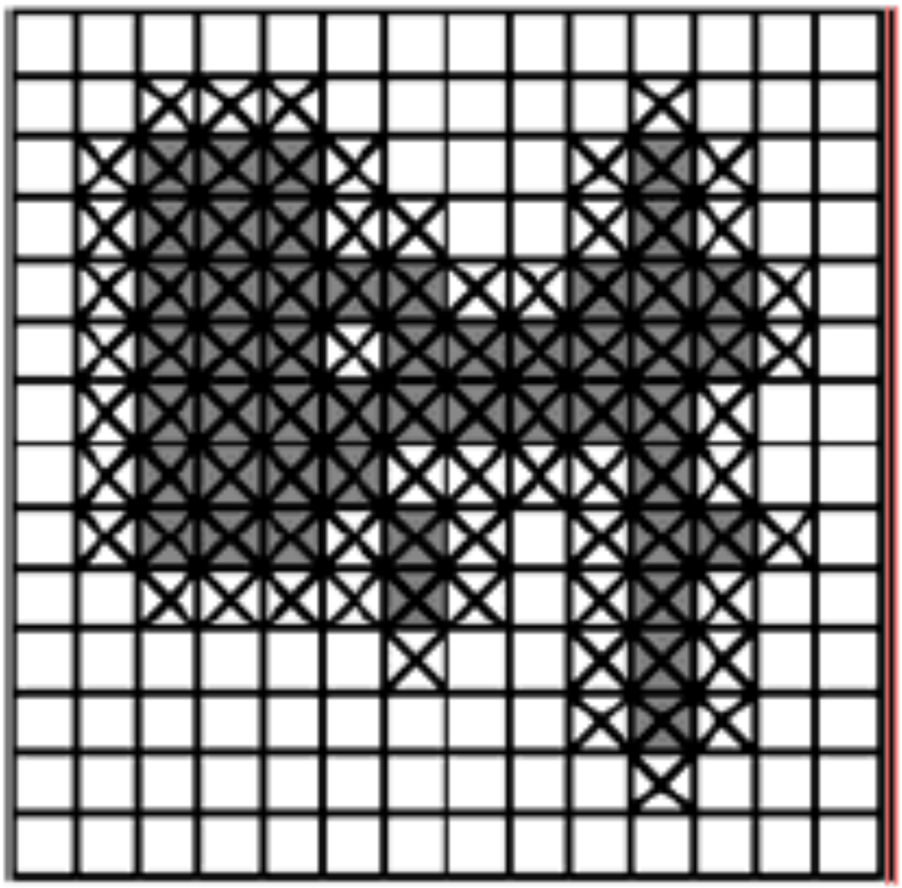
Dilation

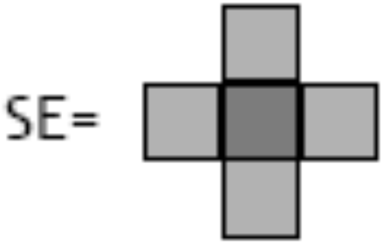


SE=



Dilation





Dilation

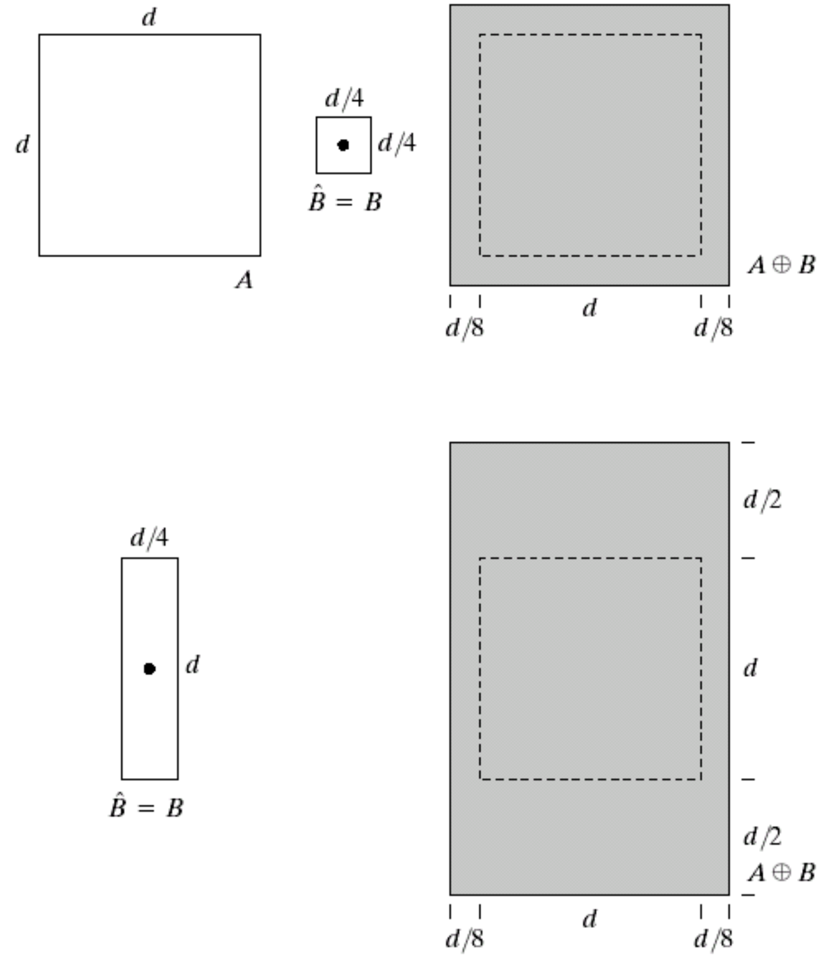
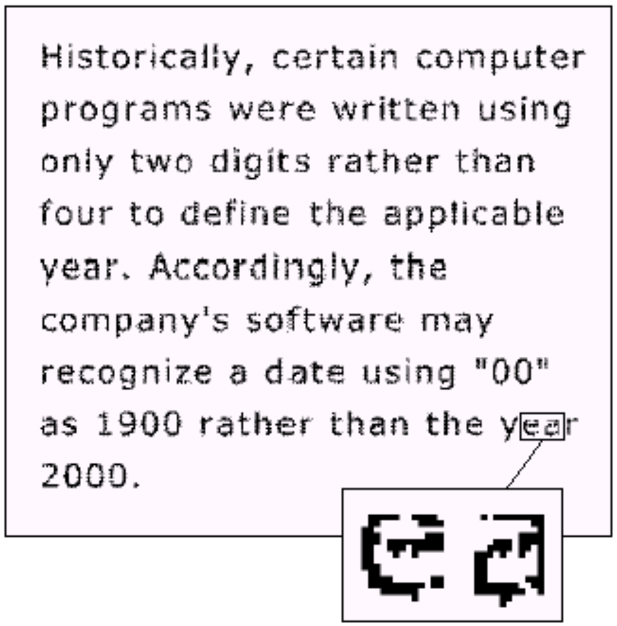


FIGURE 9.4
(a) Set A.
(b) Square structuring element (dot is the center).
(c) Dilation of A by B, shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element.

