Ray Tracing

Rendering

Issues

• Visibility
  What objects or parts in the scene are visible?
  Clipping (with respect to the view frustum)
  Done
  Occlusion (with respect to the objects in the scene)
  Hidden surface elimination

• Illumination
  Reflection, Refraction, Transparency, Shadows, etc.
Rendering Pipeline (Revisit)

Modeling Transformation

- Model 1
  - $M_1$
- Model 2
  - $M_2$
- Model n
  - $M_n$

Viewing Transformation

- 3D World Scene
- $V$

Forward Mapping Approach

- 3D View Scene
- 2D Image
- Rasterization
- 2D Scene
- Projection
Forward Ray Tracing

Modeling interaction of light with the objects/surfaces

Problem:
Many rays will not contribute to the image!
Ray Tracing

Backward Ray Tracing

Rays from camera (viewer) through each pixel to the scene

Backward Ray Tracing = Ray Tracing
Backward Ray Tracing

Primary and Secondary Rays
Backward Ray Tracing

Shadow Rays

Visibility check with respect to the light source
Ray Tracing

Two Issues

Ray-object intersection
   Visibility test: Closest to the viewer

Pixel color determination (shading)
   Illumination model
Ray Tracing

Ray Object Intersection

Sphere

\[ R_o = [X_o, Y_o, Z_o] \]
\[ R_d = [X_d, Y_d, Z_d] \]

Ray Origin

Ray Direction

\[ X_d^2 + Y_d^2 + Z_d^2 = 1 \]

Parametric Form

\[ R(t) = R_o + R_d t \quad t > 0 \]
Ray Object Intersection

Sphere

Implicit Form

Center \( S_c = [X_c, Y_c, Z_c] \)

Radius \( S_r \)

Surface Point \([X_s, Y_s, Z_s]\)

\[
(X_s - X_c)^2 + (Y_s - Y_c)^2 + (Z_s - Z_c)^2 = S_r^2
\]
Ray Object Intersection

Sphere

To solve the intersection problem the ray equation is substituted into the sphere equation and the result is solved for $t$

That is

$$(X_o + X_d t - X_c)^2 + (Y_o + Y_d t - Y_c)^2 + (Z_o + Z_d t - Z_c)^2 = S_r^2$$
Ray Tracing

Ray Object Intersection

Sphere

\[ At^2 + Bt + C = 0 \]

where

\[ A = X_d^2 + Y_d^2 + Z_d^2 = 1 \]

\[ B = 2(X_d(X_o - X_c) + Y_d(Y_o - Y_c) + Z_d(Z_o - Z_c)) \]

\[ C = (X_o - X_c)^2 + (Y_o - Y_c)^2 + (Z_o - Z_c)^2 - S_r^2 \]
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Ray Object Intersection

Sphere

\[ At^2 + Bt + C = 0 \]

\[ t_0 = \frac{-B - \sqrt{B^2 - 4AC}}{2A} \]

\[ t_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \]

Smaller positive among \( t_0 \) and \( t_1 \) gives the closest intersection point

\[ [X_i, Y_i, Z_i] = [X_o + X_d t, Y_o + Y_d t, Z_o + Z_d t] \]
Ray Object Intersection

Sphere

Normal

\[ n = \left[ \frac{(X_i - X_c)}{S_r}, \frac{(Y_i - Y_c)}{S_r}, \frac{(Z_i - Z_c)}{S_r} \right] \]
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Ray Sphere Intersection

Sum up

- Calculate A B C
- Compute the discriminant
- Calculate min (t₀, t₁)
- Compute the intersection point
- Compute the normal
Ray Object Intersection

Sphere

Geometric Approach

\[ R_0 \quad L \quad O \]

\[ R_d \quad t_{ca} \quad t_{nc} \quad d \quad r \]
Ray Tracing

Ray Object Intersection

Sphere

Geometric Approach

\[ L = O - R_0 \]
\[ t_{ca} = L^TR_d \]
\[ t_{ca} < 0 \text{ no intersection} \]
Ray Tracing

Ray Object Intersection

Sphere

Geometric Approach

\[ L = O - R_0 \]
\[ t_{ca} = L^T R_d \]
\[ t_{ca} < 0 \text{ no intersection} \]
\[ d = L^T L - t_{ca}^2 \]
\[ \text{if } d > r \text{ no intersection} \]
Ray Object Intersection

Sphere

Geometric Approach

\[ t_{hc} = \sqrt{r^2 - d^2} \]
\[ t = t_{ca} - t_{hc} \text{ and } t_{ca} + t_{hc} \text{ smaller } t \]
Ray Plane Intersection

