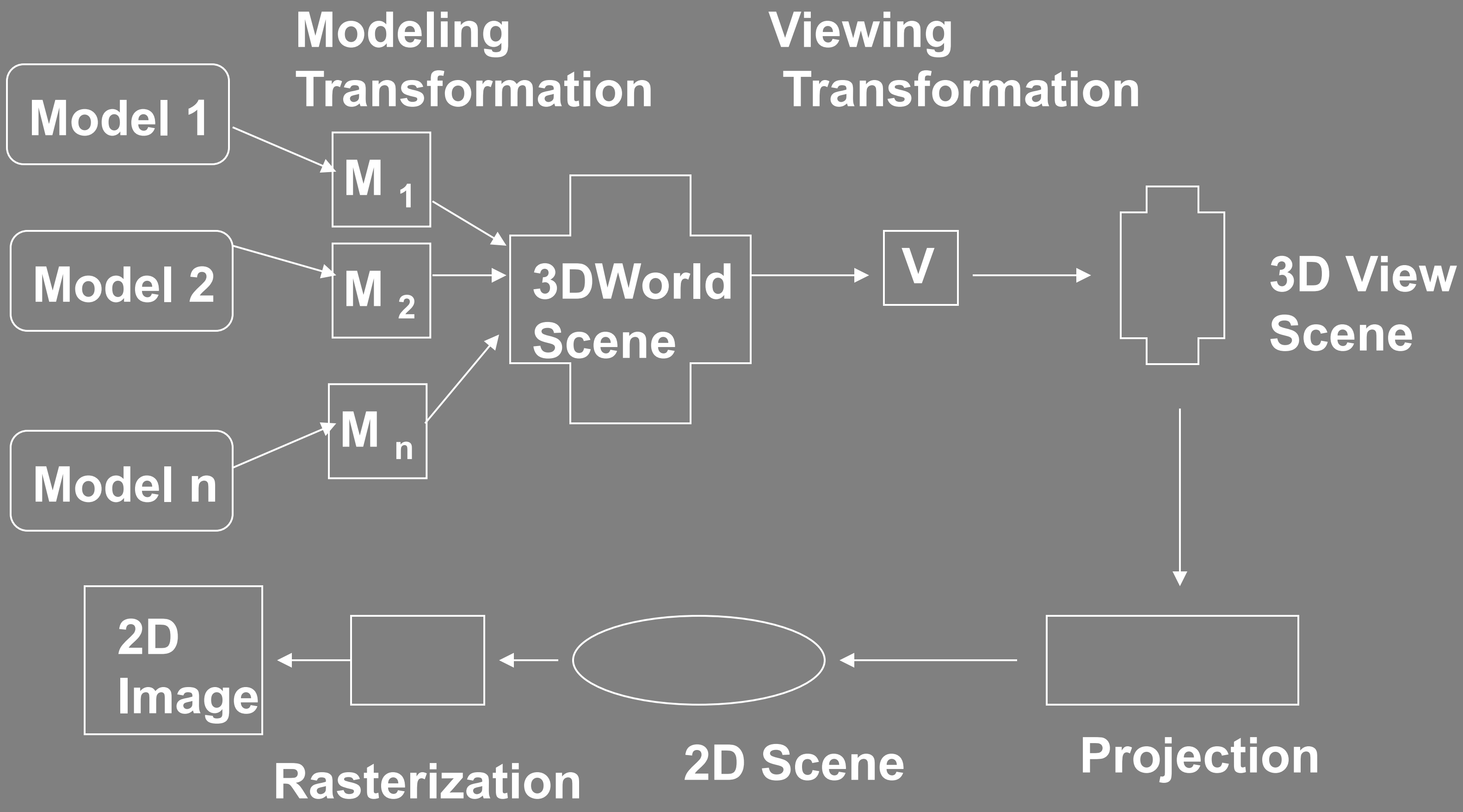
Graphics Rendering Pipeline



2D Transformation

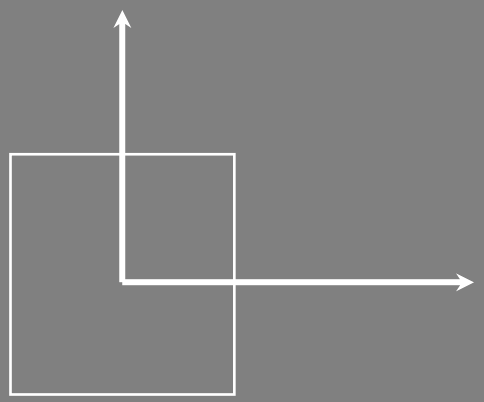
Scaling

$$P(x, y) \rightarrow P'(x', y')$$

$$x' = s_x x; y' = s_y y$$

$$[x', y'] = [x, y]\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

$$P' = P.S$$



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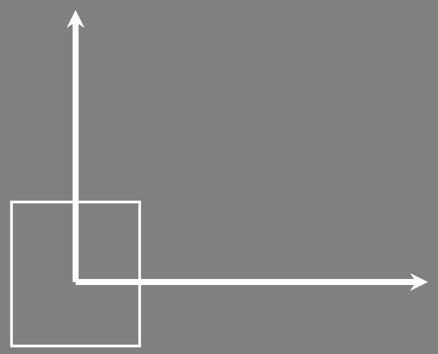


$s_x = s_y$: **uniform scaling**

$s_x \neq s_y$: **differential scaling**

2D Transformation

Rotation

$$P(x,y) \rightarrow P'(x',y')$$

$$x = r\cos(\varphi); y = r\sin(\varphi)$$

$$x' = r\cos(\varphi + \theta)$$

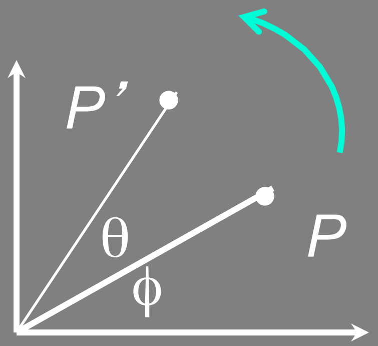
$$y' = r\sin(\varphi + \theta)$$

$$x' = x\cos(\theta) - y\sin(\theta)$$