B = structuring element

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \Phi \}$$

Dilation : Bridging gaps



a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

0	1	0
1	1	1
0	1	0

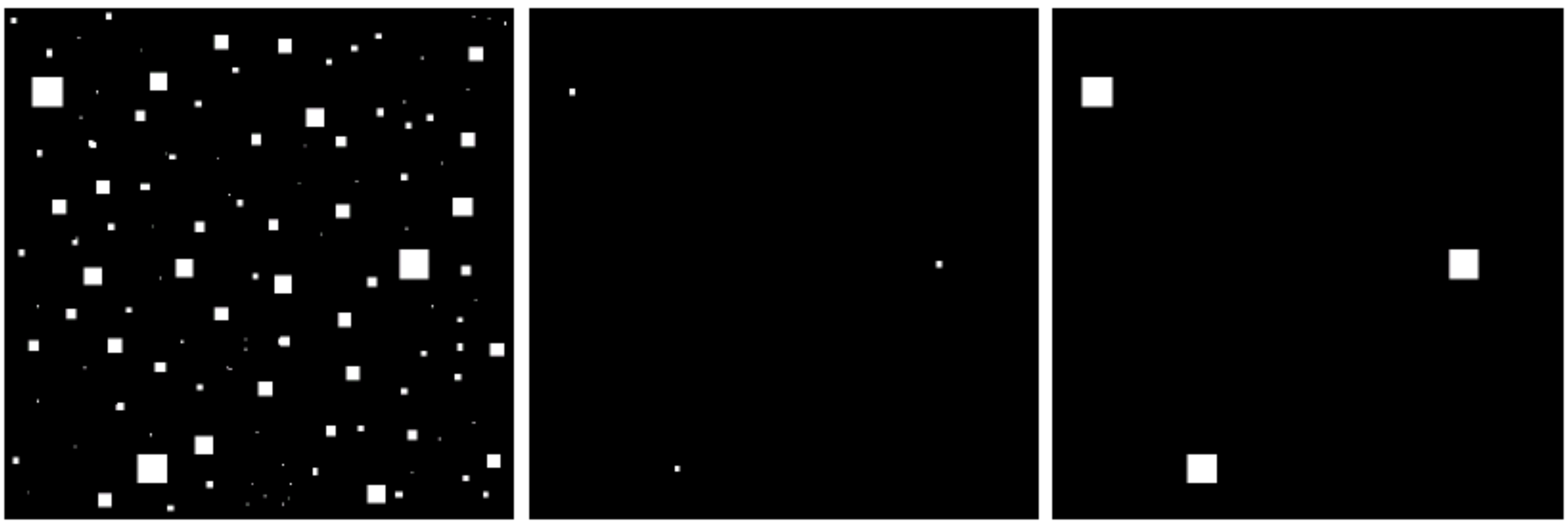
useful

- erosion
  - removal of structures of certain shape and size, given by SE
- Dilation
  - filling of holes of certain shape and size, given by SE

Combining erosion and dilation

- WANTED:
  - remove structures / fill holes
  - without affecting remaining parts
- SOLUTION:
- combine erosion and dilation
- (using same SE)

Erosion : eliminating irrelevant detail



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

structuring element B = 13x13 pixels of gray level 1

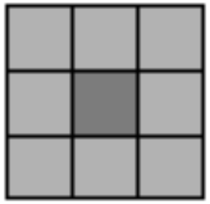
Opening

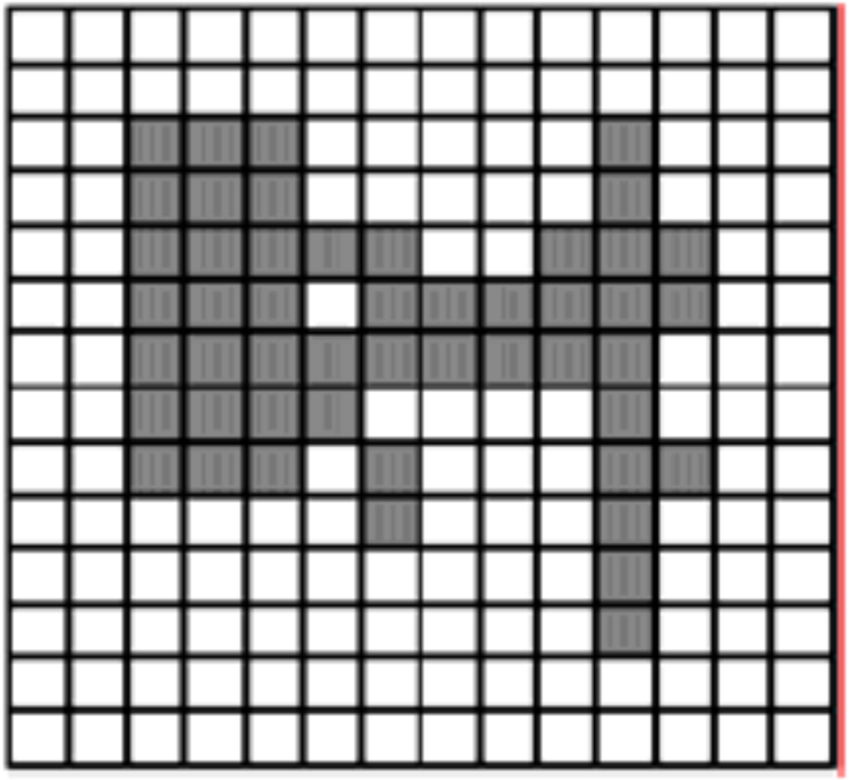
erosion followed by dilation, denoted $\circ$

$$A \circ B = (A \ominus B) \oplus B$$

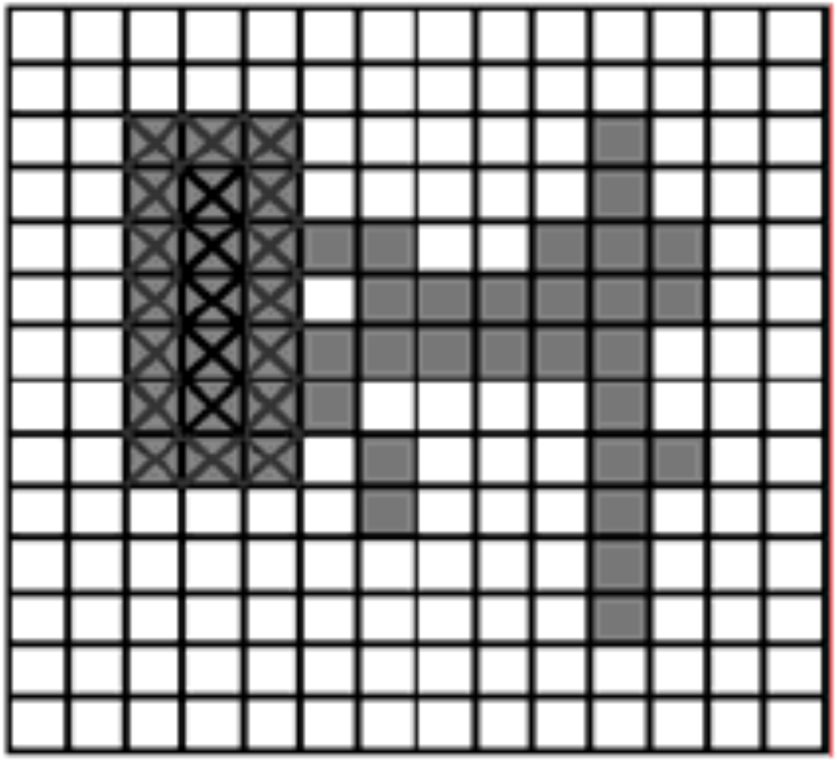
- eliminates protrusions
- breaks necks
- smoothes contour

