Curves

B-Splines

Properties

- Convex hull property
- Curve follows the shape of the defining polygon
- Maximum order = number of control points
- Invariance to affine transformation
- Variation diminishing property

Curves

B-Splines

Control Handles

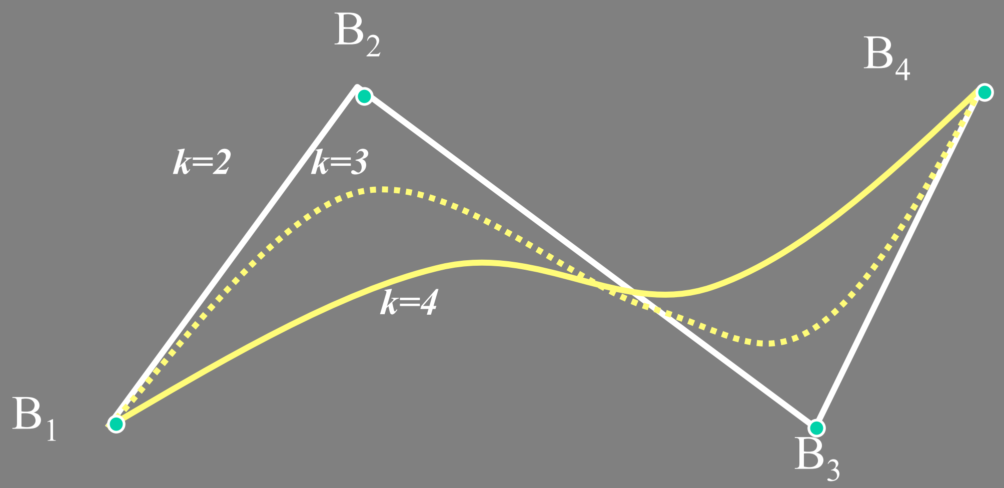
- Knot vector (uniform, open uniform, non-uniform)
- Order *k*
- Number and position of control point