$$y' = x\sin(\theta) + y\cos(\theta)$$

$$[x',y'] = [x,y]\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$P' = P.R_\theta$$



2D Transformation

General 2x2 Matrix

$$X \rightarrow X'$$

$$X' = X.T$$

$$[x', y'] = [x, y]\begin{bmatrix} a & b \\ c & d \end{bmatrix} = [(ax + cy), (bx + dy)]$$

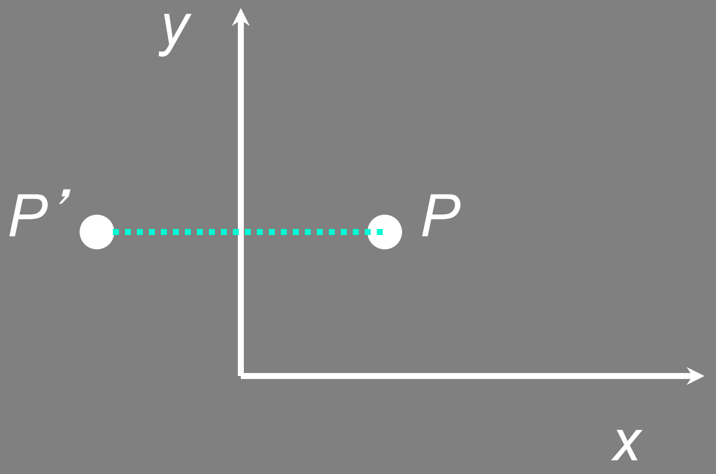
If a = d = 1 and b = c = 0
T = Identity Matrix
X' = X

2D Transformation

General 2x2 Matrix

If b = c = 0
X' = [ax, dy] : Scaling

If b = c = 0 and a = -1, d = 1
X' = [-x, y] : Reflection

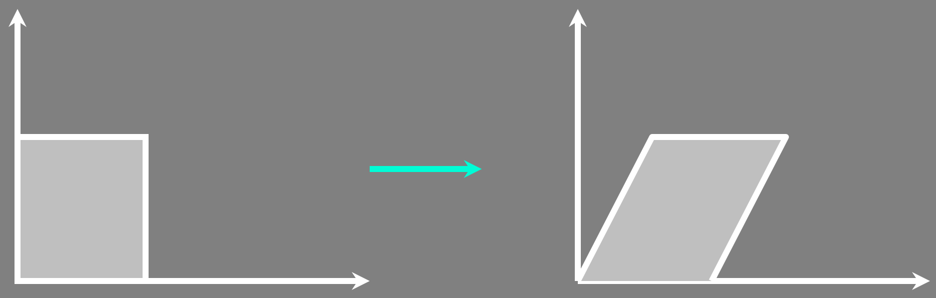


2D Transformation

General 2x2 Matrix

$$[x',y'] = [x,y]\begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix} = [x+cy, y]$$