Opening

B= 

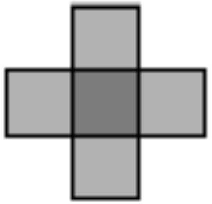


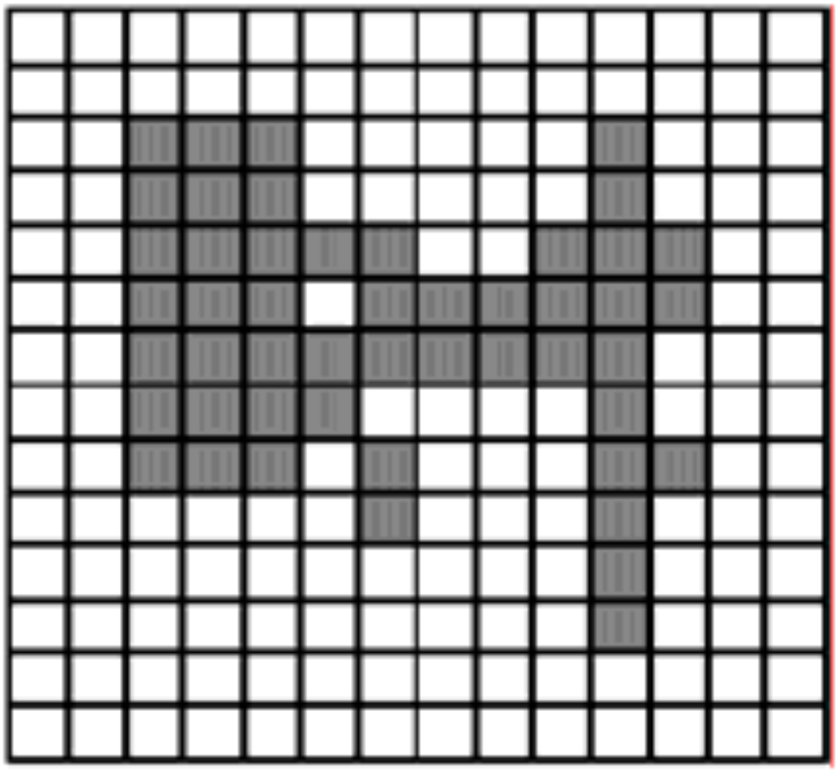
A



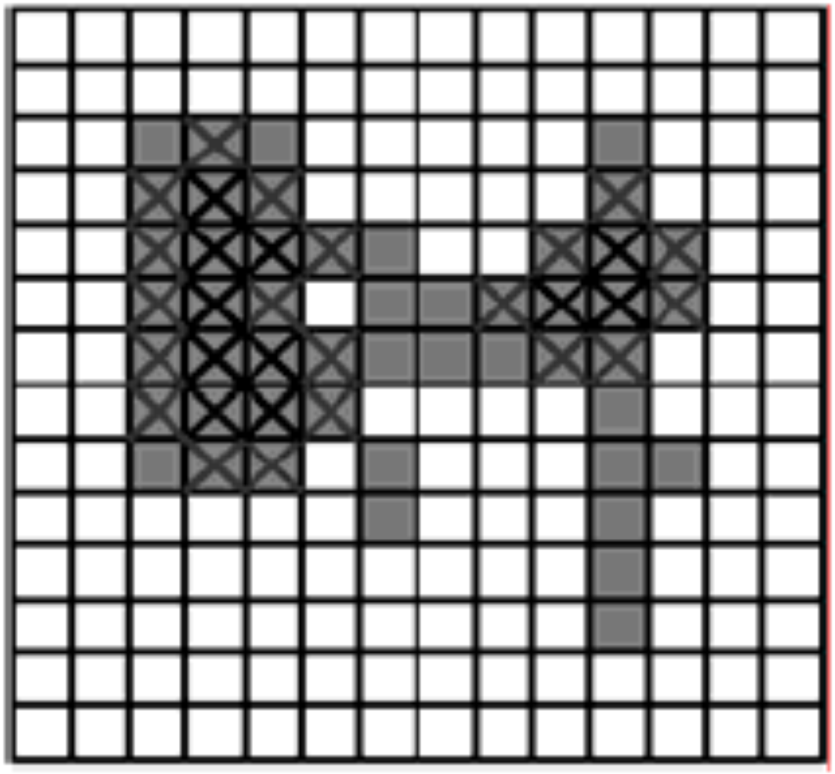
A⊖B A∘B

Opening

B= 

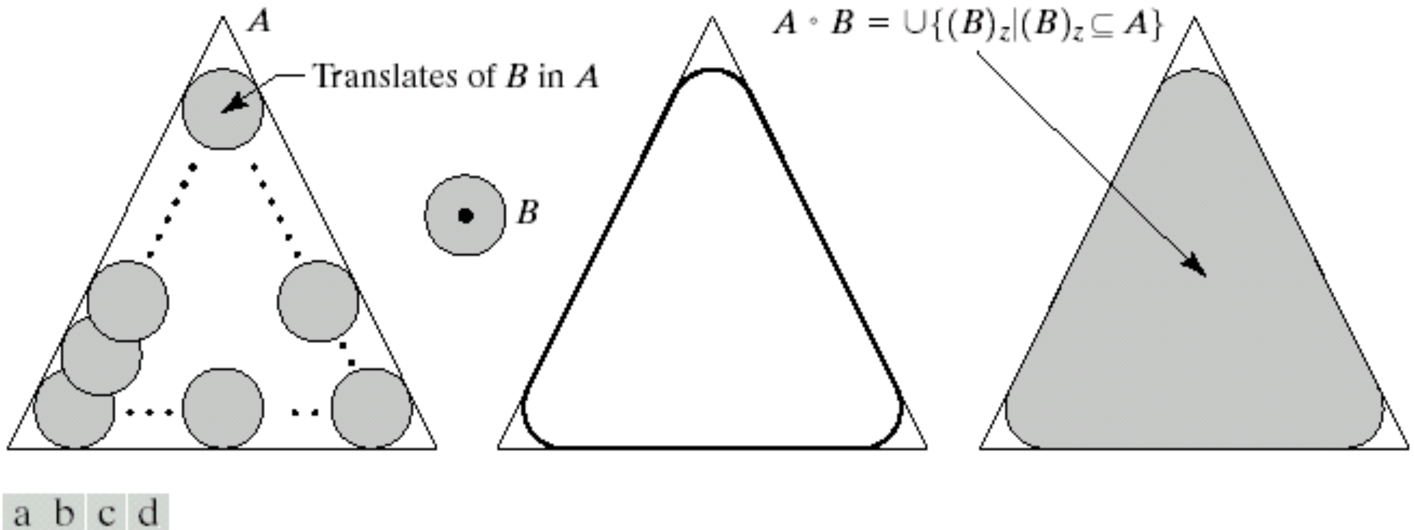


A



$A \ominus B$ $A \circ B$

Opening



a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$

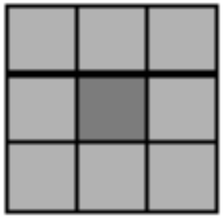
Closing

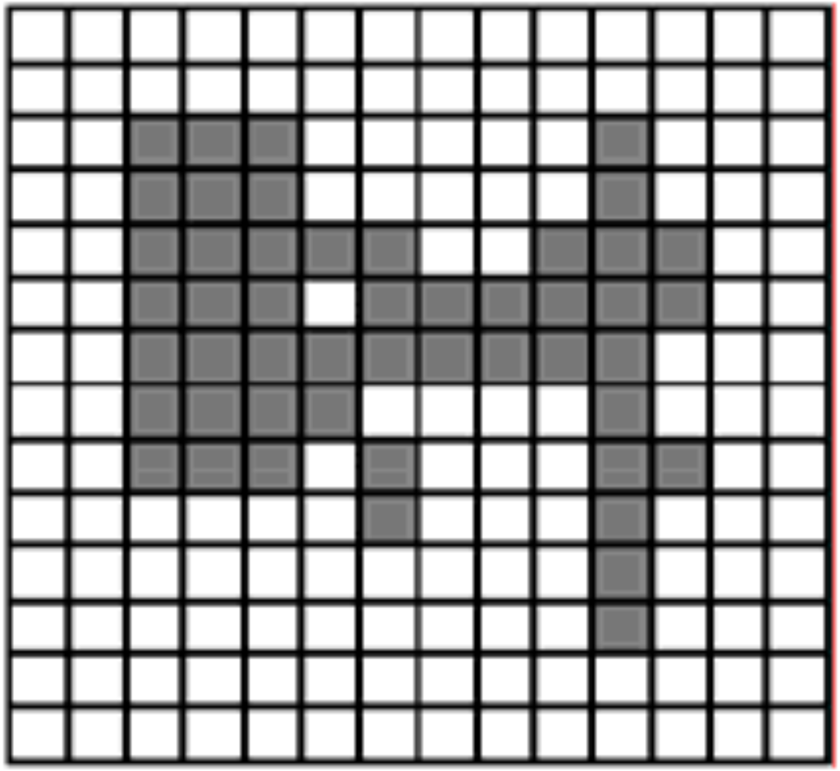
dilation followed by erosion, denoted $\bullet$