Curves

B-Splines

Order *k*

Open-uniform knot vector
Control polygon B_1 B_2 B_3 B_4



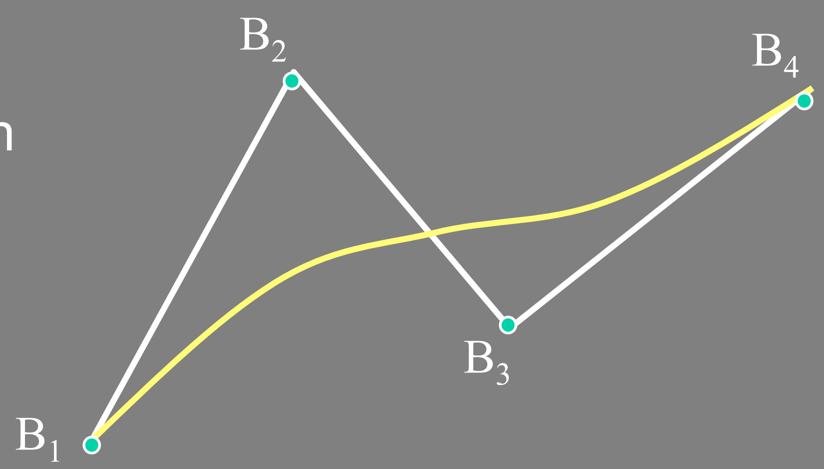
Curves

B-Splines

Multiple points

Open uniform knot vector
k=4

Control polygon
B_1 B_2 B_3 B_4



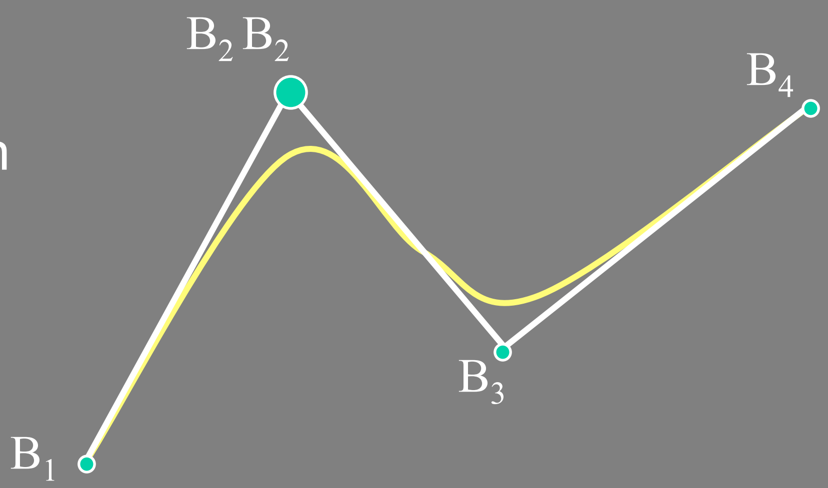
Curves

B-Splines

Multiple points

Open uniform knot vector
k=4

Control polygon
$B_1\ B_2\ B_2\ B_3\ B_4$



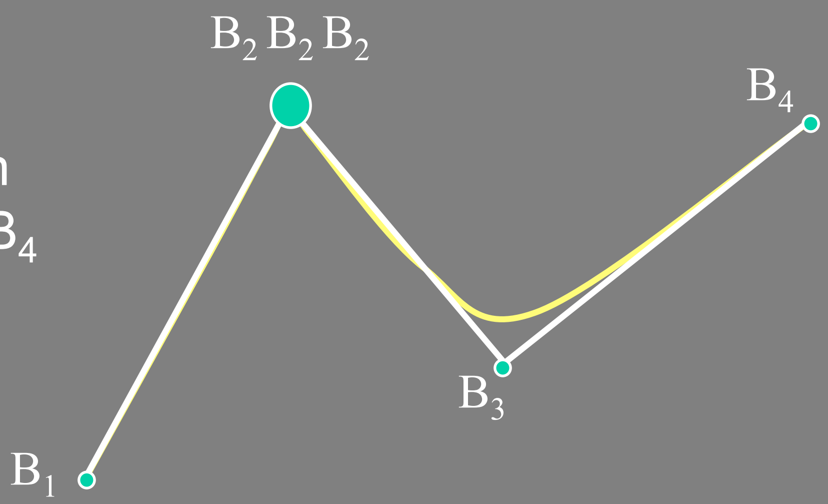
Curves

B-Splines

Multiple points

Open uniform knot vector
k=4

Control polygon
B_1 B_2 B_2 B_2 B_3 B_4

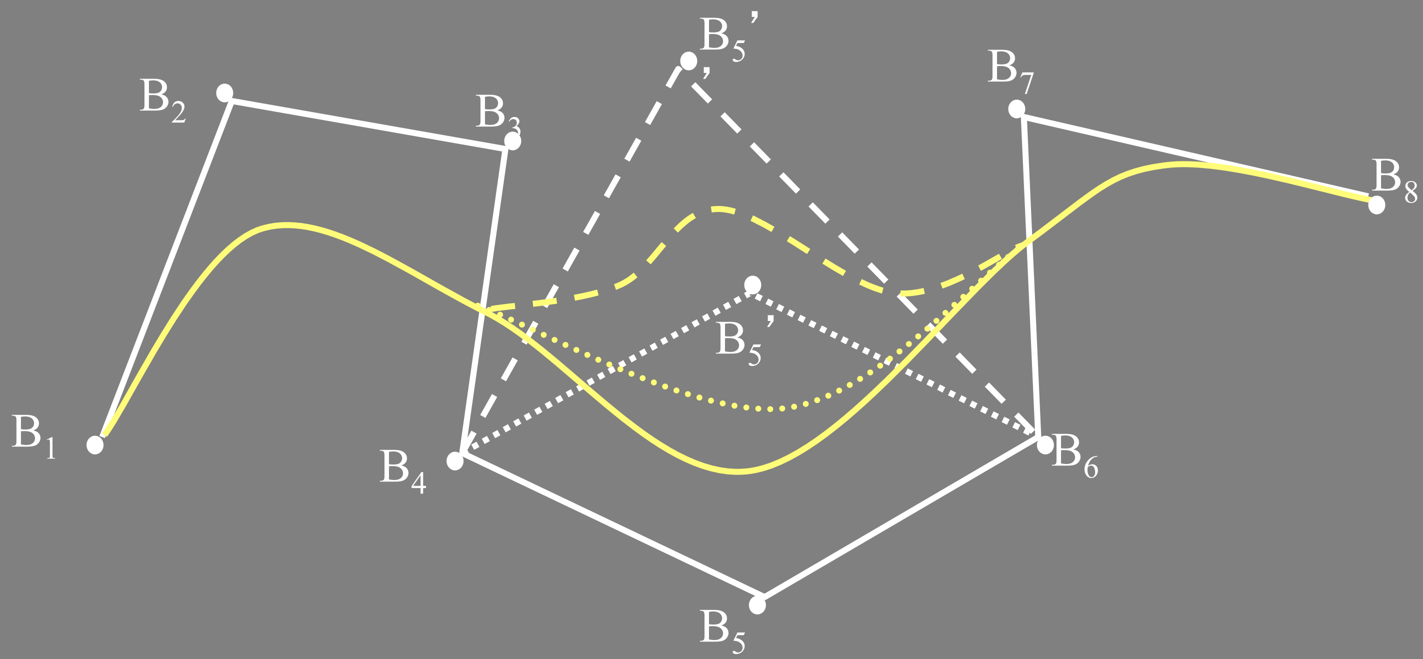


Curves

B-Splines

Local Control

Open uniform knot vector
k=4



Curves

B-Splines

Rational B-Splines

$$P(t) = \sum_{i=1}^{n+1} B_i^h N_{ik}(t)$$

Projection of a non-rational (polynomial) B-Spline in 4D into 3D

Curves

B-Splines

Rational B-Splines

