Raster Graphics

Drawing Algorithms

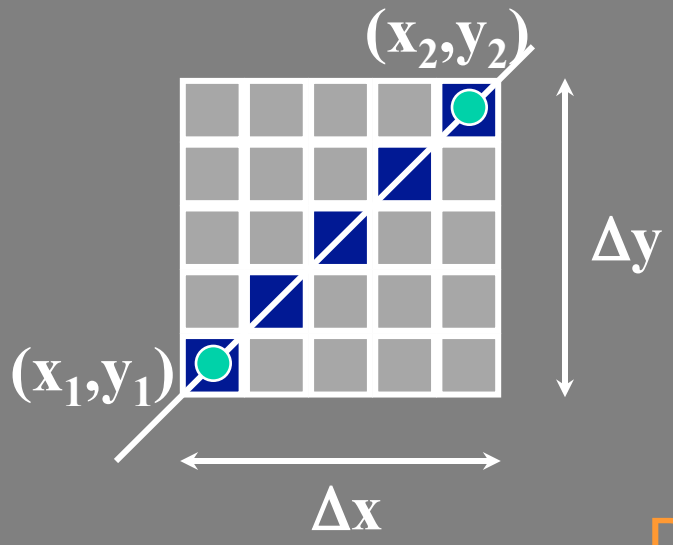
Rasterization
Pixelization
Scan Conversion

Continuous ⟶ Discrete

Raster Graphics

Line Drawing Algorithms

Line Equation:



$$y = mx + B$$

$$m = \frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

DDA (Digital Differential Analyzer)

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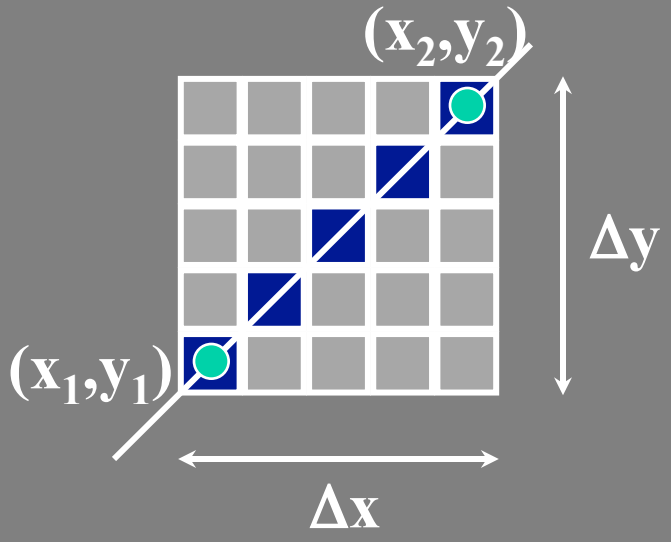
Line Drawing Algorithms

DDA (Digital Differential Analyzer)



$$y_i = mx_i + B$$
$$y_{i+1} = mx_{i+1} + B$$
$$y_{i+1} = m(x_i + \Delta x) + B$$

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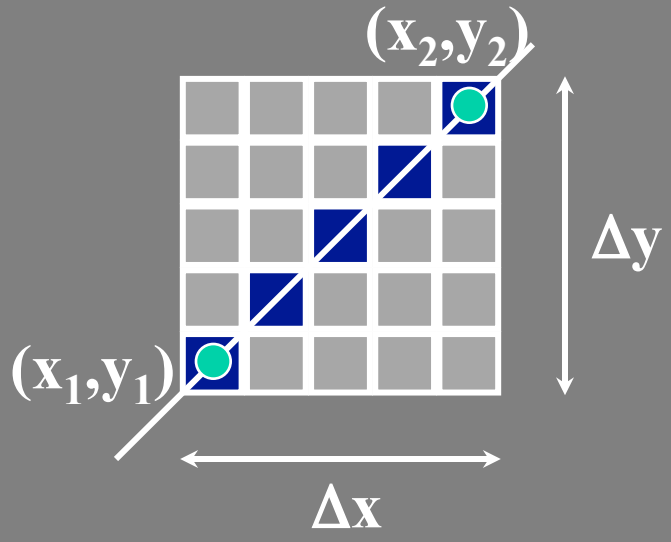
Line Drawing Algorithms

