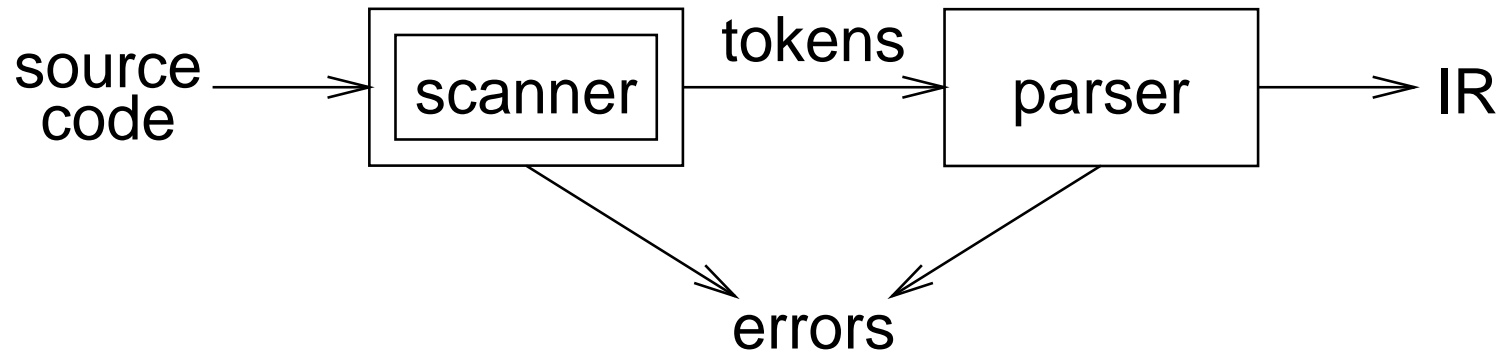


Lexical Analysis

Scanner



- maps characters into *tokens* – the basic unit of syntax

`x = x + y;`

becomes

`<id, x> = <id, x> + <id, y> ;`

- character string value for a *token* is a *lexeme*
- typical tokens: *number, id, +, -, *, /, do, end*
- eliminates white space (*tabs, blanks, comments*)
- a key issue is speed

⇒ use specialized recognizer (as opposed to `lex`)

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Specifying patterns

A scanner must recognize the units of syntax

Some parts are easy:

white space

```
<WS> ::= <WS> ' '
        | <WS> '\t'
        | ' '
        | '\t'
```

keywords and operators

specified as literal patterns: do, end

comments

opening and closing delimiters: /* ... */

Specifying patterns

A scanner must recognize the units of syntax

Other parts are much harder:

identifiers

alphabetic followed by k alphanumerics (., \$, &, ...)

numbers

integers: 0 or digit from 1-9 followed by digits from 0-9

decimals: integer ' .' digits from 0-9

reals: (integer or decimal) 'E' (+ or -) digits from 0-9

complex: '(' real ' , ' real ')'

We need a powerful notation to specify these patterns

Operations on languages

Operation	Definition
<i>union of L and M</i> written $L \cup M$	$L \cup M = \{s \mid s \in L \text{ or } s \in M\}$
<i>concatenation of L and M</i> written LM	$LM = \{st \mid s \in L \text{ and } t \in M\}$
<i>Kleene closure of L</i> written L^*	$L^* = \bigcup_{i=0}^{\infty} L^i$
<i>positive closure of L</i> written L^+	$L^+ = \bigcup_{i=1}^{\infty} L^i$

Regular expressions

Patterns are often specified as *regular languages*

Notations used to describe a regular language (or a regular set) include both *regular expressions* and *regular grammars*

Regular expressions (*over an alphabet Σ*):

1. ε is a RE denoting the set $\{\varepsilon\}$
2. if $a \in \Sigma$, then a is a RE denoting $\{a\}$
3. if r and s are REs, denoting $L(r)$ and $L(s)$, then:

(r) is a RE denoting $L(r)$

$(r) \mid (s)$ is a RE denoting $L(r) \cup L(s)$

$(r)(s)$ is a RE denoting $L(r)L(s)$

$(r)^*$ is a RE denoting $L(r)^*$

If we adopt a *precedence* for operators, the extra parentheses can go away. We assume *closure*, then *concatenation*, then *alternation* as the order of precedence.