$$P(t) = \frac{\sum_{i=1}^{n+1} B_i N_{ik}(t) h_i}{\sum_{i=1}^{n+1} h_i N_{ik}(t)} = \sum_{i=1}^{n+1} B_i R_{ik}(t)$$

$$R_{ik}(t) = \frac{h_i N_{ik}(t)}{\sum_{i=1}^{n+1} h_i N_{ik}(t)}$$

Curves

B-Splines

Rational B-Splines

- Generalization of non-rational B-Spline basis functions and curve
- Most properties and characteristics of non-rational B-spline are carry forwarded
- Additional control handle of h_i
- Enables construction of common analytical shapes

    conic sections

Curves

B-Splines

Rational B-Splines

Invariant to projective transformation

The curve is invariant with respect to the projective transformation applied to the defining vertices of the control polygon

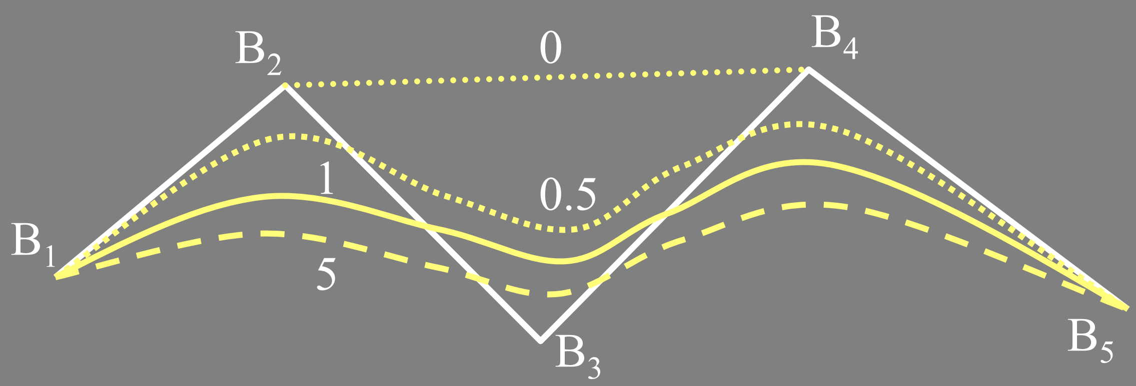
Curves

B-Splines

Rational B-Splines

n+1 = 5, k = 3

X = [0 0 0 1 2 3 3 3] H = [1 1 h_3 1 1]



