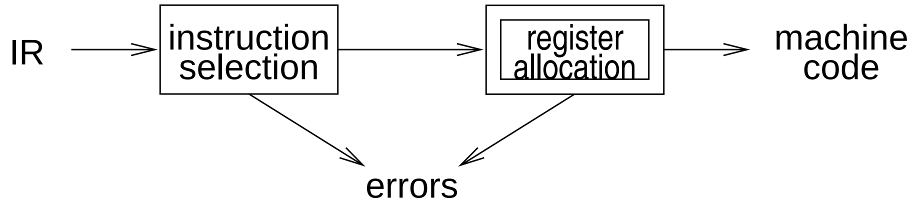
Register allocation



Register allocation:

- have value in a register when used
- limited resources
- changes instruction choices
- can move loads and stores
- optimal allocation is difficult
  $\Rightarrow$ NP-complete for $k \geq 1$ registers

Liveness analysis

Problem:

- IR contains an unbounded number of temporaries
- machine has bounded number of registers

Approach:

- temporaries with disjoint *live* ranges can map to same register
- if not enough registers then *spill* some temporaries (i.e., keep them in memory)

The compiler must perform *liveness analysis* for each temporary:

> It is *live* if it holds a value that may be needed in future

Control flow analysis

Before performing liveness analysis, need to understand the control flow by building a *control flow graph* (CFG):

- nodes may be individual program statements or basic blocks
- edges represent potential flow of control

Out-edges from node n lead to *successor* nodes, *succ*$[n]$

In-edges to node n come from *predecessor* nodes, *pred*$[n]$

Example

$$
\begin{aligned}
& a \leftarrow 0 \\
L_1: \quad & b \leftarrow a + 1 \\
& c \leftarrow c + b \\
& a \leftarrow b \times 2 \\
& \text{if } a < N \text{ goto } L_1 \\
& \text{return } c
\end{aligned}
$$

Liveness analysis

Gathering liveness information is a form of *data flow analysis* operating over the CFG:

- liveness of variables “flows” around the edges of the graph
- assignments *define* a variable, v:
  - *def*$(v) = \text{set of graph nodes that define } v$
  - *def*$[n] = \text{set of variables defined by } n$
- occurrences of v in expressions *use* it:
  - *use*$(v) = \text{set of nodes that use } v$
  - *use*$[n] = \text{set of variables used in } n$

Definitions

- v is *live* on edge e if there is a directed path from e to a *use* of v that does not pass through any *def*(v)
- v is *live-in* at node n if live on any of n's in-edges
- v is *live-out* at n if live on any of n's out-edges
- $v \in \mathit{use}[n] \Rightarrow v$ live-in at n
- v live-in at $n \Rightarrow v$ live-out at all $m \in \mathit{pred}[n]$
- v live-out at $n, v \notin \mathit{def}[n] \Rightarrow v$ live-in at n

Liveness analysis

Define:

$$\begin{aligned} \mathit{in}[n] &= \text{variables live-in at } n \\ \mathit{out}[n] &= \text{variables live-out at } n \end{aligned}$$

Then:

$$\mathit{out}[n] = \bigcup_{s \in \mathit{succ}(n)} \mathit{in}[s]$$

$$\mathit{succ}[n] = \phi \Rightarrow \mathit{out}[n] = \phi$$

Note:

$$\begin{aligned} \mathit{in}[n] &\supseteq \mathit{use}[n] \\ \mathit{in}[n] &\supseteq \mathit{out}[n] - \mathit{def}[n] \end{aligned}$$

$\mathit{use}[n]$ and $\mathit{def}[n]$ are constant (independent of control flow)
Now, $v \in \mathit{in}[n]$ iff. $v \in \mathit{use}[n]$ or $v \in \mathit{out}[n] - \mathit{def}[n]$
Thus, $\mathit{in}[n] = \mathit{use}[n] \cup (\mathit{out}[n] - \mathit{def}[n])$

Iterative solution for liveness

<u>foreach</u> n

$$in[n] \leftarrow \phi$$

$$out[n] \leftarrow \phi$$

<u>repeat</u>

<u>foreach</u> n

$$in'[n] \leftarrow in[n];$$

$$out'[n] \leftarrow out[n];$$

$$in[n] \leftarrow use[n] \cup (out[n] - def[n])$$

$$out[n] \leftarrow \bigcup_{s \in succ[n]} in[s]$$

<u>until</u> $in'[n] = in[n] \ \wedge \ out'[n] = out[n], \forall n$

Notes

- should order computation of inner loop to follow the “flow”
- liveness flows *backward* along control-flow arcs, from *out* to *in*
- nodes can just as easily be basic blocks to reduce CFG size
- could do one variable at a time, from *uses* back to *defs*, noting liveness along the way

Iterative solution for liveness

Complexity: for input program of size N

- $\leq N$ nodes in CFG
  $\Rightarrow \leq N$ variables
  $\Rightarrow N$ elements per *in/out*
  $\Rightarrow \mathrm{O}(N)$ time per set-union
- **for** loop performs constant number of set operations per node
  $\Rightarrow \mathrm{O}(N^2)$ time for **for** loop
- each iteration of **repeat** loop can only add to each set
  sets can contain at most every variable
  $\Rightarrow$ sizes of all in and out sets sum to $2N^2$,
  bounding the number of iterations of the **repeat** loop

$\Rightarrow$ worst-case complexity of $\mathrm{O}(N^4)$

- ordering can cut **repeat** loop down to 2-3 iterations
  $\Rightarrow \mathrm{O}(N)$ or $\mathrm{O}(N^2)$ in practice

Least fixed points

There is often more than one solution for a given dataflow problem (see example).

Any solution to dataflow equations is a *conservative approximation*:

- v has some later use downstream from n $\Rightarrow v \in \mathit{out}(n)$
- but not the converse

Conservatively assuming a variable is live does not break the program; just means more registers may be needed.

Assuming a variable is dead when really live *will* break things.

Many possible solutions but we want the “smallest”: the least fixpoint. The iterative algorithm computes this least fixpoint.