Examples

identifier

$letter \rightarrow (a | b | c | \dots | z | A | B | C | \dots | Z)$

$digit \rightarrow (0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9)$

$id \rightarrow letter (letter | digit)^*$

numbers

$integer \rightarrow (+ | - | \epsilon) (0 | (1 | 2 | 3 | \dots | 9) digit^*)$

$decimal \rightarrow integer . (digit)^*$

$real \rightarrow (integer | decimal) E (+ | -) digit^*$

$complex \rightarrow '(, real , real)'$

Numbers can get much more complicated

Most programming language tokens can be described with REs

We can use REs to build scanners automatically

Algebraic properties of REs

Axiom	Description
$r s = s r$	$ $ is commutative
$r (s t) = (r s) t$	$ $ is associative
$(rs)t = r(st)$	concatenation is associative
$r(s t) = rs rt$ $(s t)r = sr tr$	concatenation distributes over $ $
$\varepsilon r = r$ $r\varepsilon = r$	ε is the identity for concatenation
$r^* = (r \varepsilon)^*$	relation between $*$ and ε
$r^{**} = r^*$	$*$ is idempotent

Examples

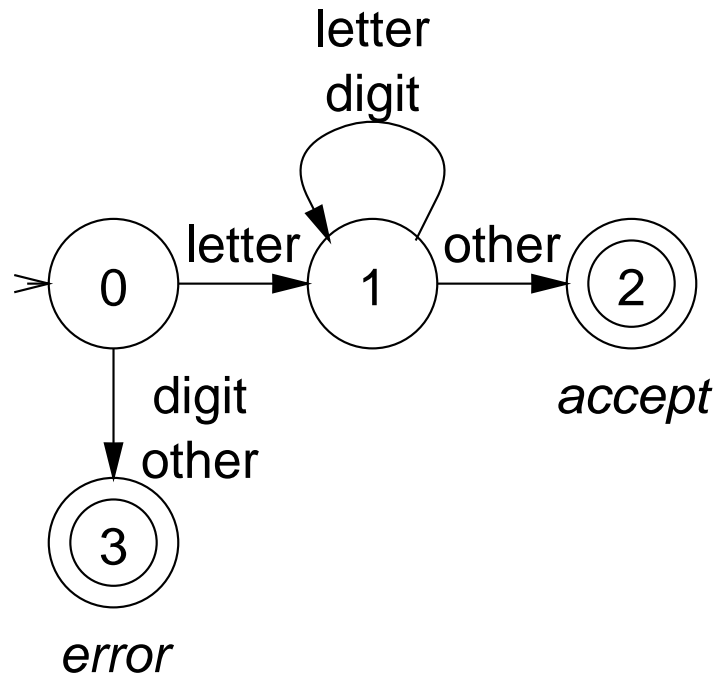
Let $\Sigma = \{a, b\}$

1. $a|b$ denotes $\{a, b\}$
2. $(a|b)(a|b)$ denotes $\{aa, ab, ba, bb\}$
i.e., $(a|b)(a|b) = aa|ab|ba|bb$
3. a^* denotes $\{\varepsilon, a, aa, aaa, \dots\}$
4. $(a|b)^*$ denotes the set of all strings of a 's and b 's (including ε)
i.e., $(a|b)^* = (a^*b^*)^*$
5. $a|a^*b$ denotes $\{a, b, ab, aab, aaab, aaaab, \dots\}$

Recognizers

From a regular expression we can construct a
deterministic finite automaton (DFA)

Recognizer for *identifier*:



identifier

letter $\rightarrow (a | b | c | \dots | z | A | B | C | \dots | Z)$

digit $\rightarrow (0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9)$

id $\rightarrow letter (letter | digit)^*$

Code for the recognizer

```
char ← next_char();
state ← 0;          /* code for state 0 */
done ← false;
token_value ← ""   /* empty string */
while( not done ) {
    class ← char_class[char];
    state ← next_state[class,state];
    switch(state) {
        case 1:      /* building an id */
            token_value ← token_value + char;
            char ← next_char();
            break;
        case 2:      /* accept state */
            token_type = identifier;
            done = true;
            break;
        case 3:      /* error */
            token_type = error;
            done = true;
            break;
    }
}
return token_type;
```