DDA (Digital Differential Analyzer)



if $\Delta x = 1$

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i + m$

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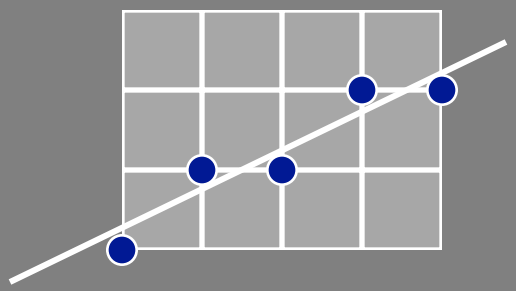
Line Drawing Algorithms

DDA (Digital Differential Analyzer)



$(x_i, \text{round}(y_i))$

```
y = y1
for (x = x1; x ≤ x2; x++)
{
   Writepixel(x, round (y));
   y+ = m;
}
```

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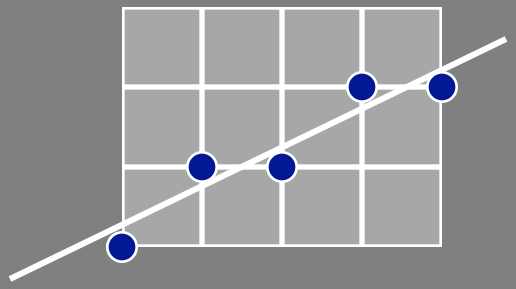
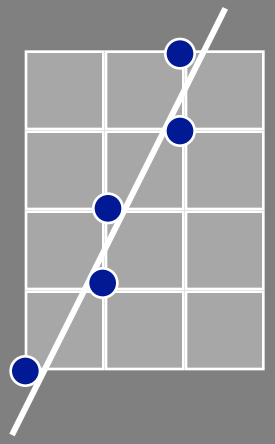
Line Drawing Algorithms

DDA (Digital Differential Analyzer)

m <= 1



m > 1



Exchange the role of x and y

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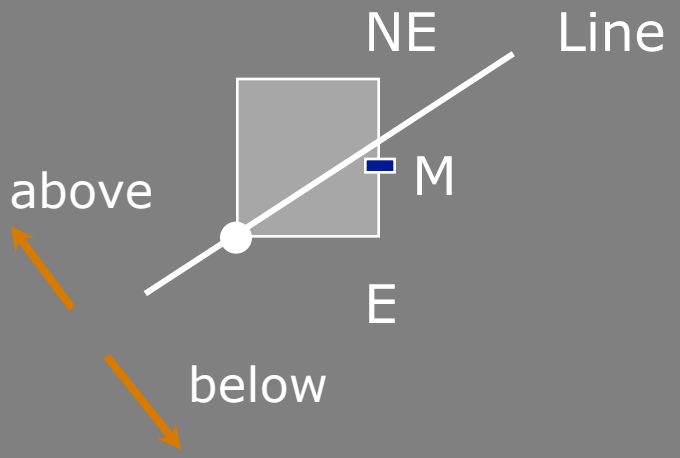
Line Drawing Algorithms

Midpoint Line Algorithm

Find on what side of the line the mid point is:

If below then
NE is closer to line
If above then
E is closer to line



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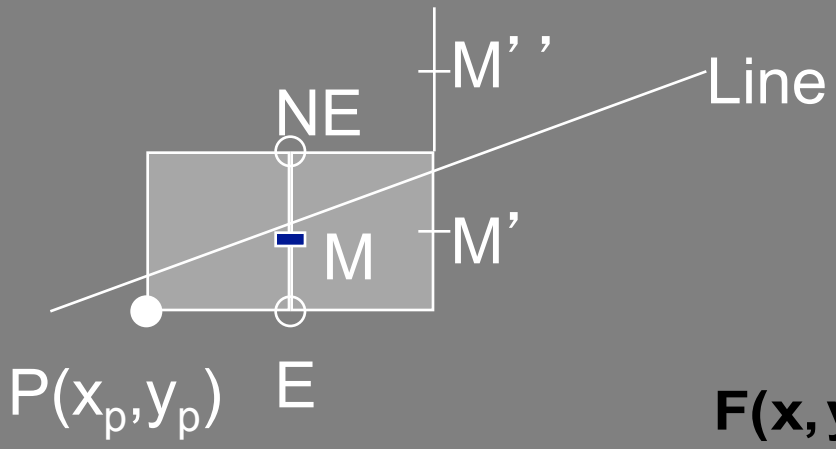
Midpoint Line Algorithm