$$A \bullet B = (A \oplus B) \ominus B$$

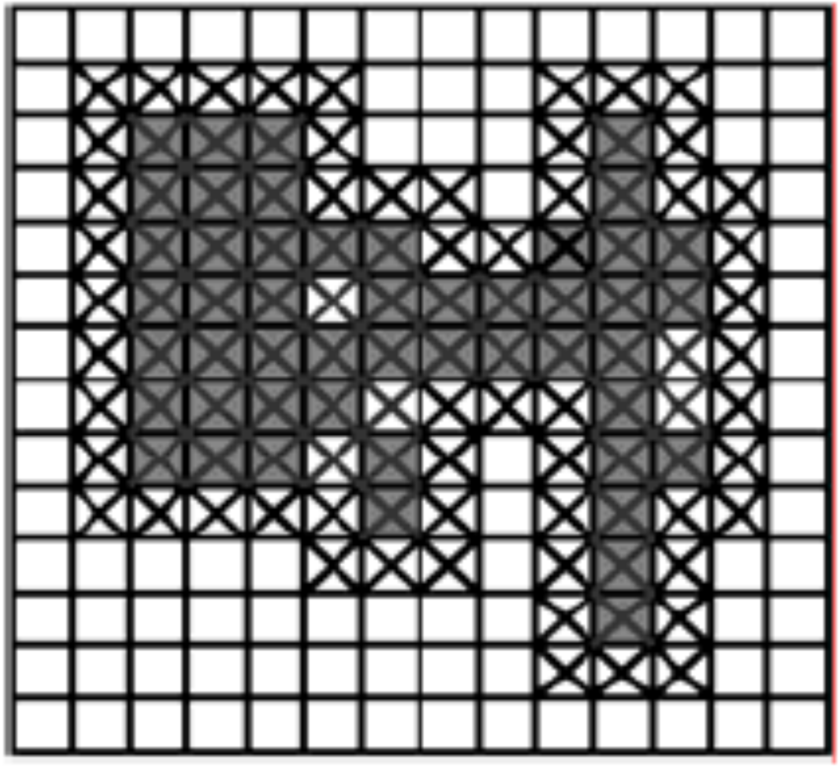
- smooth contour
- fuse narrow breaks and long thin gulfs
- eliminate small holes
- fill gaps in the contour

Closing

B= 

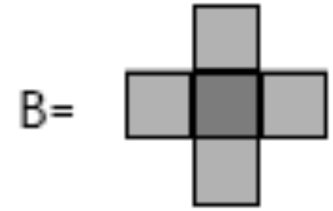


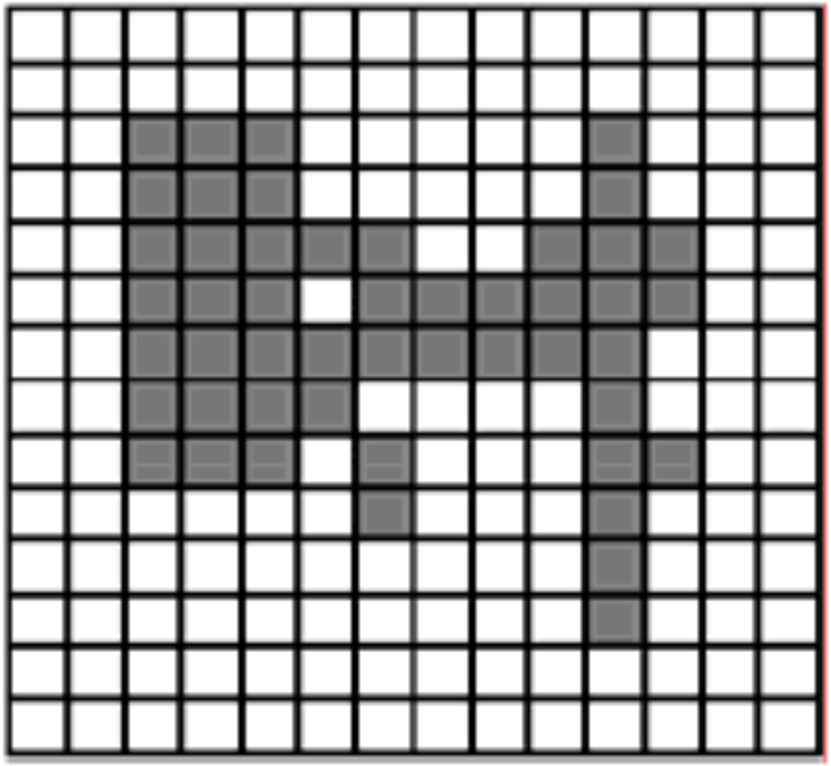
A



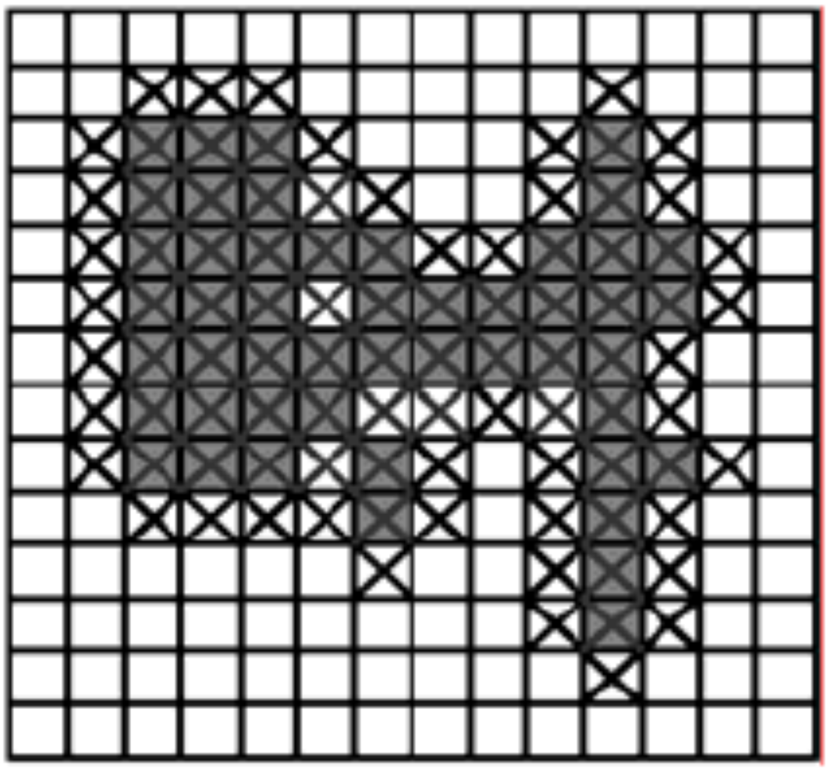
$A \oplus B \quad A \bullet B$

Closing★



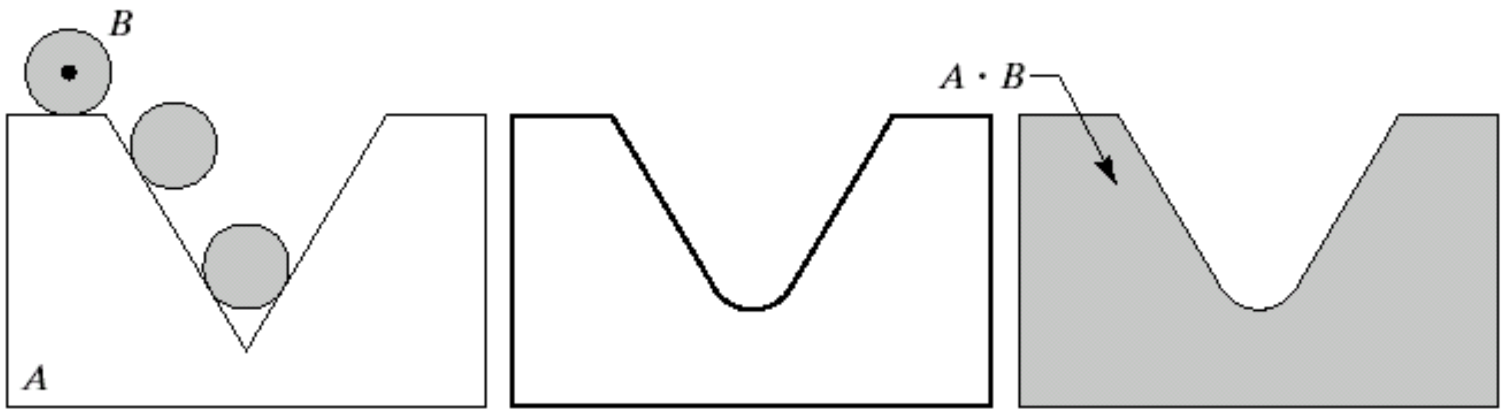


A



A⊕B A • B

Closing



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

$$A \bullet B = (A \oplus B) \ominus B$$

Properties

Opening

(i) $A \circ B$ is a subset (subimage) of A

(ii) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$

(iii) $(A \circ B) \circ B = A \circ B$

Closing

(i) A is a subset (subimage) of $A \bullet B$

(ii) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$

(iii) $(A \bullet B) \bullet B = A \bullet B$

<u>Note:</u> repeated openings/closings has no effect!