Shearing in X

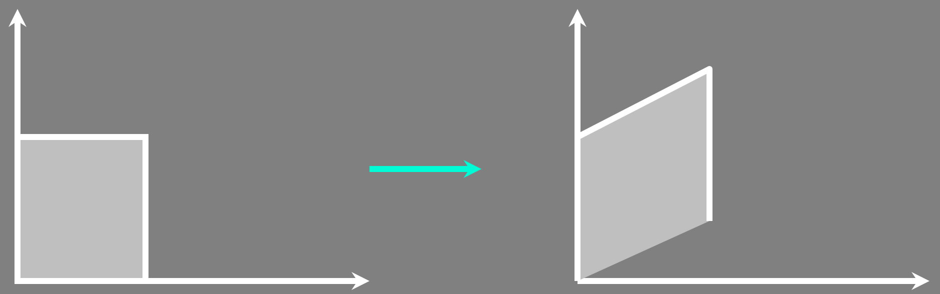


2D Transformation

General 2x2 Matrix

$$[x', y'] = [x, y]\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} = [x, bx + y]$$

Shearing in Y



2D Transformation

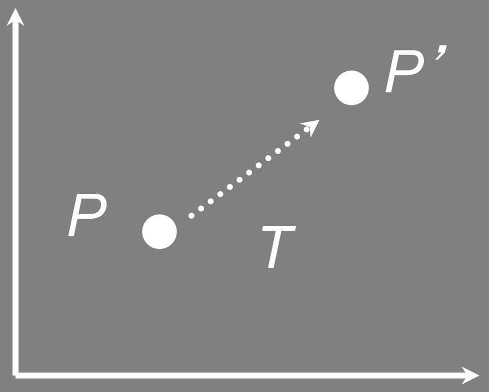
Translation

$$P(x, y) \rightarrow P'(x', y')$$

$$x' = x + t_x ; y' = y + t_y$$

$$P' = P + T$$

$$P = [x, y]; T = [t_x, t_y]$$



2D Transformation

Homogenous Coordinates

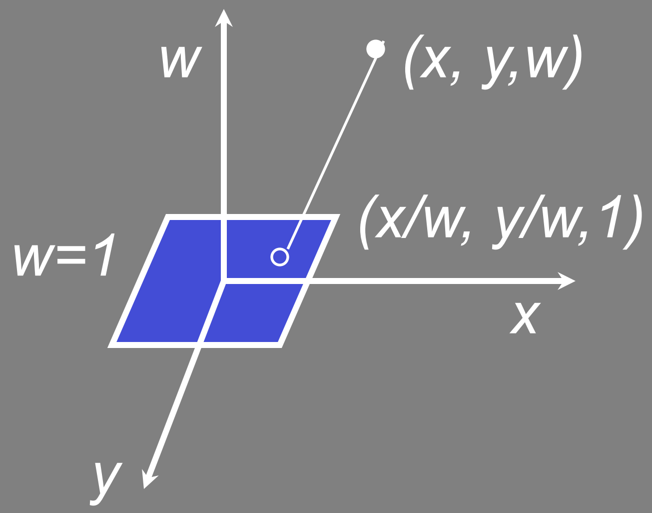
Scale/Rotate/Reflect/Shear: X' = XT
Translate: X' = X + T

$$P = [x, y]$$
$$P_h = [x, y, w]$$

Multiple values for the same point
e.g., (2, 3, 6) and (4, 6, 12) are same points

2D Transformation

Homogenous Coordinates



2D Transformation

Homogenous Coordinates

Unifying representation for transformation
Transformation matrix from 2x2 to 3x3

$$X \rightarrow X'$$

$$X' = X.T$$

$$[x', y', w] = [x, y, 1] \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ l & m & 1 \end{bmatrix}$$

$$= [(ax + cy + l), (bx + dy + m), 1]$$

2D Transformation

Homogenous Coordinates

Translation

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l & m & 1 \end{bmatrix}$$

$$X' = XT$$

$$[x', y'] = [x + l, y + m]$$

2D Transformation

Homogenous Coordinates

Scaling/Rotation/Shear

$$T = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2D Transformation

Homogenous Coordinates

Successive translations: $T_1 = (l_1, m_1)$, $T_2 = (l_2, m_2)$

After T_1

$$X' = X\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & m_1 & 1 \end{bmatrix}$$