Tables for the recognizer

Two tables control the recognizer

char_class:		<i>a - z</i>	<i>A - Z</i>	<i>0 - 9</i>	other
	value	letter	letter	digit	other

next_state:	class	0	1	2	3
	letter	1	1	—	—
	digit	3	1	—	—
	other	3	2	—	—

To change languages, we can just change tables

Automatic construction

Scanner generators automatically construct code from RE-like descriptions

- construct a DFA
- use state minimization techniques
- emit code for the scanner
(table driven or direct code)

A key issue in automation is an interface to the parser

`lex` is a scanner generator supplied with UNIX

- emits C code for scanner
- provides macro definitions for each token
(used in the parser)

Grammars for regular languages

Can we place a restriction on the *form* of a grammar to ensure that it describes a regular language?

Provable fact:

For any RE r , \exists a grammar g such that $L(r) = L(g)$

Grammars that generate regular sets are called *regular grammars*:

They have productions in one of 2 forms:

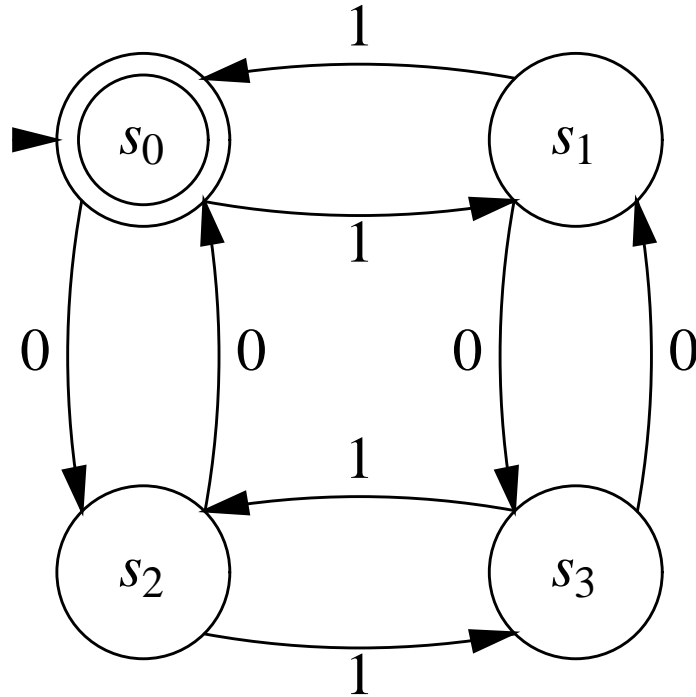
1. $A \rightarrow aA$
2. $A \rightarrow a$

where A is any non-terminal and a is any terminal symbol

These are also called *type 3 grammars* (Chomsky)

More regular languages

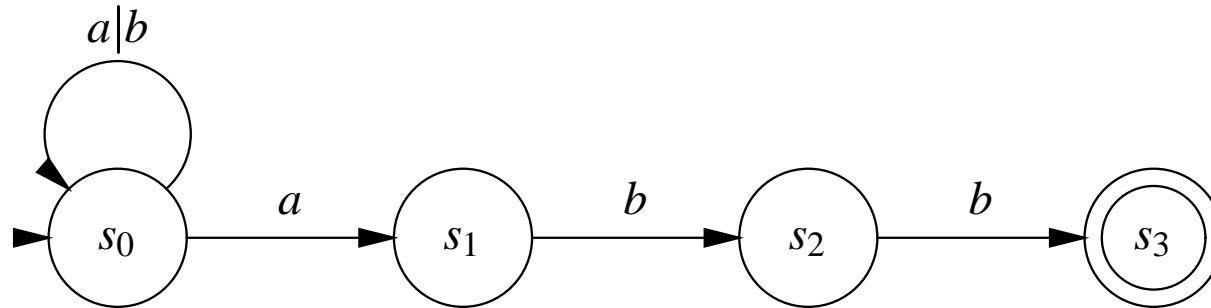
Example: the set of strings containing an even number of zeros and an even number of ones



The RE is $(00 \mid 11)^*((01 \mid 10)(00 \mid 11)^*(01 \mid 10)(00 \mid 11)^*)^*$

More regular expressions

What about the RE $(a | b)^*abb$?