Duality

- Opening and closing are dual with respect to complementation and reflection

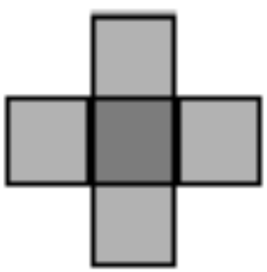
$$(A \bullet B)^c = (A^c \circ \hat{B})$$

A

$A \ominus B$

$(A \ominus B)^C$

$$B = \hat{B}$$

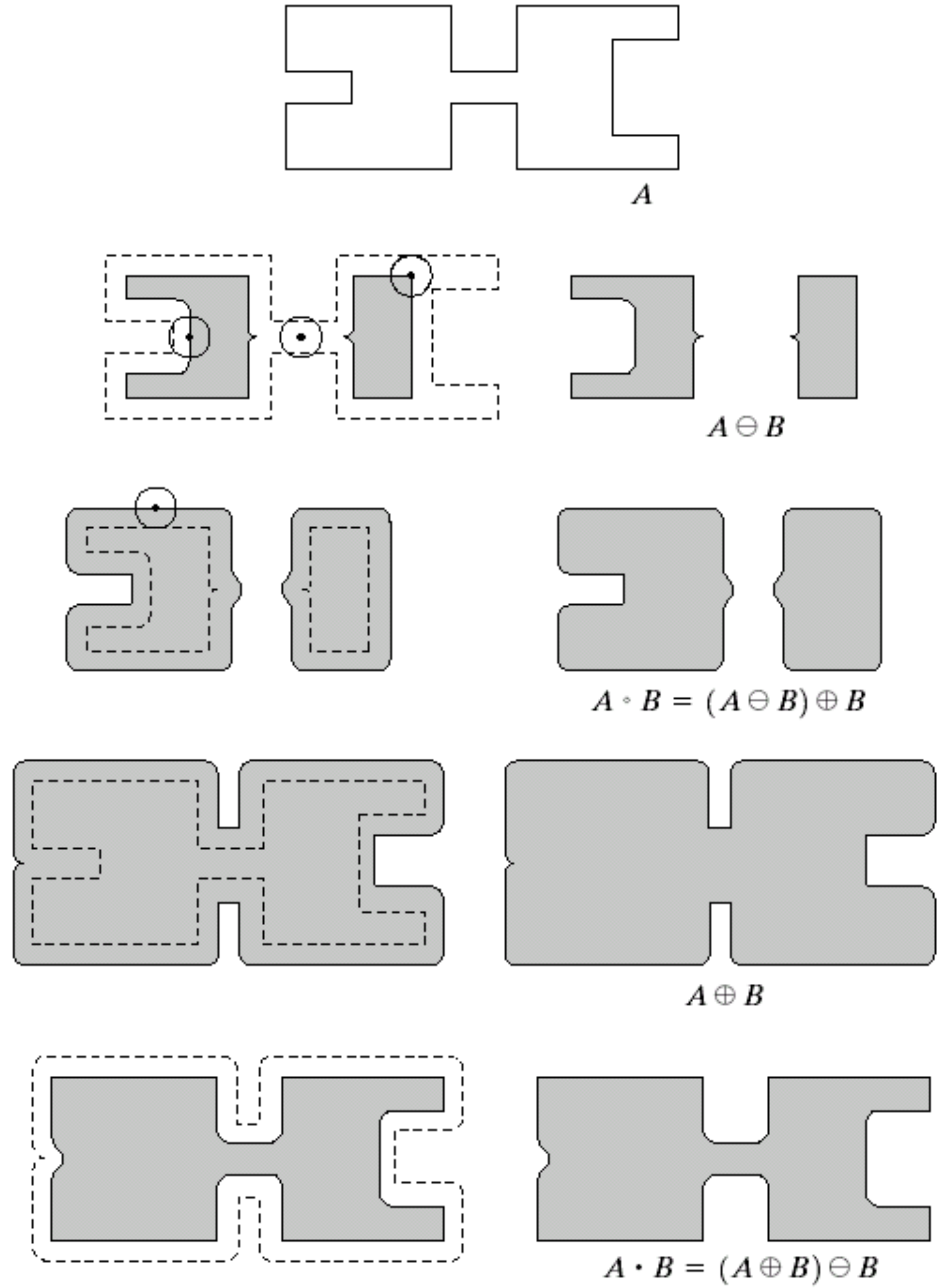


A^C

$A^C \oplus B$

	a
b	c
d	e
f	g
h	i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



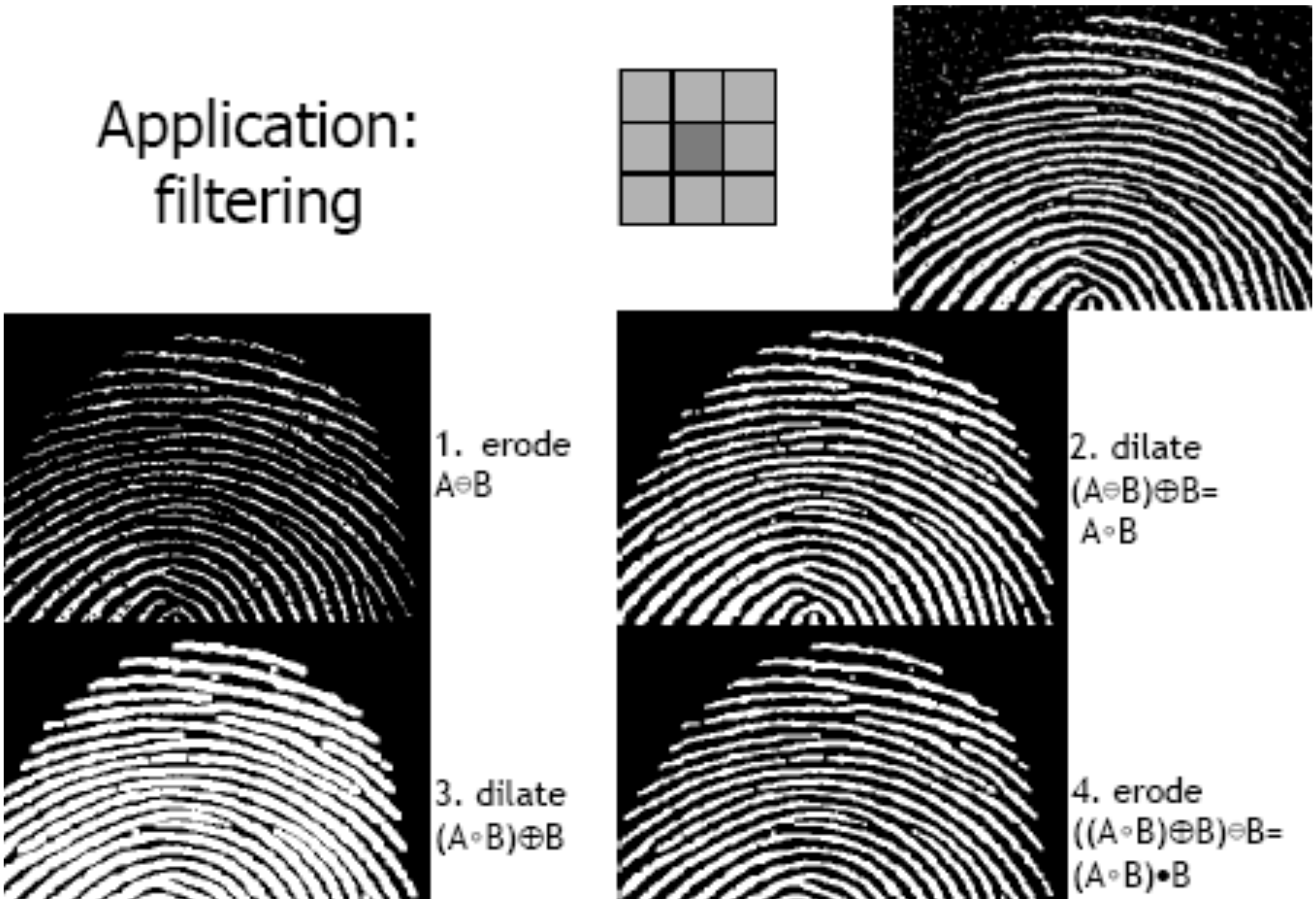
Useful: open & close

A

opening of A
→removal of small protrusions, thin connections, ...

