Raster Graphics

Drawing Algorithms

Rasterization
Pixelization
Scan Conversion

Continuous → Discrete
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Line Drawing Algorithms

Line Equation:

\[ y = mx + B \]

\[ m = \frac{dy}{dx} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} \]

DDA (Digital Differential Analyzer)
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Line Drawing Algorithms

DDA (Digital Differential Analyzer)

$y_i = mx_i + B$
$y_{i+1} = mx_{i+1} + B$
$y_{i+1} = m(x_i + \Delta x) + B$
if $\Delta x = 1$

$x_{i+1} = x_i + 1$

$y_{i+1} = y_i + m$
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Line Drawing Algorithms

DDA (Digital Differential Analyzer)

\[ y = y_1 \]

\[ \text{for} \ (x = x_1; \ x \leq x_2; \ x++) \]
\[ \{ \]
\[ \text{Writepixel}(x, \ \text{round} \ (y)); \]
\[ y+ = m; \]
\[ \} \]
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Line Drawing Algorithms

DDA (Digital Differential Analyzer)

\[ m \leq 1 \quad \text{and} \quad m > 1 \]

Exchange the role of x and y
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Line Drawing Algorithms

Midpoint Line Algorithm

Find on what side of the line the mid point is:

If below then NE is closer to line
If above then E is closer to line
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Midpoint Line Algorithm

\[ P(x_p, y_p) \]

\[ F(x, y) = 0 : (x, y) \text{ on line} \]
\[ F(x, y) > 0 : (x, y) \text{ below line} \]
\[ F(x, y) < 0 : (x, y) \text{ above line} \]
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Midpoint Line Algorithm

Consider line

\[ y = \frac{dy}{dx} x + B \]

\[ F(x, y) = ax + by + c = 0 \]

\[ xdy - ydx + Bdx = 0 \]

\[ (a = dy, b = -dx, c = Bdx) \]
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Midpoint Line Algorithm

\[ F(M) = F(x_p + 1, y_p + \frac{1}{2}) = d \]

\[ d = a(x_p + 1) + b(y_p + \frac{1}{2}) + c \]

if \( d > 0 \) M is below the line, choose NE

if \( d < 0 \) M is above the line, choose E
Midpoint Line Algorithm

When \( E \):

\[
\begin{align*}
  d_{\text{new}} &= F(M') = F(x_p + 2, y_p + \frac{1}{2}) \\
  d_{\text{new}} &= a(x_p + 2) + b(y_p + \frac{1}{2}) + c \\
  d_{\text{old}} &= a(x_p + 1) + b(y_p + \frac{1}{2}) + c \\
  \Delta_E &= d_{\text{new}} - d_{\text{old}} = a = dy
\end{align*}
\]
Midpoint Line Algorithm

When NE:

\[ d_{new} = F(M'') = F(x_p + 2, y_p + \frac{3}{2}) \]

\[ d_{new} = a(x_p + 2) + b(y_p + \frac{3}{2}) + c \]

\[ d_{old} = a(x_p + 1) + b(y_p + \frac{1}{2}) + c \]

\[ \Delta_{NE} = d_{new} - d_{old} = a + b = dy - dx \]
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Midpoint Line Algorithm

At start:

\[ d_{start} = F(x_0 + 1, y_0 + \frac{1}{2}) = a(x_0 + 1) + b(y_0 + \frac{1}{2}) + c \]

\[ d_{start} = ax_0 + by_0 + c + a + \frac{b}{2} \]

\[ d_{start} = a + \frac{b}{2} = dy - \frac{dx}{2} \quad \text{(division)} \]

\[ F(x, y) = 2(ax + by + c) \]
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Midpoint Line Algorithm

dx = x2 - x1; dy = y2 - y1;
d = 2dy - dx; ΔE = 2dy; ΔNE = 2(dy - dx);
x = x1; y = y1;
Writepixel(x, y);
While (x < x2)
if d ≤ 0
    d+ = ΔE; x+ = 1;
else
    d+ = ΔNE; x+ = 1; y+ = 1;
end
Writepixel(x, y);
end While
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Midpoint Circle Algorithm

\[(x, y)\]

\[x^2 + y^2 = R^2\]
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Midpoint Circle Algorithm

\( x^2 + y^2 = R^2 \)
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Midpoint Circle Algorithm

(x, y)
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Midpoint Circle Algorithm

8-way symmetry: drawing in one octant is enough
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Midpoint Circle Algorithm

Consider II octant

\[ F(x, y) = x^2 + y^2 - R^2 \]

For a given point \((x, y)\):
- \(F(x, y) = 0\) : \((x, y)\) on circle
- \(F(x, y) > 0\) : \((x, y)\) outside circle
- \(F(x, y) < 0\) : \((x, y)\) inside circle
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Midpoint Circle Algorithm

Consider II octant

Evaluate $F(M)$
If $< 0$ (M inside circle)  
→ Choose E
If $> 0$ (M outside circle)  
→ Choose SE
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Midpoint Circle Algorithm

Consider II octant

\[ d_{\text{old}} = F(M) = F(x_p + 1, y_p - \frac{1}{2}) \]
\[ = (x_p + 1)^2 + (y_p - \frac{1}{2})^2 - R^2 \]

When \( E (d_{\text{old}} < 0) \)

\[ d_{\text{new}} = F(M') = F(x_p + 2, y_p - \frac{1}{2}) \]
\[ = (x_p + 2)^2 + (y_p - \frac{1}{2})^2 - R^2 \]

\[ \Delta_E = d_{\text{new}} - d_{\text{old}} = 2x_p + 3 \]
Midpoint Circle Algorithm

Consider II octant

When SE \( (d_{\text{old}} \geq 0) \)

\[
d_{\text{new}} = F(M'') = F(x_p + 2, y_p - \frac{3}{2})
\]

\[
= (x_p + 2)^2 + (y_p - \frac{3}{2})^2 - R^2
\]

\[
\Delta_{SE} = d_{\text{new}} - d_{\text{old}} = 2x_p - 2y_p + 5
\]
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Midpoint Circle Algorithm

Consider II octant

Initial Condition
(0, R) start point,
next mid point = \((1, R - \frac{1}{2})\)

\[ F(1, R - \frac{1}{2}) = \frac{5}{4} - R \]
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Midpoint Circle Algorithm

Consider II octant

\[ x = 0; \ y = R; d = \frac{5}{4} - R; \]

Writepixel(x, y);
While (y > x) do
if \( d < 0 \)
\[ d+ = 2x + 3; \ x+ = 1; \]
else
\[ d+ = 2x - 2y + 5; \ x+ = 1; \ y- = 1; \]
end
Writepixel(x, y);
end While
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Ellipse Drawing Algorithm

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]