State s_0 has multiple transitions on a !

\Rightarrow *nondeterministic finite automaton*

	a	b
s_0	$\{s_0, s_1\}$	$\{s_0\}$
s_1	—	$\{s_2\}$
s_2	—	$\{s_3\}$

Finite automata

A *non-deterministic finite automaton* (NFA) consists of:

1. a set of *states* $S = \{s_0, \dots, s_n\}$
2. a set of input symbols Σ (the alphabet)
3. a transition function *move* mapping state-symbol pairs to sets of states
4. a distinguished *start state* s_0
5. a set of distinguished *accepting* or *final* states F

A *Deterministic Finite Automaton* (DFA) is a special case of an NFA:

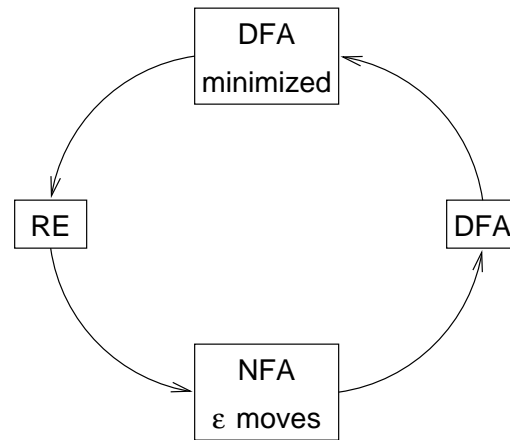
1. no state has a ε -transition, and
2. for each state s and input symbol a , there is at most one edge labelled a leaving s

A DFA *accepts* x iff. \exists a *unique* path through the transition graph from s_0 to a final state such that the edges spell x .

DFAs and NFAs are equivalent

1. DFAs are clearly a subset of NFAs
2. Any NFA can be converted into a DFA, by simulating sets of simultaneous states:
 - each DFA state corresponds to a set of NFA states
 - possible exponential blowup

Constructing a DFA from a regular expression



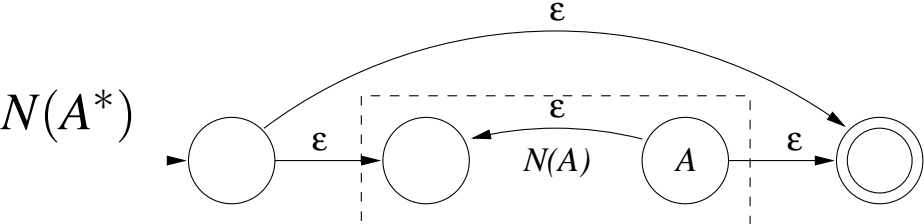
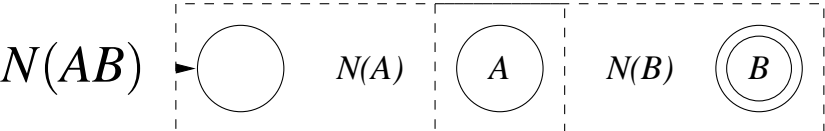
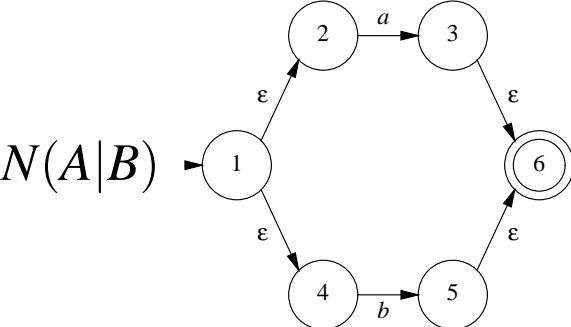
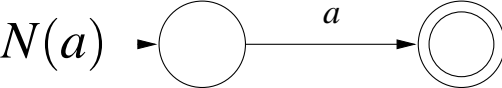
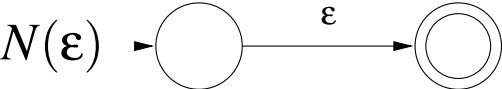
RE \rightarrow NFA w/ ϵ moves
build NFA for each term
connect them with ϵ moves