F(x, y) = 0 : (x, y) on line
F(x, y) > 0 : (x, y) below line
F(x, y) < 0 : (x, y) above line

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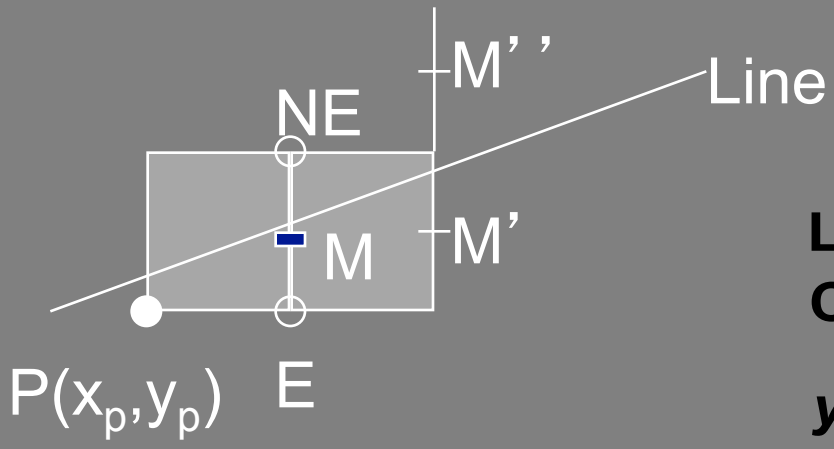
Midpoint Line Algorithm



Let 0 <= m < 1

Consider line

$$y = \frac{dy}{dx}x + B$$

$$F(x,y) = ax + by + c = 0$$

$$xdy - ydx + Bdx = 0$$

(a = dy, b = -dx, c = Bdx)

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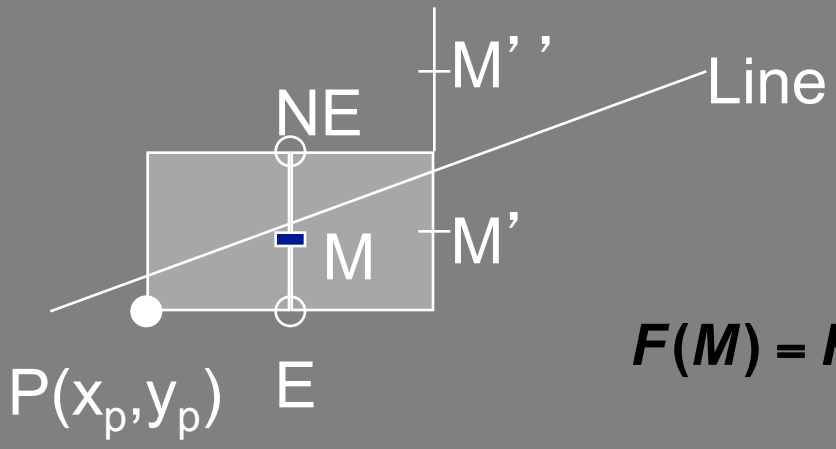
Midpoint Line Algorithm



$$F(M) = F(x_p + 1, y_p + \frac{1}{2}) = d$$

$$d = a(x_p + 1) + b(y_p + \frac{1}{2}) + c$$

if d > 0 M is below the line, choose NE

if d < 0 M is above the line, choose E

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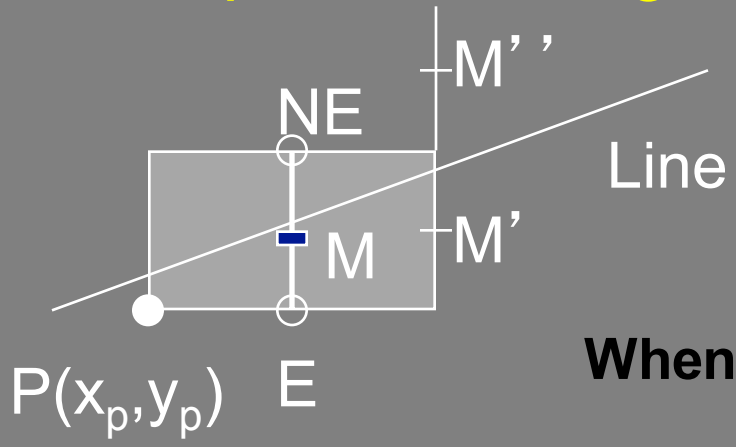
Midpoint Line Algorithm