After T_1 and T_2

$$X'' = X'\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_2 & m_2 & 1 \end{bmatrix}$$

$$\boldsymbol{X'' = X\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & m_1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_2 & m_2 & 1 \end{bmatrix} = X\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 + l_2 & m_1 + m_2 & 1 \end{bmatrix}}$$

Successive translations are additive

2D Transformation

Homogenous Coordinates

Successive scaling: $S_1 = (s_{x1}, s_{y1})$, $S_2 = (s_{x2}, s_{y2})$

After S_1

$$X' = X\begin{bmatrix} s_{x1} & 0 & 0 \\ 0 & s_{y1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After S_1 and S_2

$$X'' = X'\begin{bmatrix} s_{x2} & 0 & 0 \\ 0 & s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{X'' = X\begin{bmatrix} s_{x1} & 0 & 0 \\ 0 & s_{y1} & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} s_{x2} & 0 & 0 \\ 0 & s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = X\begin{bmatrix} s_{x1}s_{x2} & 0 & 0 \\ 0 & s_{y1}s_{y2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

Successive scaling is multiplicative

2D Transformation

Homogenous Coordinates

Successive rotations: R(θ), R(ϕ)

After R(θ)

$$X' = X\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After R(θ) and R(ϕ)

$$X'' = X'\begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{X'' = X\begin{bmatrix} \cos(\theta+\varphi) & \sin(\theta+\varphi) & 0 \\ -\sin(\theta+\varphi) & \cos(\theta+\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

Successive rotations are additive

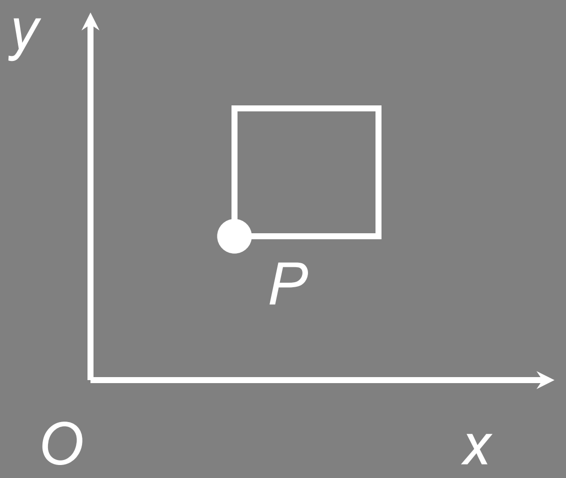
2D Transformation

Composition of transformation

Rotation about arbitrary point

Rotation about origin is known

- Translate such that P becomes O
- Rotate (about O)
- Translate back to P



2D Transformation

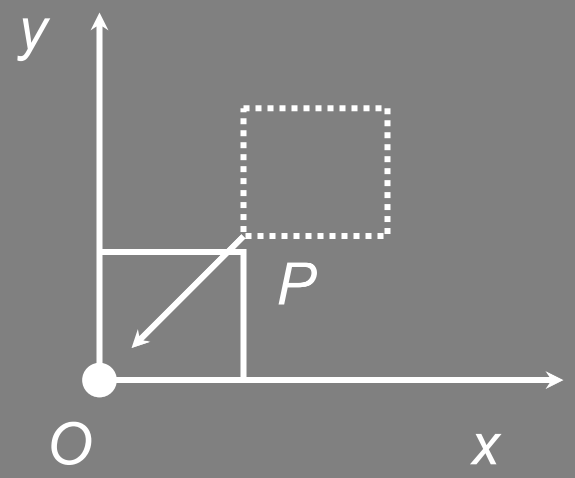
Composition of transformation

Rotation about arbitrary point

Rotation about origin is known

- Translate such that P becomes O

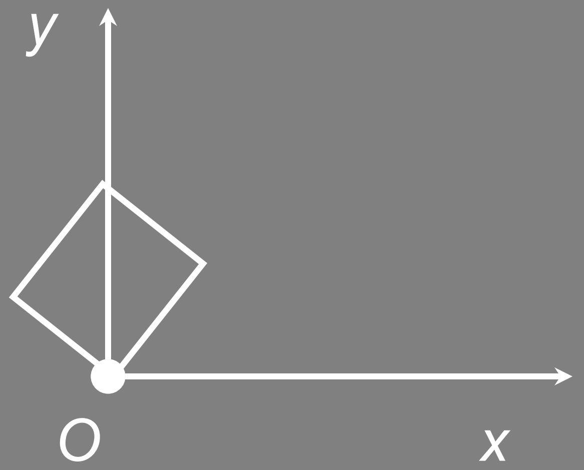
$$X' = X \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -l & -m & 1 \end{bmatrix}$$



