Deformation
Deformation
Deformation causes change in the shape keeping typically the same topology

Geometric deformation (does not account for any law of physics)

Local or global deformation
Deformation: A transformation/mapping of the positions of every particle in the original object to those in the deformed body

Each particle represented by a point \( p \) is moved by \( \phi(\cdot) \):
\[
p \rightarrow \phi(t, p)
\]
where \( p \) represents the original position and \( \phi(t, p) \) represents the position at time \( t \).
Deformation
Deformation

• Changing an object’s shape
  – Usually refers to non-simulated algorithms
  – Usually relies on user guidance

• Easiest when the number of faces and vertices of a shape is preserved, and the shape topology is not changed either
  – Define the movements of vertices
Deformation

\((x, y, z)\)  \(\varphi(x, y, z)\)
Deformation

- If vertex $i$ is displaced by $(x, y, z)$ units
  - Displace each neighbor, $j$, of $i$ by
    - $(x, y, z) \ast f(i, j)$

- $f(i,j)$ is typically a function of distance
  - Euclidean distance
  - Number of edges from $i$ to $j$
  - Distance along surface (i.e., geodesics)
Deformation

Original Mesh → Destination Mesh

Destination Mesh ← Original Mesh
Deformation

Displacement of seed vertex

Attenuated displacement propagated to adjacent vertices
Deformation

\[ f(i) = 1.0 - \left( \frac{i}{n+1} \right)^{k+1}; k \geq 0 \]

\[ f(i) = \left( 1.0 - \frac{i}{n+1} \right)^{-k+1}; k < 0 \]
Deformation

\[ P_{u0} = (1-u)P_{00} + uP_{10} \]
\[ P_{u1} = (1-u)P_{01} + uP_{11} \]
\[ P_{uv} = (1-v)P_{u0} + vP_{u1} \]
\[ = (1-u)(1-v)P_{00} + (1-v)uP_{01} + u(1-v)P_{10} + uvP_{11} \]
Deformation
Deformation

Working at a coarser level

Working at a finer level
Local Deformation

Hierarchical B-Spline Surfaces
Local Deformation

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Hierarchical B-Spline Surfaces
Global Deformation

A Barr SIGGRAPH 1984
Global Deformation

Original object

\[ s(z) = \frac{(\text{maxz} - z)}{\text{maxz} - \text{minz}} \]

\[ x' = s(z)x \]
\[ y' = s(z)y \]
\[ z' = z \]

Tapered object

\[ P' = M(p)p \]
Global Deformation

\[ k = \text{twist factor} \]
\[ x' = x\cos(kz) - y\sin(kz) \]
\[ y' = x\sin(kz) + y\cos(kz) \]
\[ z' = z \]
Global Deformation
\[ s(z) = \frac{(\max z - z)}{(\max z - \min z)} \]

\[
\begin{bmatrix}
  x' \\
  y' \\
  z'
\end{bmatrix} = \begin{bmatrix}
  s(z) & 0 & 0 \\
  0 & s(z) & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
\]

\[ x' = s(z) \cdot x \]
\[ y' = s(z) \cdot y \]
\[ z' = z \]

Figure 3.63 Global tapering

\[ k = \text{twist factor} \]
\[ x' = x \cdot \cos (k \cdot z) - y \cdot \sin (k \cdot z) \]
\[ y' = x \cdot \sin (k \cdot z) + y \cdot \cos (k \cdot z) \]
\[ z' = z \]

Figure 3.64 Twist about an axis
\[ \theta = \begin{cases} 
    z - z_{\min} & z < z_{\max} \\
    z_{\max} - z_{\min} & \text{otherwise} 
\end{cases} \]

\[ C_\theta = \cos \theta \]
\[ S_\theta = \sin \theta \]
\[ R = y_0 - y \]

\[ x' = x \]

\[ y' = \begin{cases} 
    y & \text{if } z < z_{\min} \\
    y_0 - (R \cdot C_\theta) & \text{if } z_{\min} \leq z \leq z_{\max} \\
    y_0 - (R \cdot C_\theta) + (z - z_{\max}) \cdot S_\theta & \text{if } z > z_{\max} 
\end{cases} \]

\[ z' = \begin{cases} 
    z & \text{if } z < z_{\min} \\
    z_{\min} + (R \cdot S_\theta) & \text{if } z_{\min} \leq z \leq z_{\max} \\
    z_{\min} + (R \cdot S_\theta) + (z - z_{\max}) \cdot C_\theta & \text{if } z > z_{\max} 
\end{cases} \]

Figure 3.65 Global bend operation
• Physical Analogy: A clear, flexible plastic parallelepiped with one or more objects embedded in it.
• The embedded objects are also flexible so that they deform with the object.
Parametric surfaces are free-form surfaces.