When E :

$$d_{new} = F(M') = F(x_p + 2, y_p + \frac{1}{2})$$

$$d_{new} = a(x_p + 2) + b(y_p + \frac{1}{2}) + c$$

$$d_{old} = a(x_p + 1) + b(y_p + \frac{1}{2}) + c$$

$$\Delta_E = d_{new} - d_{old} = a = dy$$

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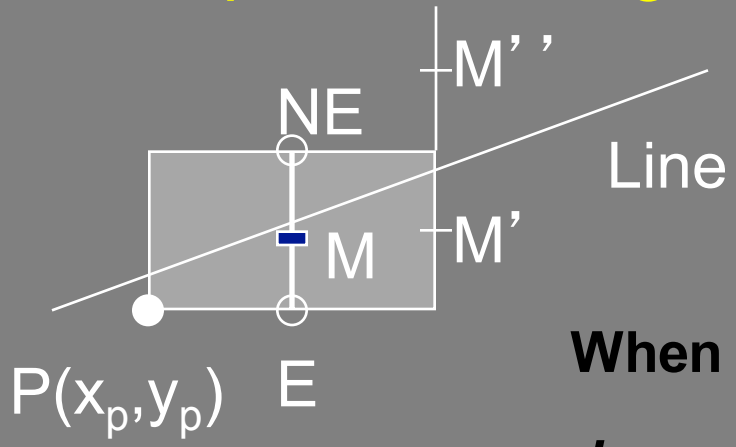
Midpoint Line Algorithm



When NE :

$$d_{new} = F(M'') = F(x_p + 2, y_p + \frac{3}{2})$$

$$d_{new} = a(x_p + 2) + b(y_p + \frac{3}{2}) + c$$

$$d_{old} = a(x_p + 1) + b(y_p + \frac{1}{2}) + c$$

$$\Delta_{NE} = d_{new} - d_{old} = a + b = dy - dx$$

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Midpoint Line Algorithm

At start :

$$d_{start} = F(x_0 + 1, y_0 + \frac{1}{2}) = a(x_0 + 1) + b(y_0 + \frac{1}{2}) + c$$

$$d_{start} = ax_0 + by_0 + c + a + \frac{b}{2}$$

$$d_{start} = a + \frac{b}{2} = \mathrm{dy} - \frac{dx}{2} \quad \text{(division)}$$

$$F(x, y) = 2(ax + by + c)$$

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Midpoint Line Algorithm

```
dx = x2 - x1; dy = y2 - y1;
d = 2dy - dx; ΔE = 2dy; ΔNE = 2(dy - dx);
x = x1; y = y1;
Writepixel(x, y);
While (x < x2)
if d ≤ 0
    d+ = ΔE; x+ = 1;
else
    d+ = ΔNE; x+ = 1; y+ = 1;
end
Writepixel(x, y);
end While
```

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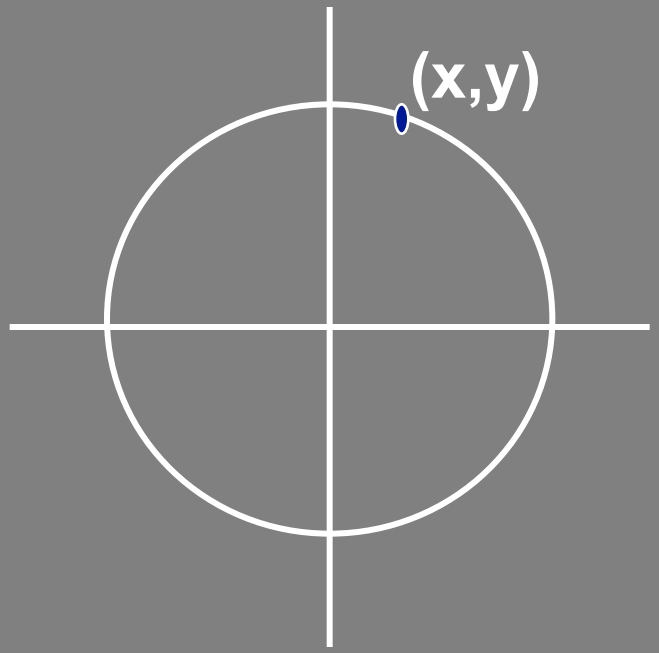
Midpoint Circle Algorithm



