Subdivision
Subdivision

Bezier Curves (Recall)

Subdivision using control points yields the curve
Beziers Curves

Subdivision

Curve $P(t)$ $0 \leq t \leq 1$ is subdivided into two curves
- $Q(u)$ $0 \leq u \leq 1$
- $R(v)$ $0 \leq v \leq 1$
Curves

Bezier Curves

Using de Casteljau Algorithm

\[ c_0 = b_0 \]
\[ c_1 = \frac{b_0 + b_1}{2} \]
\[ c_2 = \frac{b_0 + 2b_1 + b_2}{4} \]
\[ c_3 = \frac{b_0 + 3b_1 + 3b_2 + b_3}{8} \]
\[ d_0 = c_3 \]
\[ d_1 = \frac{b_1 + 2b_2 + b_3}{4} \]
\[ d_2 = \frac{b_2 + b_3}{2} \]
\[ d_3 = b_3 \]
Curves

Bezier Curves

Subdivision

\[ Q(u) \equiv R(v) \]

\[ c_0, c_1, c_2, c_3 = d_0, d_1, d_2, d_3 \]
Subdivision (Curves)

Idea: [Zorin and Schröder, 2000]

Figure 2.1: Example of subdivision for curves in the plane. On the left 4 points connected with straight line segments. To the right of it a refined version: 3 new points have been inserted “inbetween” the old points and again a piecewise linear curve connecting them is drawn. After two more steps of subdivision the curve starts to become rather smooth.
Subdivision (Surfaces)

Idea: [Zorin and Schröder, 2000]

Figure 2.2: Example of subdivision for a surface, showing 3 successive levels of refinement. On the left an initial triangular mesh approximating the surface. Each triangle is split into 4 according to a particular subdivision rule (middle). On the right the mesh is subdivided in this fashion once again.
Subdivision

Limit Surface
Input Mesh - Output refined (continuous) Mesh

\[ M_i = f(M_i) \]
\[ S = \lim(M_i) \]

[DeRose et al., 1998]
Subdivision

• Do away with the explicit parametric representation
• Base a curve or surface solely on its control points and their connectivity.
• Provide a simple mechanism which produces a larger, more refined set of control points from the current set.
• Iterate refinement until the appropriate level of detail is achieved.
Curves

- Set of rules $S$ that take a curve as input and produce a more highly refined curve as output
- Recursively applying $S$ yields a sequence of curves which should converge to some limit shape
Subdivision

Rules

• Typically chosen to be linear combinations of neighboring vertices
• Rules usually depend only on local topology of shape
Subdivision

Curves

For Example
Example
Example
Example
Example
Example
Example
Example
Example
Example
Subdivision

Chaiken’s Algorithm

[Real Time Rendering]
Chaiken’s Algorithm

Approximation using initial control points
Basically follow $\frac{3}{4} \frac{1}{4}$ rule
C1 continuous
Limiting curve is quadratic B-spline

Extension for surfaces: Doo-Sabin Subdivision
Subdivision

Interpolating Curve: Dyn et. al 1987
Subdivision

Interpolating Curve: Dyn et. al 1987

\[
P_{2i}^{k+1} = P_i^k
\]

\[
P_{2i+1}^{k+1} = (1/2 + w) (P_i^k + P_{i+1}^k) - w (P_{i-1}^k + P_{i+2}^k)
\]

w: tension parameter

[Real Time Rendering]
Subdivision: Surfaces

Loop Subdivision

Figure 3.1: Refinement of a triangular mesh. New vertices are shown as black dots. Each edge of the control mesh is split into two, and new vertices are reconnected to form 4 new triangles, replacing each triangle of the mesh.

[Zorin and Schröder, 2000]
Subdivision: Surfaces

Loop Subdivision Rules

[Zorin and Schröder, 2000]
Subdivision: Surfaces

Loop Subdivision Rules

New Vertex  Updated Vertex

Edge Mask  Vertex Mask

[Zorin and Schröder, 2000]
Subdivision: Surfaces

Catmull Clark Subdivision

• Introduce new points
  - At face centre
  - At mid edges
• Adjust position of original points
• Repeat until sufficient details

[Zorin and Schröder, 2000]
Subdivision: Surfaces

Catmull Clark Subdivision

Face

Edge

Update
Subdivision: Surfaces

Some Results

Loop
Catmull Clark

[Zorin and Schröder, 2000]
Subdivision: Surfaces

Some Results

Loop  Catmull Clark

[Zorin and Schröder, 2000]
Subdivision: Surfaces

Manipulation/Deformation

[Zorin and Schröder, 2000]
Subdivision: Surfaces

Very popular in Computer Graphics:

Pixar’s:

Geri’s Game 1998
Monster Inc 2000