The flexibility in this technique of deformation allows us deform the model in a free-form manner.

Any surface patches
Global or local deformation
Continuity in local deformation
Volume preservation
Free Form Deformation

\[ V = \sum_{i} W_i X_i \]
Free Form Deformation

\[ V = \sum_{i} W_i X_i \]

\[ V = \sum_{i} w_i x_i = \sum_{i} \binom{d}{i} (1-t)^{d-i} t^i x_i \]
Free Form Deformation
Free Form Deformation

\[ v = \frac{2}{3} \]

\[ 2(1 - u)^2 (1 - v) v \]

\[ (1 - u)^2 (1 - v)^2 \]

\[ 2(1 - u) u (1 - v)^2 \]

\[ u^2 (1 - v)^2 \]
Free Form Deformation
Basic idea: deform space by deforming a lattice around an object

• The deformation is defined by moving the control points of the lattice
• Imagine it as if the object were enclosed by rubber
• The key is how to define
  – Local coordinate system
  – The mapping
Free Form Deformation

\[ X = X_0 + s S + t T + u U. \]

\[ s = \frac{T \times U \cdot (X - X_0)}{T \times U \cdot S}, \quad t = \frac{S \times U \cdot (X - X_0)}{S \times U \cdot T}, \quad u = \frac{S \times T \cdot (X - X_0)}{S \times T \cdot U} \]

\[ 0 < s < 1, \ 0 < t < 1 \text{ and } 0 < u < 1. \]

\[ X_{fd} = \sum_{i=0}^{l} \left( \begin{array}{c} l \\ i \end{array} \right) (1-s)^{l-i} s^i \left[ \sum_{j=0}^{m} \left( \begin{array}{c} m \\ j \end{array} \right) (1-t)^{m-j} t^j \left[ \sum_{k=0}^{n} \left( \begin{array}{c} n \\ k \end{array} \right) (1-u)^{n-k} u^k P_{ijk} \right] \right] \]
Free Form Deformation
Free Form Deformation

Point in a cell is repositioned within the corresponding cell in the deformed lattice, in the same relative position within the cell.
Can enforce $C^k$ continuity

- Surface $(s, t, u) = (s(v, w), t(v, w), u(v, w))$
- Two adjacent FFDs $X_1(s_1, t_1, u_1) \& X_2(s_2, t_2, u_2)$ with common boundary $s_1 = s_2 = 0$
- Conditions for first derivative continuity
  - $\frac{\partial X_1(0, t, u)}{\partial s} = \frac{\partial X_2(0, t, u)}{\partial s}$
  - $\frac{\partial X_1(0, t, u)}{\partial t} = \frac{\partial X_2(0, t, u)}{\partial t}$
  - $\frac{\partial X_1(0, t, u)}{\partial u} = \frac{\partial X_2(0, t, u)}{\partial u}$
Can enforce Ckcontinuity
Can enforce Ckcontinuity
Free Form Deformation

Volume Preservation
Non Parallelopipeded Lattice
Non Parallelopipeded Lattice
Non Parallelopiped Lattice
Extended Free Form Deformation

Some Results
Animation
Free Form Deformation
Animation
Free Form Deformation

Object traversing the logical FFD coordinate space

Object traversing the distorted space
Animation
Free Form Deformation

Initial configuration

Surface distorted after joint articulation
Free Form Deformation
Animation
Free Form Deformation
Direct
Free Form Deformation

![Diagram of a deformed sphere and a face.](image-url)
Direct Free Form Deformation
Direct Free Form Deformation


References