$$x^2 + y^2 = R^2$$

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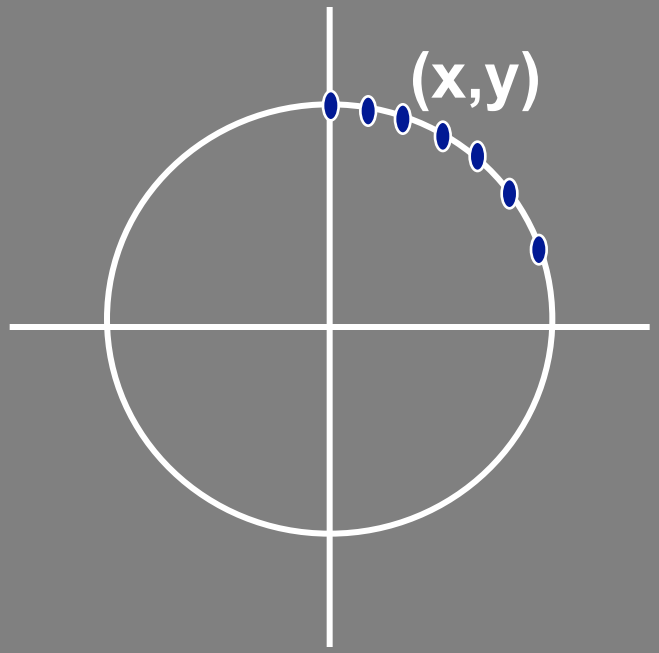
Midpoint Circle Algorithm



$$x^2 + y^2 = R^2$$

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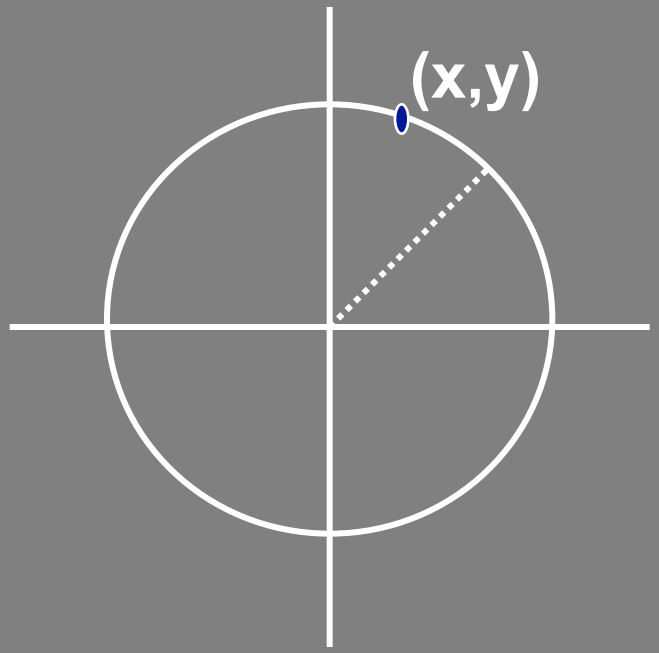
Midpoint Circle Algorithm



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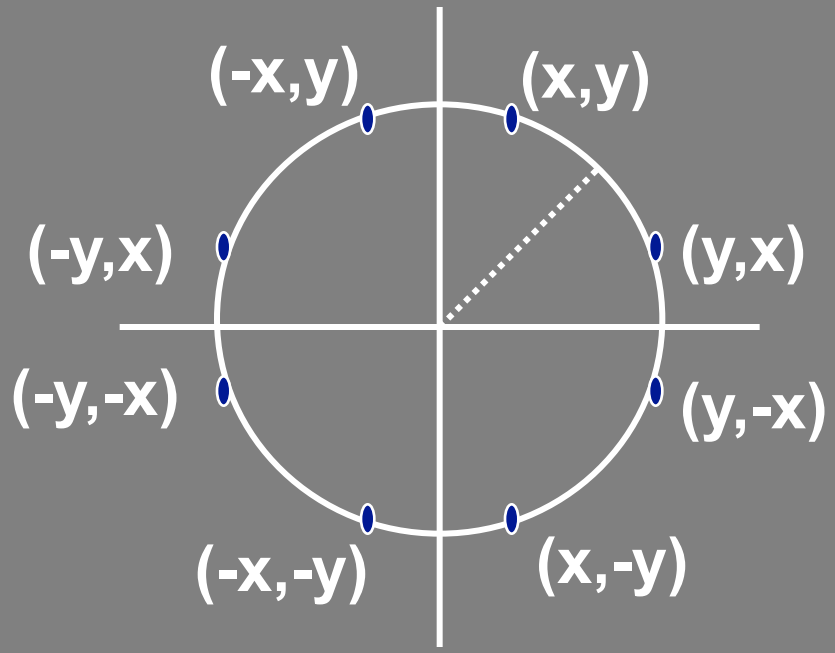
Midpoint Circle Algorithm