NFA w/ ϵ moves to DFA
construct the simulation
the “subset” construction

DFA \rightarrow minimized DFA
merge compatible states

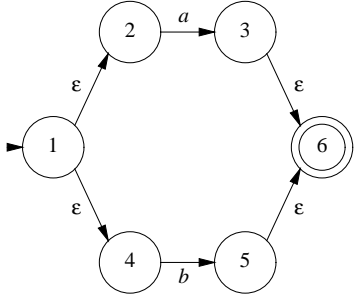
DFA \rightarrow RE

RE to NFA

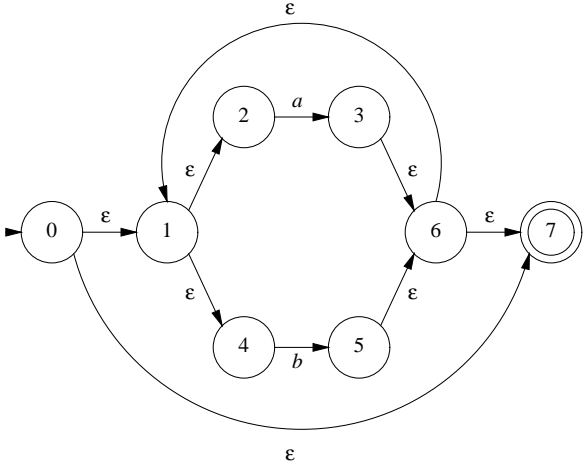


RE to NFA: example

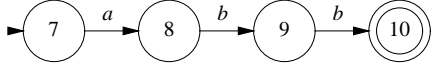
$a|b$



$(a|b)^*$



abb



NFA to DFA: the subset construction

Input: NFA N

Output: A DFA D with states $Dstates$ and transitions $Dtrans$ such that $L(D) = L(N)$

Method: Let s be a state in N and T be a set of states, and using the following operations:

Operation	Definition
ϵ -closure(s)	set of NFA states reachable from NFA state s on ϵ -transitions alone
ϵ -closure(T)	set of NFA states reachable from some NFA state s in T on ϵ -transitions alone
$move(T, a)$	set of NFA states to which there is a transition on input symbol a from some NFA state s in T

add state $T = \epsilon$ -closure(s_0) unmarked to $Dstates$

while \exists unmarked state T in $Dstates$

 mark T

for each input symbol a

$U = \epsilon$ -closure($move(T, a)$)