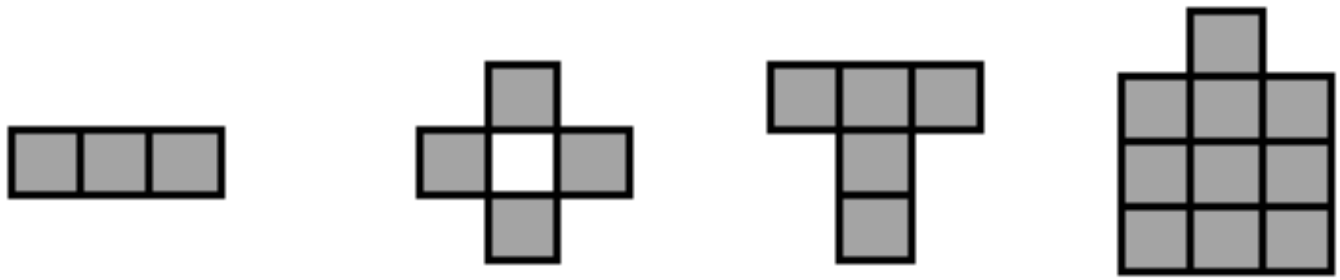
closing of A
→ removal of holes

Application: filtering



Hit-or-Miss Transformation (HMT)

- find location of one shape among a set of shapes ”template matching

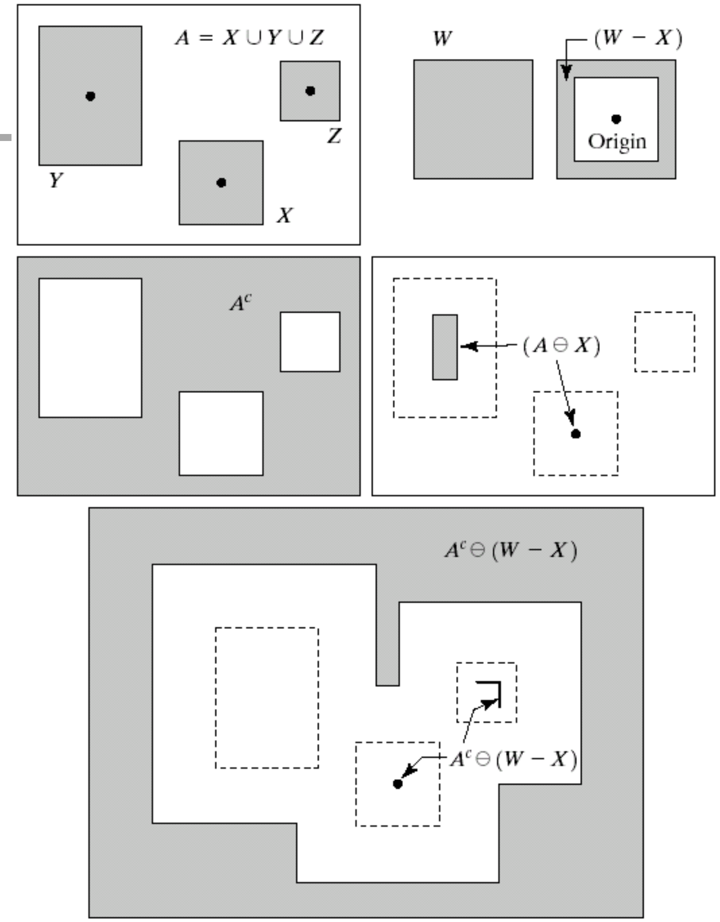


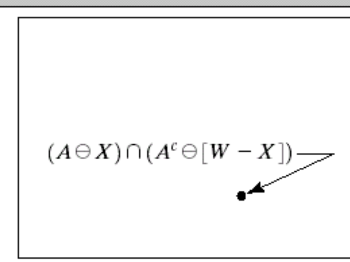
- composite SE: object part (B1) and background part (B2)
- does B1 ***fits the object while, simultaneously,*** B2 misses the object, i.e., ***fits the background?***

Hit-or-Miss Transformation

a b
c d
e
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$.
(c) Complement of A. (d) Erosion of A by X.
(e) Erosion of A^c by $(W - X)$.
(f) Intersection of (d) and (e), showing the location of the origin of X, as desired.





$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

Boundary Extraction

a b
c d

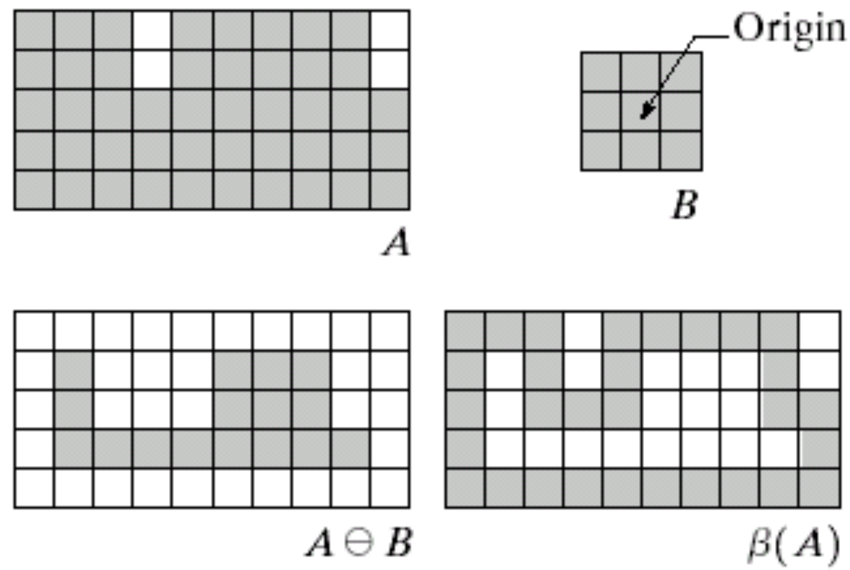
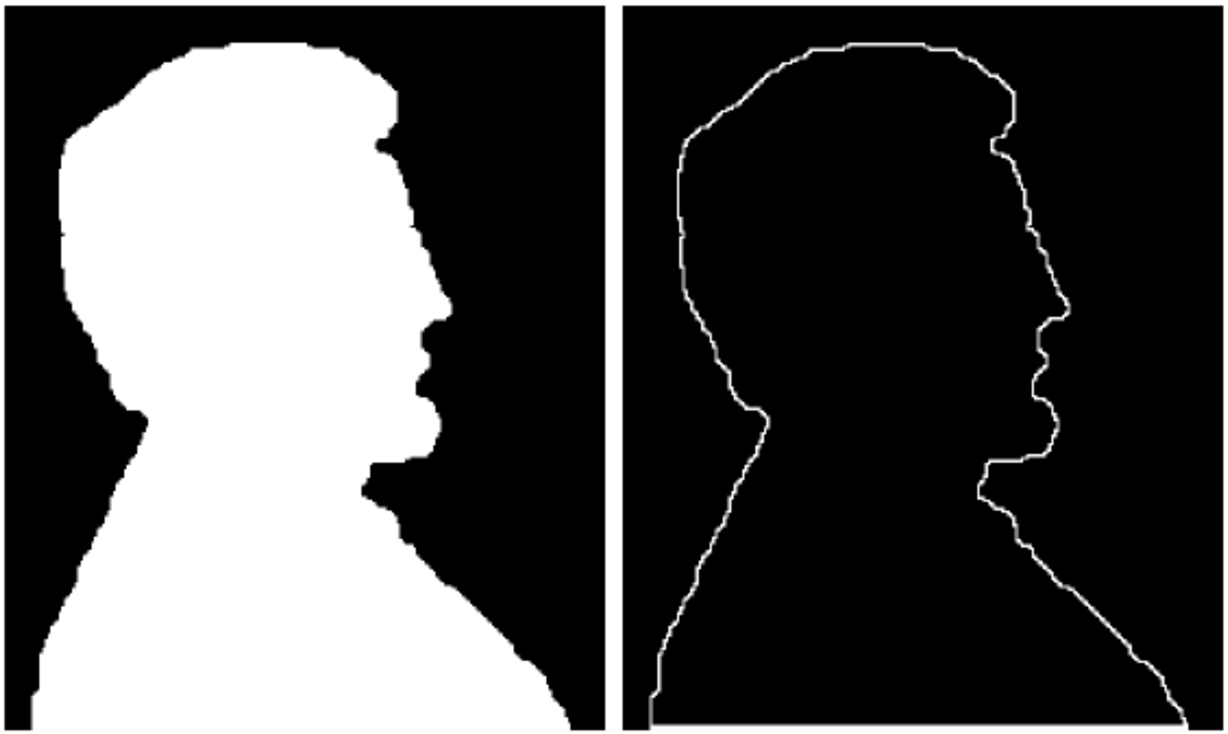


FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

$$\beta(A) = A - (A \ominus B)$$

Example

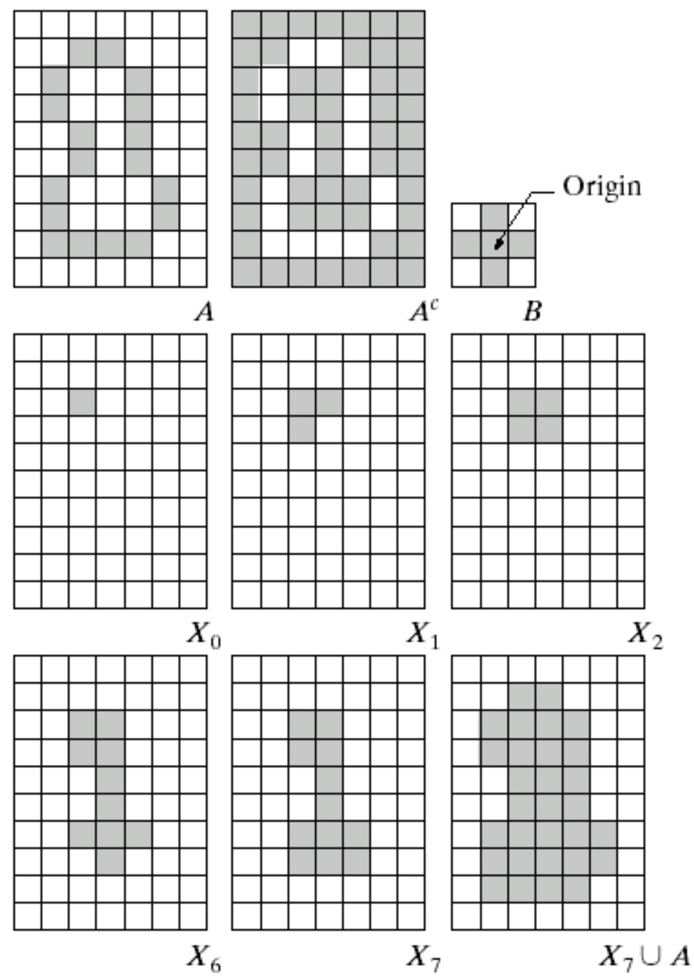


a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

Region Filling

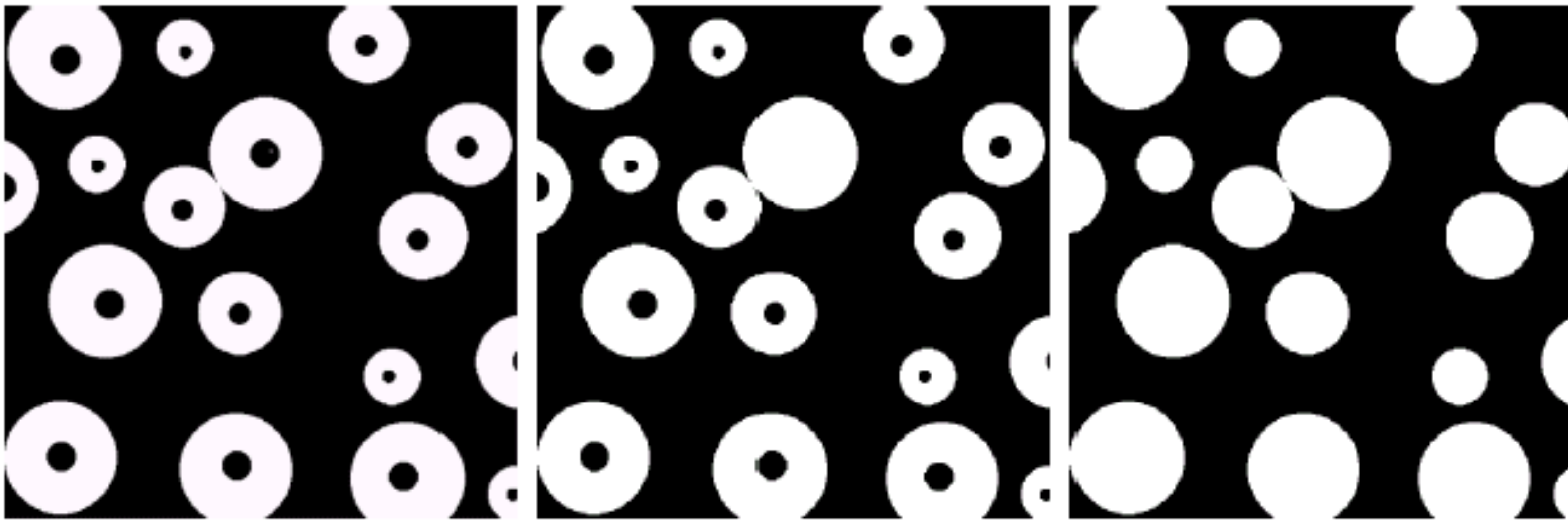
$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3,\ldots$$



a	b	c
d	e	f
g	h	i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

Example

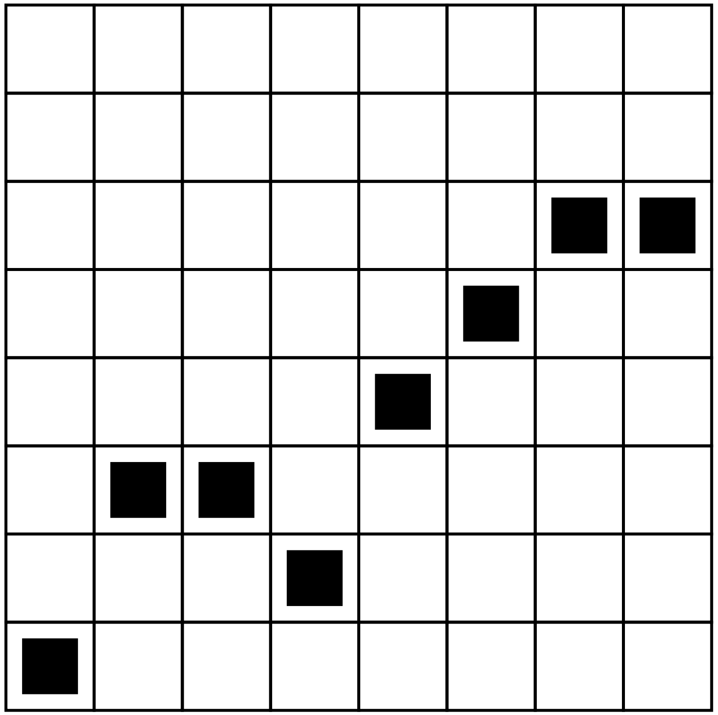


a b c

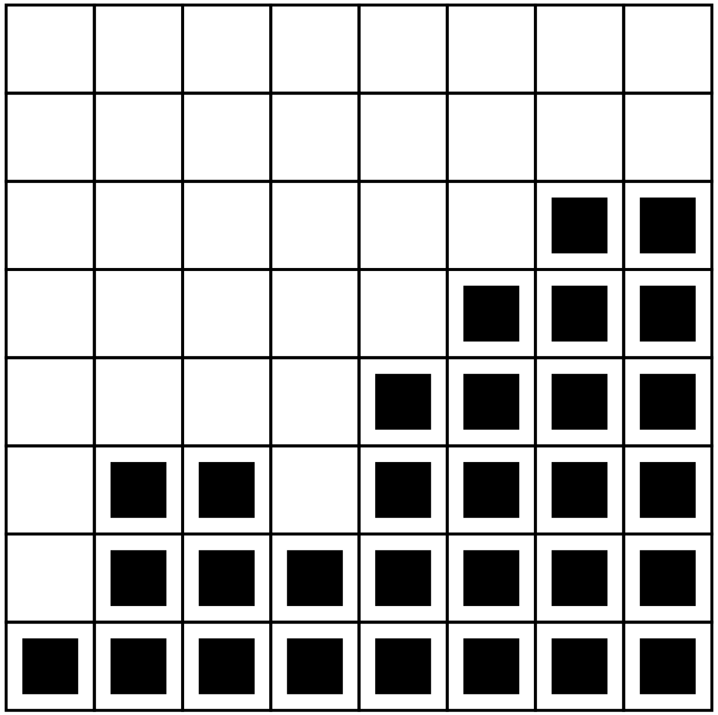
FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Umbra