8-way symmetry:
drawing in one octant is enough

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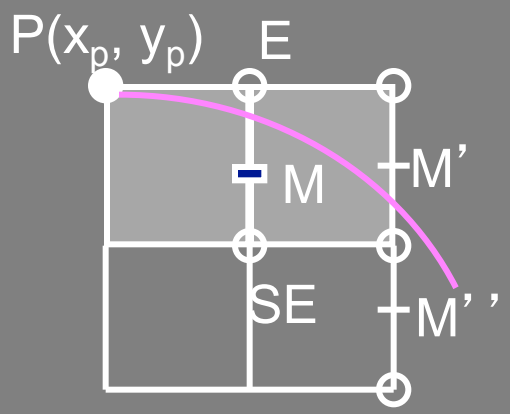
Midpoint Circle Algorithm

Consider II octant



$$F(x,y) = x^2 + y^2 - R^2$$

For a given point (x, y)

F(x, y) = 0 : (x, y) on circle

F(x, y) > 0 : (x, y) outside circle

F(x, y) < 0 : (x, y) inside circle

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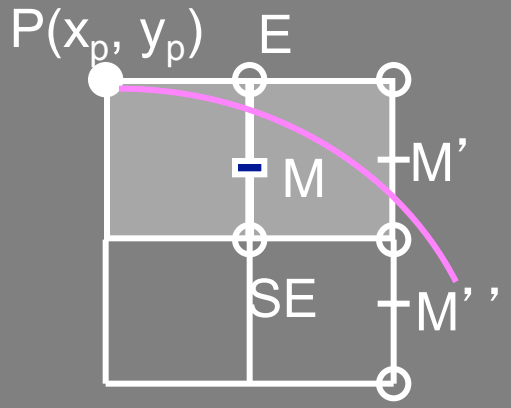
Midpoint Circle Algorithm

Consider II octant



Evaluate F(M)

If < 0 (M inside circle)

→ Choose E

If > 0 (M outside circle)

→ Choose SE

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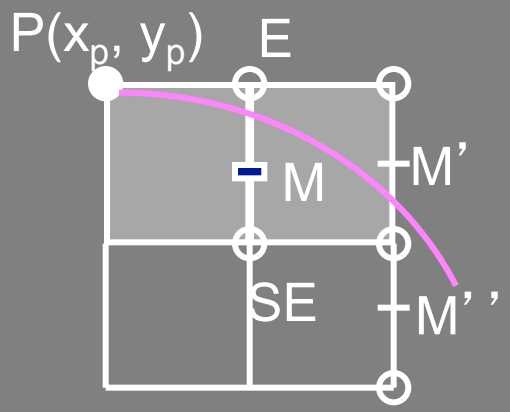
Midpoint Circle Algorithm

Consider II octant



$$d_{old} = F(M) = F(x_p + 1, y_p - \frac{1}{2})$$

$$= (x_p + 1)^2 + (y_p - \frac{1}{2})^2 - R^2$$

When E ($d_{old} < 0$)

$$d_{new} = F(M') = F(x_p + 2, y_p - \frac{1}{2})$$

$$= (x_p + 2)^2 + (y_p - \frac{1}{2})^2 - R^2$$

$$\Delta_E = d_{new} - d_{old} = 2x_p + 3$$

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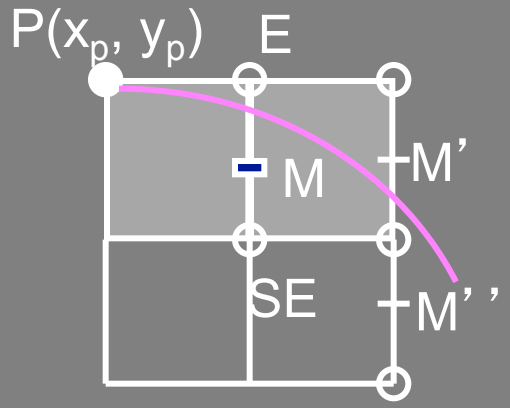
Midpoint Circle Algorithm