2D Transformation

Composition of transformation

Rotation about arbitrary point



Rotation about origin is known

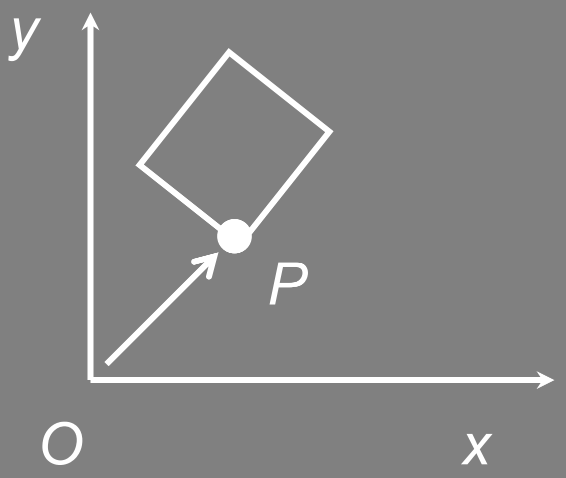
- Rotate about O

$$X'' = X' \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2D Transformation

Composition of transformation

Rotation about arbitrary point



Rotation about origin is known

- Translate back to P

$$X''' = X''\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l & m & 1 \end{bmatrix}$$

2D Transformation

Composition of transformation

Composite transformation

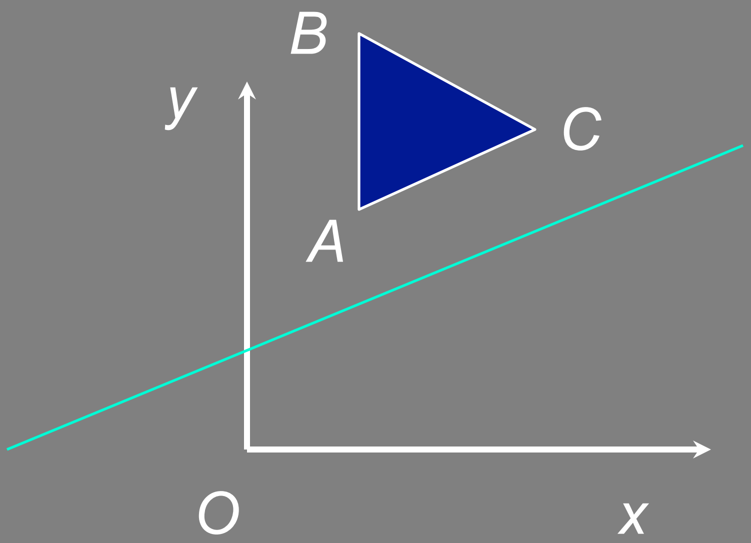
$$X_f = X\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -l & -m & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l & m & 1 \end{bmatrix}$$

$$= X\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -m(\cos\theta - 1) + n\sin\theta & -n(\cos\theta - 1) - m\sin\theta & 1 \end{bmatrix}$$