Curves

B-Splines

Rational B-Splines

NURBS : Non Uniform Rational B-Spline

Most generalized form of B-Spline

Non Uniform Knot Vector

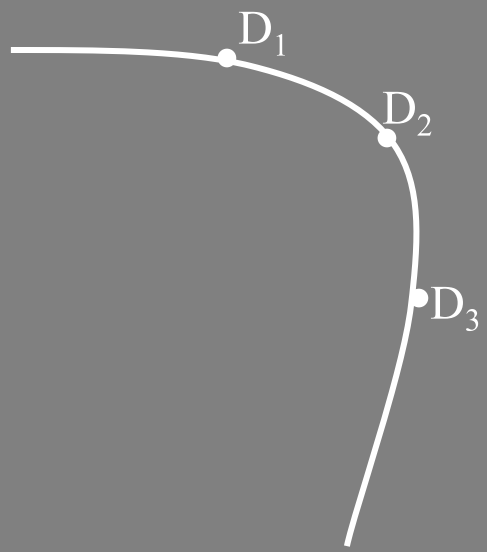
Rational Basis Functions

Curves

B-Splines

Curve Fitting

Find a B-Spline passing through D_1, D_2 … D_j



Curves

B-Splines

Curve Fitting

$$P(t) = \sum_{i=1}^{n+1} B_i N_{i,k}(t)$$

$$D_1(t_1) = N_{1,k}(t_1)B_1 + N_{2,k}(t_1)B_2 + \ldots + N_{n+1,k}(t_1)B_{n+1}$$

$$D_2(t_2) = N_{1,k}(t_2)B_1 + N_{2,k}(t_2)B_2 + \ldots + N_{n+1,k}(t_2)B_{n+1}$$

$$\vdots$$

$$D_j(t_j) = N_{1,k}(t_j)B_1 + N_{2,k}(t_j)B_2 + \ldots + N_{n+1,k}(t_j)B_{n+1}$$

Curves

B-Splines

Curve Fitting

$$[D] = [N]\,[B]$$

$$[N] = \begin{bmatrix} N_{1,k}(t_1) & \cdots & \cdots & N_{n+1,k}(t_1) \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ N_{1,k}(t_j) & \cdots & \cdots & N_{n+1,k}(t_j) \end{bmatrix}$$

$$[B] = [N]^{-1}[D]$$

When $j = n+1$ => N square matrix

Curves

B-Splines

Curve Fitting

$$[D] = [N]\,[B]$$

$$[N]^T[D] = [N]^T[N][B]$$

$$[B] = \left[[N]^T[N]\right]^{-1}[N]^T[D]$$

Curves

B-Splines

Curve Fitting

Parameter value t_j

For l^{th} data point

$$\frac{t_l}{t_{max}} = \frac{\sum_{s=2}^{l} \left| D_s - D_{s-1} \right|}{\sum_{s=2}^{j} \left| D_s - D_{s-1} \right|} \qquad l \geq 2$$

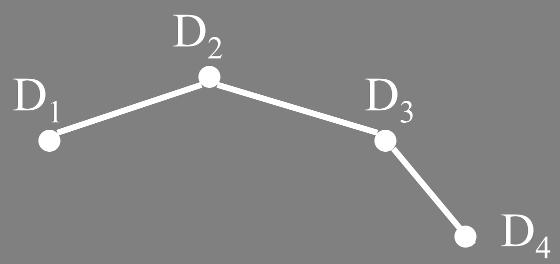
$t_1 = 0$, t_{max} = maximum value of the knot vector

Curves

B-Splines

Curve Fitting

Parameter value t_j



$$\frac{t_2}{t_{max}} = \frac{|D_2 - D_1|}{|D_2 - D_1| + |D_3 - D_2| + |D_4 - D_3|}$$