Consider II octant



When SE ($d_{old} \geq 0$)

$$d_{new} = F(M'') = F(x_p + 2, y_p - \frac{3}{2})$$

$$= (x_p + 2)^2 + (y_p - \frac{3}{2})^2 - R^2$$

$$\Delta_{SE} = d_{new} - d_{old} = 2x_p - 2y_p + 5$$

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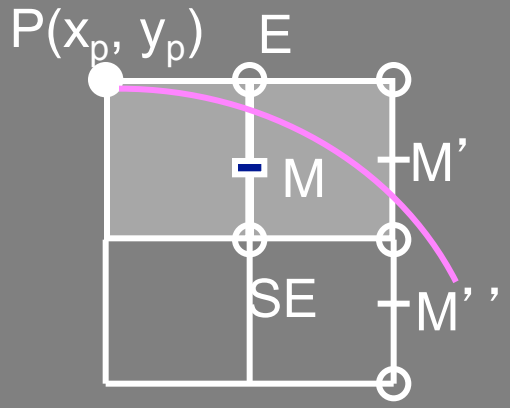
Midpoint Circle Algorithm

Consider II octant



***Initial* Condition**

(0, R) start point,

$$\text{next mid point} = (1, R - \frac{1}{2})$$

$$F(1, R - \frac{1}{2}) = \frac{5}{4} - R$$

Raster Graphics

Midpoint Circle Algorithm

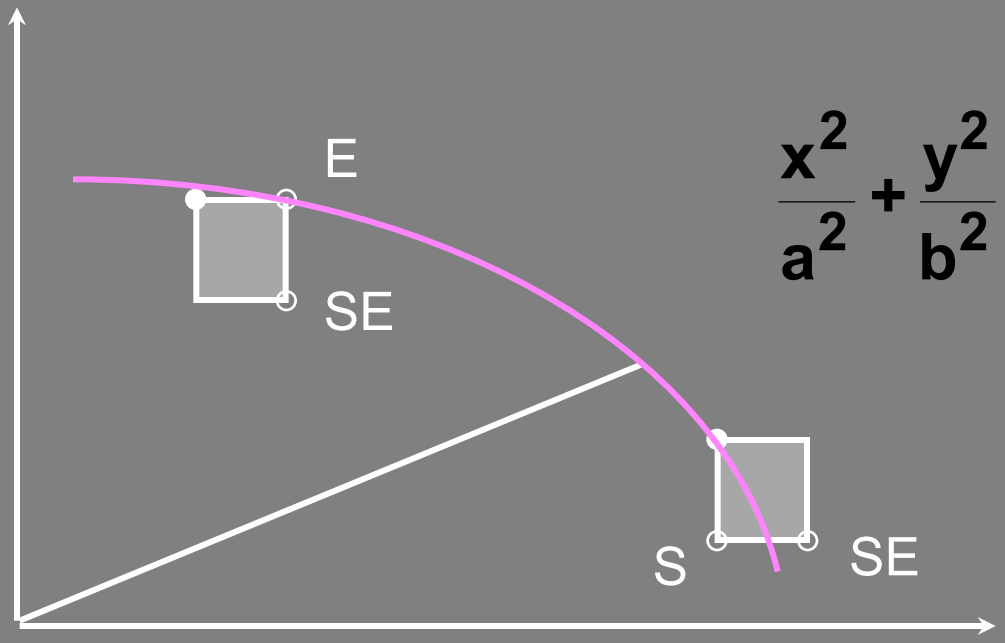
Consider II octant

$$x = 0; y = R; d = \frac{5}{4} - R;$$

```
Writepixel(x, y);
While (y > x) do
if d < 0
    d+ = 2x + 3; x+ = 1;
else
    d+ = 2x - 2y + 5; x+ = 1; y- = 1;
end
Writepixel(x, y);
end While
```

Raster Graphics

Ellipse Drawing Algorithm



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$