Some Fundamental Algorithms (contd)
Sine function as series

- Problem
  Evaluate \( \sin(x) \) as a series expansion i.e., upto \( n \) terms

\[
\sin(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots
\]

- Instead of computing upto \( n \) terms, terminate based on an error
Sine function as series

term = x
sum = x
i = 1
y = x^x
error = 0.00001
while (abs(term) > error) do
  i = i + 2
  term = -term * y / (i * (i - 1))
  sum = sum + term
end-while
output sum
Square Root (x)

• Problem

Find square root of x.

1) Start with a **guess**, \( g \)
2) If \( g \times g \) is **close enough** to \( x \), stop and say \( g \) is the answer
3) Otherwise make a **new guess** by averaging \( g \) and \( x / g \)
4) Using the new guess, **repeat** process until close enough

<table>
<thead>
<tr>
<th>( g )</th>
<th>( g \times g )</th>
<th>( x / g )</th>
<th>( (g + x / g) / 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
<td>16/3</td>
<td>4.17</td>
</tr>
<tr>
<td>4.17</td>
<td>17.36</td>
<td>3.837</td>
<td>4.0035</td>
</tr>
<tr>
<td>4.0035</td>
<td>16.0277</td>
<td>3.997</td>
<td>4.000002</td>
</tr>
</tbody>
</table>

Fibonacci series

• Problem

Generate and print first n terms of Fibonacci sequence, which looks like

0, 1, 1, 2, 3, 5, 8, 13 …. 

An improved version:

0, 1, 1, 2, 3, 5, 8, 13 …. 

a=0  a=a+b
b=1  b=a+b
Fibonacci series

input n
a=0
b=1
i=2  # keeps track of number of terms decided
while i<n do
    output(a,b)
    a=a+b
    b=a+b
    i=i+2
end-while
if i==n then output (a,b) else output a

An improved version!
Reverse Digits of \( (x) \)

- Problem
  Given a positive integer, reverse the order of its digit.

Input: 17653
Output: 35671

\[ 17653 = 1 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 3 \]

Let \( \text{div} \) performs integer division and \( \text{mod} \) provides remainder
Reverse Digits of (x)

• Problem
  Given a positive integer, reverse the order of its digit.

  \[
  17653 \ mod \ 10 = 3 \quad 17653 \ div \ 10 = 1765 \\
  \text{(first digit for the reversed number)} \quad \text{(for next iteration)} \\
  1765 \ mod \ 10 = 5 \quad 1765 \ div \ 10 = 176 \\
  176 \ mod \ 10 = 6 \quad 176 \ div \ 10 = 17 \\
  17 \ mod \ 10 = 7 \quad 17 \ div \ 10 = 1 \\
  1 \ mod \ 10 = 1 \quad 1 \ div \ 10 = 0
  \]
Reverse Digits of (x)

input n
reverse=0
while (n > 0) do
   reverse = reverse * 10 + n mod 10
   n = n div 10
end-while
output reverse
Greatest Common Divisor (gcd)

- **Problem**
  Given two positive non-zero integers $n$ and $m$ find their greatest common divisor

  $m = 18$, $n = 30$
  
  $\gcd = 6$
Greatest Common Divisor (gcd)

Problem

Given two positive non-zero integers n and m find their greatest common divisor

```
input n, m
variable r
while r > 0
  r = n mod m
  n = m
  m = r
end while
output n
```
Smallest Exact Divisor

• Problem
  Given an integer n find its smallest exact divisor other than 1
  Option 1
    start from 2, 3, 4, 5, … till n
    check if n is exactly divided
    input n
    for i = 2 to n
      if n mod i == 0
        break
    end for
    output i
Smallest Exact Divisor

• Problem

Given an integer n find its smallest exact divisor other than 1

Option 2

When a number s exactly divides n, there exists another number b that also exactly divides n

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<table>
<thead>
<tr>
<th>s</th>
<th>b</th>
<th>sx×b=n</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>b=s so here sx×s=n</td>
</tr>
</tbody>
</table>
Smallest Exact Divisor

Option 2

If n is even the smallest divisor is 2!

Input n
if n mod 2 == 0
  s = 2
else
  d = 3
  r = sqrt(n)
  while ((n mod d != 0) and (d<r))
    d = d+2
  end while
  if n mod d == 0
    s=d
  else
    s=1
end if