2D Transformation

Composition of transformation

Reflection about an arbitrary line

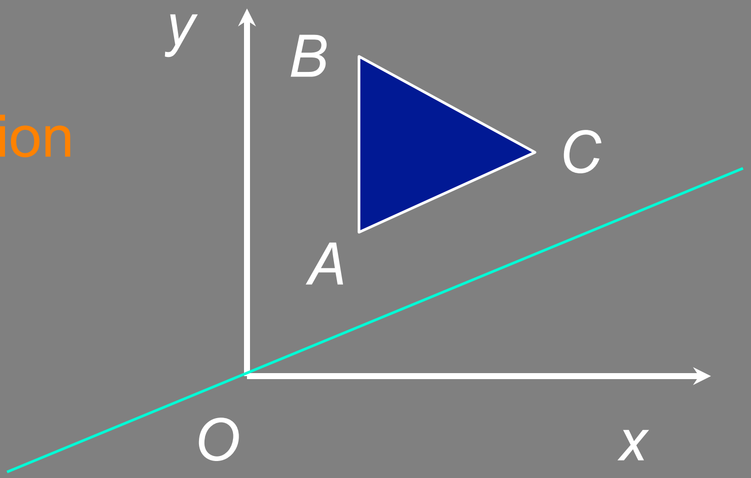


2D Transformation

Composition of transformation

Reflection about an arbitrary line

Translation

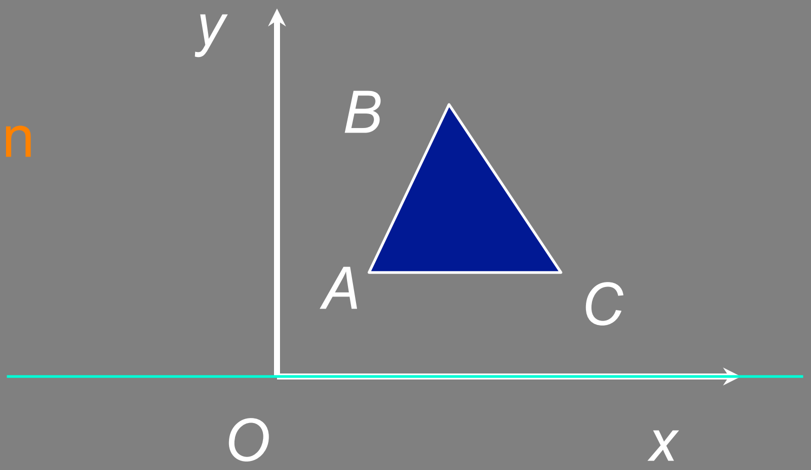


2D Transformation

Composition of transformation

Reflection about an arbitrary line

Rotation

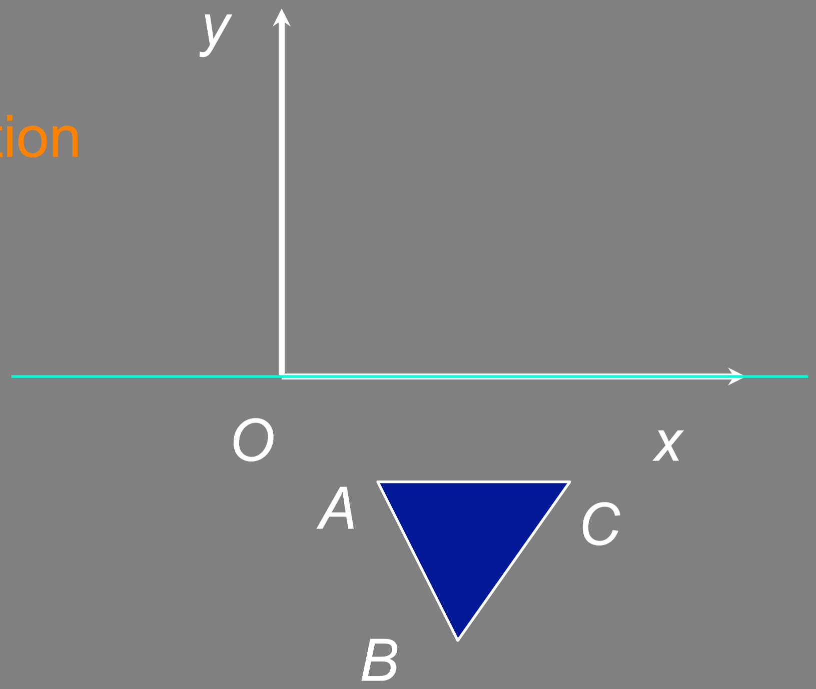


2D Transformation

Composition of transformation

Reflection about an arbitrary line

Reflection

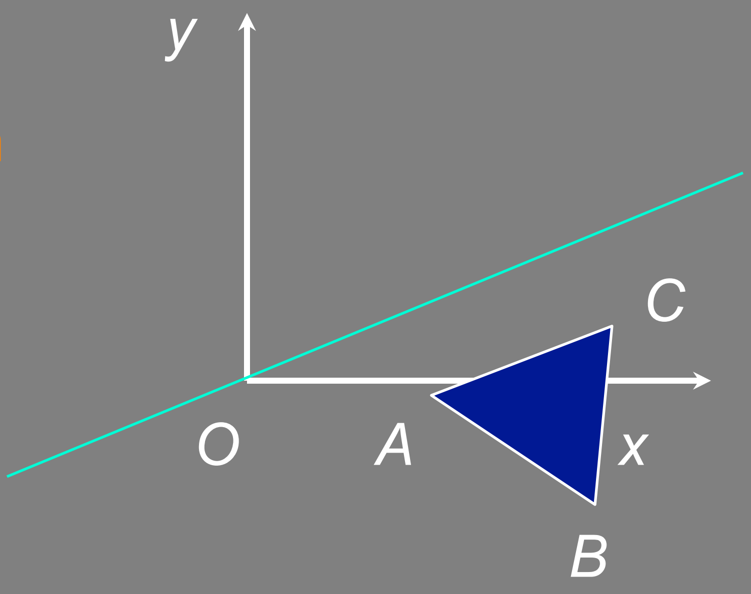


2D Transformation

Composition of transformation

Reflection about an arbitrary line

Rotation

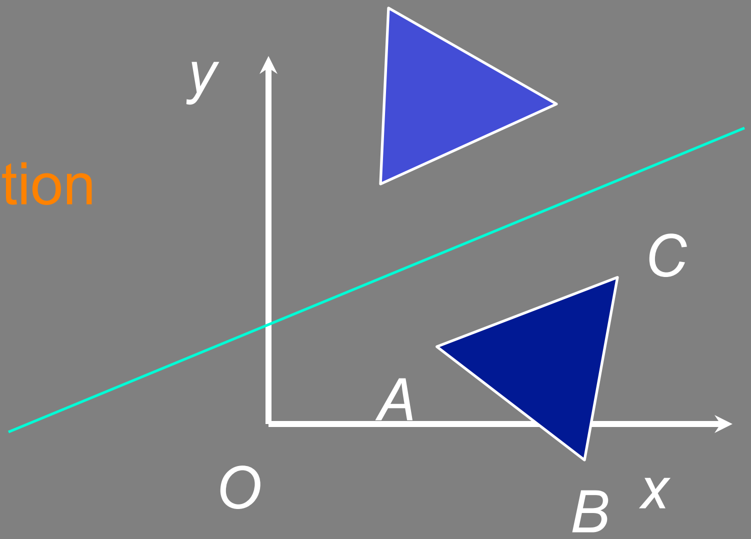


2D Transformation

Composition of transformation

Reflection about an arbitrary line

Translation



2D Transformation