$$U(f) = \{(x, y) : y \leq f(x)\}$$



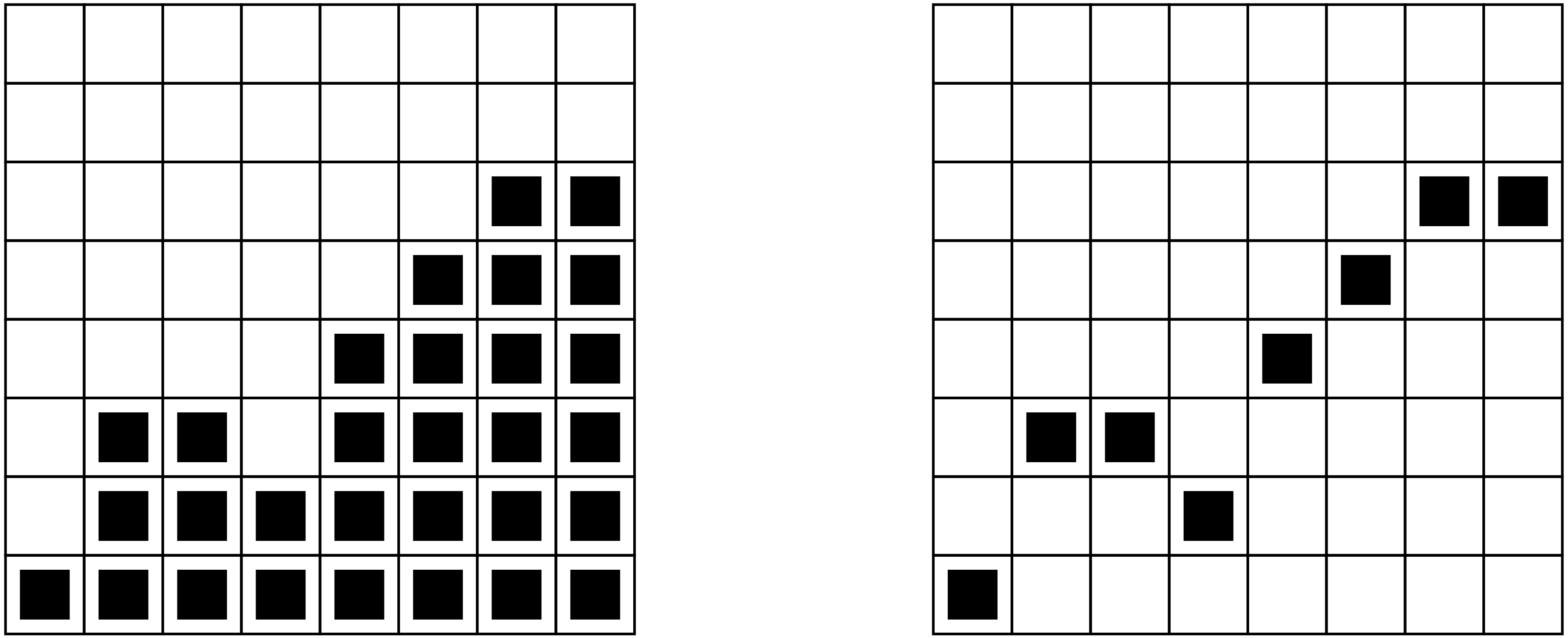
f



$U(f)$

Top surface

$$T(R) = \{(x, y) : y \geq z \text{ for all } (x, z) \in R\}$$



R $\qquad$ $T(R)$

Gray level erode and dilate

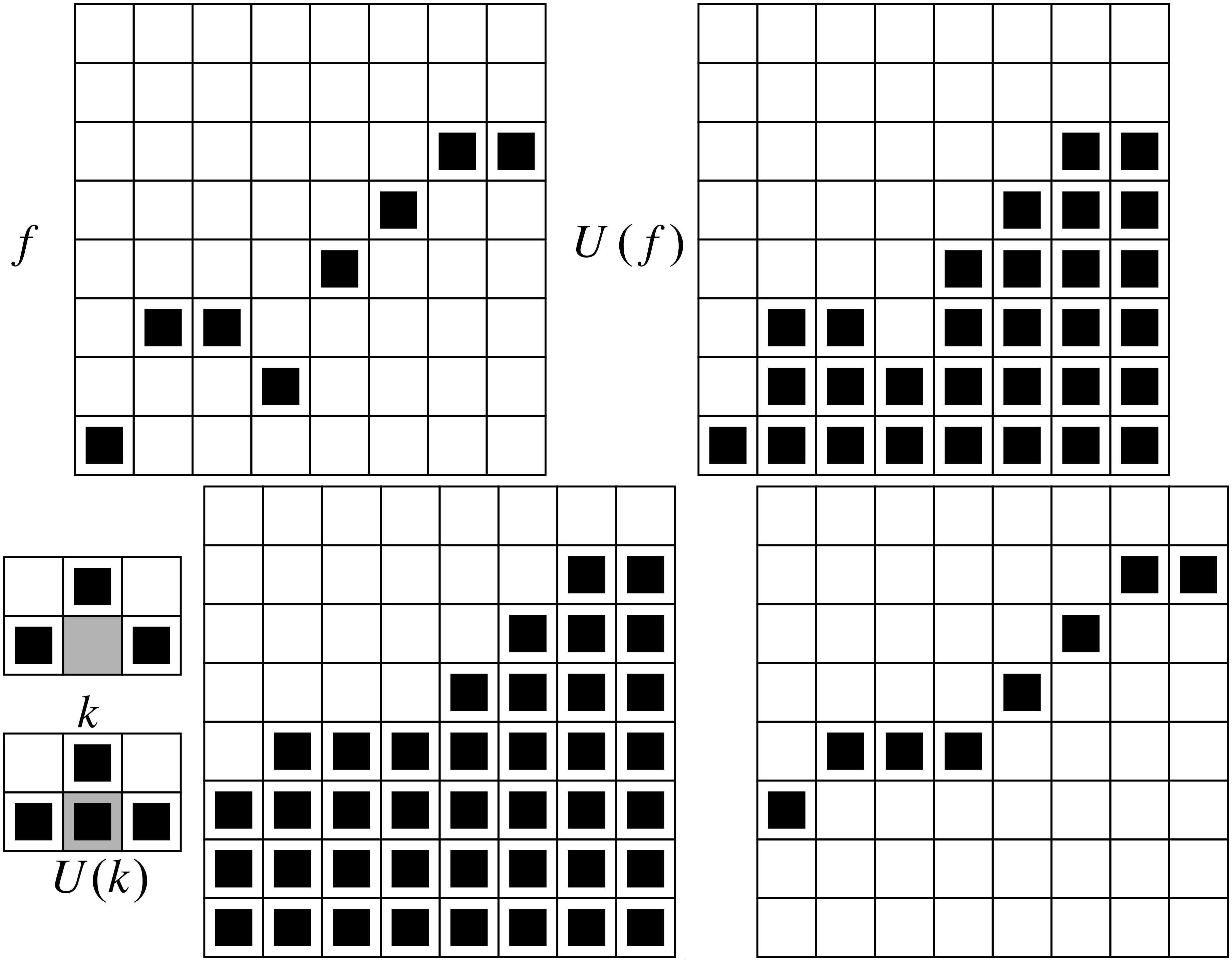
- Erosion:

$$f \ominus k = T(U(f) \ominus U(k))$$

- Dilation:

$$f \oplus k = T(U(f) \oplus U(k))$$

- Open and close defined as before.


f
$U(f)$
k
$U(k)$
$U(f) \oplus U(k)$
$T(U(f) \oplus U(k))$