Ray Tracing

Rendering

Issues

• Visibility
  What objects or parts in the scene are visible?
  Clipping (with respect to the view frustum)
    Done
  Occlusion (with respect to the objects in the scene)
    Hidden surface elimination

• Illumination
  Reflection, Refraction, Transparency, Shadows, etc.
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Rendering Pipeline (Revisit)

Forward Mapping Approach

Modeling Transformation

Model 1

Model 2

Model n

3D World Scene

Viewing Transformation

V

3D View Scene

2D Image

Rasterization

2D Scene

Projection
Ray Tracing

Forward Ray Tracing

Modeling interaction of light with the objects/surfaces

Problem:
Many rays will not contribute to the image!
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Backward Ray Tracing

Rays from camera (viewer) through each pixel to the scene

Backward Ray Tracing = Ray Tracing
Ray Tracing

Backward Ray Tracing

Primary and Secondary Rays
Ray Tracing

Backward Ray Tracing

Shadow Rays

Visibility check with respect to the light source
Ray Tracing

Ray Casting

Viewer

View Plane

A

B

C

D

E

F
Ray Tracing

Two Issues

Ray-object intersection
   Visibility test: Closest to the viewer

Pixel color determination (shading)
   Illumination model
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Ray Object Intersection

Sphere

**Ray Origin**
\[ R_o = [X_o \ Y_o \ Z_o] \]

**Ray Direction**
\[ R_d = [X_d \ Y_d \ Z_d] \]

**Parametric Form**
\[ R(t) = R_o + R_d t \quad t > 0 \]

\[ X_d^2 + Y_d^2 + Z_d^2 = 1 \]
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Ray Object Intersection

Sphere

Implicit Form

Center $S_c = [X_c \ Y_c \ Z_c]$
Radius $S_r$
Surface Point $[X_s \ Y_s \ Z_s]$

$$(X_s - X_c)^2 + (Y_s - Y_c)^2 + (Z_s - Z_c)^2 = S_r^2$$
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Ray Object Intersection

Sphere

To solve the intersection problem the ray equation is substituted into the sphere equation and the result is solved for $t$

That is

$$(X_o + X_d t - X_c)^2 + (Y_o + Y_d t - Y_c)^2 + (Z_o + Z_d t - Z_c)^2 = S_r^2$$
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Ray Object Intersection

Sphere

\[ At^2 + Bt + C = 0 \]

where

\[ A = X_d^2 + Y_d^2 + Z_d^2 = 1 \]
\[ B = 2(X_d(X_o - X_c) + Y_d(Y_o - Y_c) + Z_d(Z_o - Z_c)) \]
\[ C = (X_o - X_c)^2 + (Y_o - Y_c)^2 + (Z_o - Z_c)^2 - S_r^2 \]
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Ray Object Intersection

Sphere

\[ At^2 + Bt + C = 0 \]

\[
t_0 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}
\]

\[
t_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A}
\]

Smaller positive among \( t_0 \) and \( t_1 \) gives the closest intersection point

\[
[ X_i, Y_i, Z_i ] = [ X_o + X_d t, Y_o + Y_d t, Z_o + Z_d t ]
\]
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Ray Object Intersection

Sphere

Normal

\[ n = \begin{bmatrix} \frac{(X_i - X_c)}{S_r}, \frac{(Y_i - Y_c)}{S_r}, \frac{(Z_i - Z_c)}{S_r} \end{bmatrix} \]
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Ray Sphere Intersection

Sum up

- Calculate A B C
- Compute the discriminant
- Calculate min (t₀, t₁)
- Compute the intersection point
- Compute the normal
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Ray Object Intersection

Sphere

Geometric Approach

\[ R_0 \quad L \quad d \quad r \quad O \quad t_{ca} \quad t_{hc} \quad R_d \]
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Ray Object Intersection

Sphere

Geometric Approach

\[ L = O - R_0 \]
\[ t_{ca} = L^T R_d \]
\[ t_{ca} < 0 \text{ no intersection} \]
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Ray Object Intersection

Sphere

Geometric Approach

\[ L = O - R_0 \]
\[ t_{ca} = L^T R_d \]
\[ t_{ca} < 0 \text{ no intersection} \]
\[ d = L^T L - t_{ca}^2 \]
if \( d > r \) no intersection
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Ray Object Intersection

Sphere

Geometric Approach

\[ t_{hc} = \sqrt{r^2 - d^2} \]

\[ t = t_{ca} - t_{hc} \text{ and } t_{ca} + t_{hc} \]

smaller \( t \)
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Ray Plane Intersection