Composition of transformation

Given T_1 and T_2

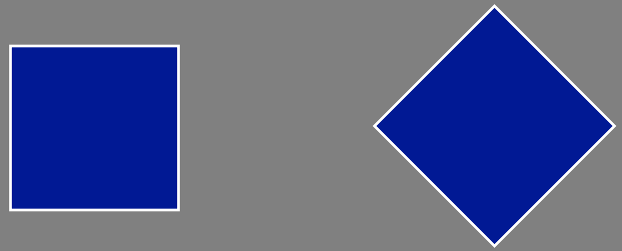
In general,

$$T = T_1 T_2 \neq T_2 T_1$$

2D Transformation

Rigid Transformations

- Square remains square
- Preserves length and angles
- Sequence of rotations and translations

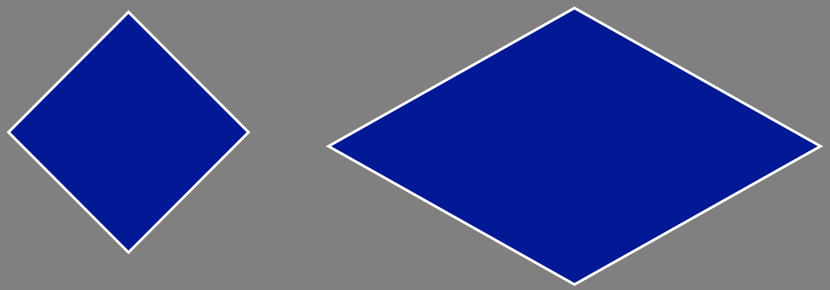


$$T = \begin{bmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ l & m & 1 \end{bmatrix}$$

2D Transformation

Affine Transformations

- Preserves parallelism
- Sequence of rotations, translations, scaling and shear



$$T = \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ l & m & 1 \end{bmatrix}$$

- Linear transformation is when no translation

2D Transformation

General Transformation

- 3x3 matrix

$$T = \begin{bmatrix} a & b & p \\ c & d & q \\ l & m & s \end{bmatrix}$$