Ray

\[ \mathbf{R}_o = [X_0, Y_0, Z_0] \] \hspace{1cm} \text{(ray origin)}
\[ \mathbf{R}_d = [X_d, Y_d, Z_d] \] \hspace{1cm} \text{(ray direction)}
\[ X_d^2 + Y_d^2 + Z_d^2 = 1 \] \hspace{1cm} \text{(normalized)}
\[ \mathbf{R}(t) = \mathbf{R}_o + \mathbf{R}_d t \quad t > 0 \]

Plane

\[ \mathbf{P} : A x + B y + C z + D = 0 \]
\[ A^2 + B^2 + C^2 = 1 \]
\[ \mathbf{P}_{\text{normal}} = \mathbf{P}_n = [A, B, C] \]
\[ D : \text{Distance from origin} \]
Ray Plane Intersection

Substituting ray equation in plane’s equation

\[ A (X_o + X_d t) + B (Y_o + Y_d t) + C (Z_o + Z_d t) + D = 0 \]

Solving for \( t \)

\[ t = -\frac{AX_0 + BY_0 + CZ_0 + D}{AX_d + BY_d + CZ_d} \]

\[ t = -\frac{P_n \cdot R_0 + D}{P_n \cdot R_d} \]
Ray Plane Intersection

Let

\[ V_d = P_n \cdot R_d = AX_d + BY_d + CZ_d \]

If \( V_d = 0 \) then the ray is parallel to the plane (no intersection)

\( V_d > 0 \) normal is pointing away from the ray (may be used for back-face culling)
Ray Plane Intersection

Let

\[ V_0 = -(P_n \cdot R_0 + D) = (AX_0 + BY_0 + CZ_0 + D) \]

\[ t = \frac{V_0}{V_d} \]

If \( t < 0 \) then plane is behind ray’s origin
else compute intersection

\[ r_i = [X_i, Y_i, Z_i] = [X_0 + XDt, Y_0 + YDt, Z_0 + ZDt] \]
\[ r_{\text{normal}} = P_n \]
Polygon Intersection

Containment Test

Parity Test: If the number of intersection is odd then point is inside (special case for vertices)
Triangle Intersection

Containment Test

Triangle: Barycentric Coordinates

\[ P = uV_1 + vV_2 + wV_3 \]
Triangle Intersection

Containment Test

Triangle: Barycentric Coordinates

\[ P = uV_1 + vV_2 + wV_3 \]
Triangle Intersection

Containment Test

Triangle: Barycentric Coordinates

\[ u = \frac{A_1}{A} , \quad v = \frac{A_2}{A} , \quad w = \frac{A_3}{A} \]

\[ u + v + w = 1 \]

\[ u \geq 0 , \quad v \geq 0 , \quad w \geq 0 \]

\[ P = uV_1 + vV_2 + wV_3 \]
Ray Quadric Intersection

Quadrics:
Cylinders, Cone, Sphere, Ellipsoids, Paraboloids, Hyperboloids, etc.

Implicit form \( f(X,Y,Z) = 0 \)
\[
Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 26y + Hz^2 + 2lz + J = 0
\]

Ray: \textit{Parametric form}
\[
R_0 = [ X_o \ Y_o \ Z_o ] \quad \text{(ray origin)}
R_d = [ X_d \ Y_d \ Z_d ] \quad \text{(ray direction)}
X_d^2 + Y_d^2 + Z_d^2 = 1 \quad \text{(normalized)}
R(t) = R_o + R_dt \quad t > 0
\]
Ray Quadric Intersection

Matrix Form

\[ f(X, Y, Z) = 0 \]

\[
\begin{bmatrix}
X & Y & Z & 1
\end{bmatrix}
\begin{bmatrix}
A & B & C & D \\
B & E & F & G \\
C & F & H & I \\
D & G & I & J
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} = 0
\]
Ray Quadric Intersection

Substituting

\[ A_q t^2 + B_q t + C_q = 0 \]

If \( A_q \neq 0 \)

\[
t_0 = \frac{-B_q - \sqrt{B_q^2 - 4A_qC_q}}{2A_q}
\]

\[
t_1 = \frac{-B_q + \sqrt{B_q^2 - 4A_qC_q}}{2A_q}
\]

If \( A_q = 0 \)

\[
t = -\frac{C_q}{B_q}
\]
Ray Tracing

Ray Quadric Intersection

Normal

\[ n = \begin{bmatrix} \frac{\partial F}{\partial X_i}, & \frac{\partial F}{\partial Y_i}, & \frac{\partial F}{\partial Z_i} \end{bmatrix} \]

\[ n_x = 2(AX_i + BY_i + CZ_i + D) \]
\[ n_y = 2(BX_i + CY_i + CZ_i + G) \]
\[ n_z = 2(CX_i + FY_i + HZ_i + I) \]
Ray Tracing

Ray Box Intersection

3D Clipping: Cyrus Beck/Liang Barsky