Ray

\[
\begin{align*}
R_o &= [X_o \ Y_o \ Z_o] \quad \text{(ray origin)} \\
R_d &= [X_d \ Y_d \ Z_d] \quad \text{(ray direction)} \\
X_d^2 + Y_d^2 + Z_d^2 &= 1 \quad \text{(normalized)} \\
R(t) &= R_o + R_d t \quad t > 0
\end{align*}
\]

Plane

\[
\begin{align*}
P : Ax + By + Cz + D &= 0 \\
A^2 + B^2 + C^2 &= 1 \\
P_{\text{normal}} &= P_n = [A \ B \ C] \\
D : \text{Distance from origin}
\end{align*}
\]
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Ray Plane Intersection

Substituting ray equation in plane’s equation

\[ A (X_o + X_d t) + B (Y_o + Y_d t) + C (Z_o + Z_d t) + D = 0 \]

Solving for \( t \)

\[
t = -\frac{AX_0 + BY_0 + CZ_0 + D}{AX_d + BY_d + CZ_d}
\]

\[
t = -\frac{P_n \cdot R_0 + D}{P_n \cdot R_d}
\]
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Ray Plane Intersection

Let

\[ V_d = P_n \cdot R_d = AX_d + BY_d + CZ_d \]

If \( V_d = 0 \) then the ray is parallel to the plane (no intersection)

\( V_d > 0 \) normal is pointing away from the ray (may be used for back-face culling)
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Ray Plane Intersection

Let

\[ V_0 = -(P_n \cdot R_0 + D) = (AX_0 + BY_0 + CZ_0 + D) \]

\[ t = \frac{V_0}{V_d} \]

If \( t < 0 \) then plane is behind ray’s origin
else compute intersection

\[ r_i = [X_i, Y_i, Z_i] = [X_o + X_d t, Y_o + Y_d t, Z_o + Z_d t] \]
\[ r_{normal} = P_n \]
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Polygon Intersection

Containment Test

Parity Test: If the number of intersection is odd then point is inside (special case for vertices)
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Triangle Intersection

Containment Test

Triangle: Barycentric Coordinates

\[ P = uV_1 + vV_2 + wV_3 \]
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Triangle Intersection

Containment Test

Triangle: Barycentric Coordinates

\[ P = uV_1 + vV_2 + wV_3 \]
Ray Tracing

Triangle Intersection

Containment Test

Triangle: Barycentric Coordinates

\[ u = \frac{A_1}{A}, \quad v = \frac{A_2}{A}, \quad w = \frac{A_3}{A} \]
\[ u + v + w = 1 \]
\[ u \geq 0, \quad v \geq 0, \quad w \geq 0 \]
\[ P = uV_1 + vV_2 + wV_3 \]
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Ray Quadric Intersection

Quadrics:

Cylinders, Cone, Sphere, Ellipsoids, Paraboloids, Hyperboloids, etc.

Implicit form $f(X, Y, Z) = 0$

$Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 2Gy + Hz^2 + 2lz + J = 0$

Ray: Parametric form

$Ro = [ X_o \ Y_o \ Z_o ]$ (ray origin)

$Rd = [ X_d \ Y_d \ Z_d ]$ (ray direction)

$X_d^2 + Y_d^2 + Z_d^2 = 1$ (normalized)

$R(t) = R_o + R_d t \quad t > 0$
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Ray Quadric Intersection

Matrix Form

\[ f(X, Y, Z) = 0 \]

\[
\begin{bmatrix}
X & Y & Z & 1
\end{bmatrix}
\begin{bmatrix}
A & B & C & D \\
B & E & F & G \\
C & F & H & I \\
D & G & I & J
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} = 0
\]
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Ray Quadric Intersection

Substituting

\[ A_q t^2 + B_q t + C_q = 0 \]

If \( A_q \neq 0 \)

\[
t_0 = \frac{-B_q - \sqrt{B_q^2 - 4A_qC_q}}{2A_q}
\]

\[
t_1 = \frac{-B_q + \sqrt{B_q^2 - 4A_qC_q}}{2A_q}
\]

If \( A_q = 0 \)

\[
t = -\frac{C_q}{B_q}
\]
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Ray Quadric Intersection

Normal

\[
n = \begin{bmatrix}
\frac{\partial F}{\partial X_i}, & \frac{\partial F}{\partial Y_i}, & \frac{\partial F}{\partial Z_i}
\end{bmatrix}
\]

\[
n_x = 2(AX_i + BY_i + CZ_i + D)
\]

\[
n_y = 2(BX_i + EY_i + FZ_i + G)
\]

\[
n_z = 2(CX_i + FY_i + HZ_i + I)
\]
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Ray Box Intersection

3D Clipping: Cyrus Beck/Liang Barsky