if $U \notin Dstates$ **then** add U to $Dstates$ unmarked

$Dtrans[T, a] = U$

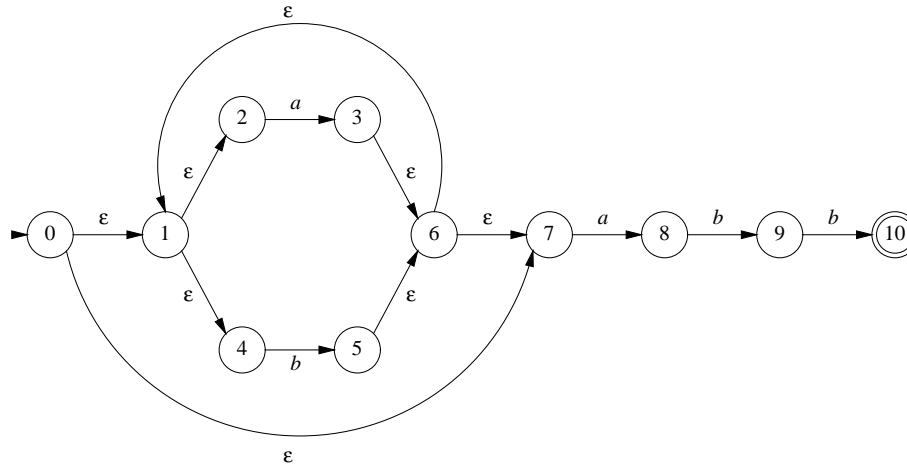
endfor

endwhile

ϵ -closure(s_0) is the start state of D

A state of D is final if it contains at least one final state in N

NFA to DFA using subset construction: example 1



$$A = \{0, 1, 2, 4, 7\}$$

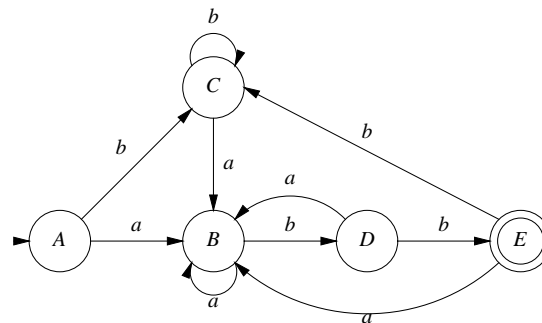
$$B = \{1, 2, 3, 4, 6, 7, 8\}$$

$$C = \{1, 2, 4, 5, 6, 7\}$$

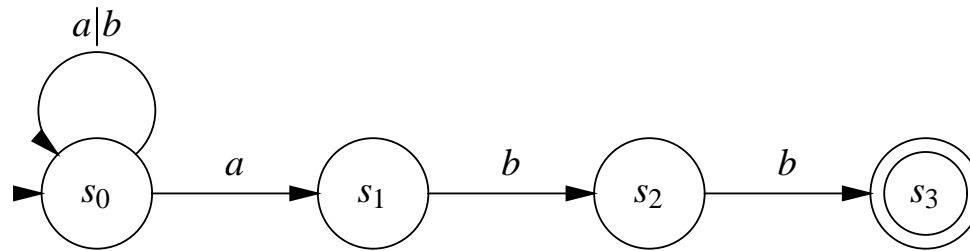
$$D = \{1, 2, 4, 5, 6, 7, 9\}$$

$$E = \{1, 2, 4, 5, 6, 7, 10\}$$

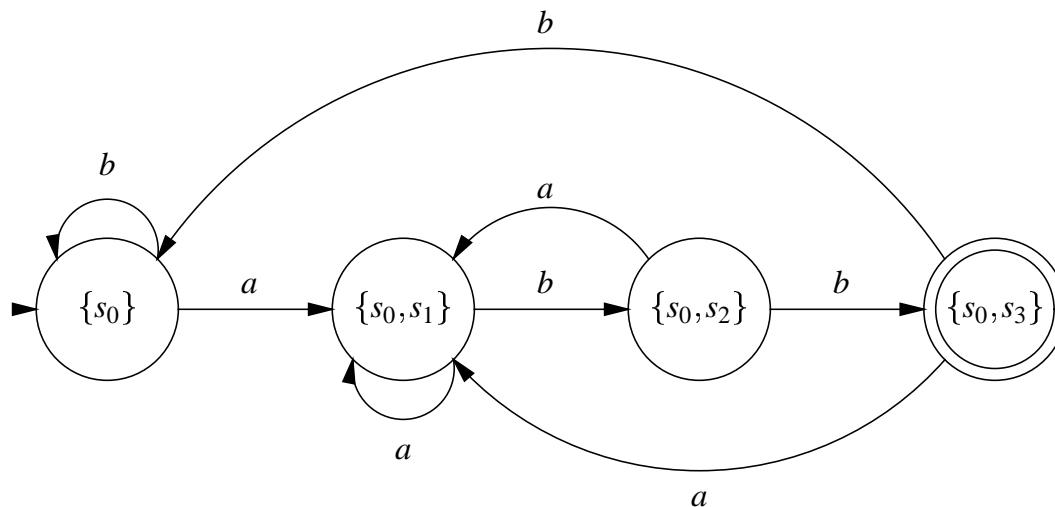
	<i>a</i>	<i>b</i>
<i>A</i>	<i>B</i>	<i>C</i>
<i>B</i>	<i>B</i>	<i>D</i>
<i>C</i>	<i>B</i>	<i>C</i>
<i>D</i>	<i>B</i>	<i>E</i>
<i>E</i>	<i>B</i>	<i>C</i>



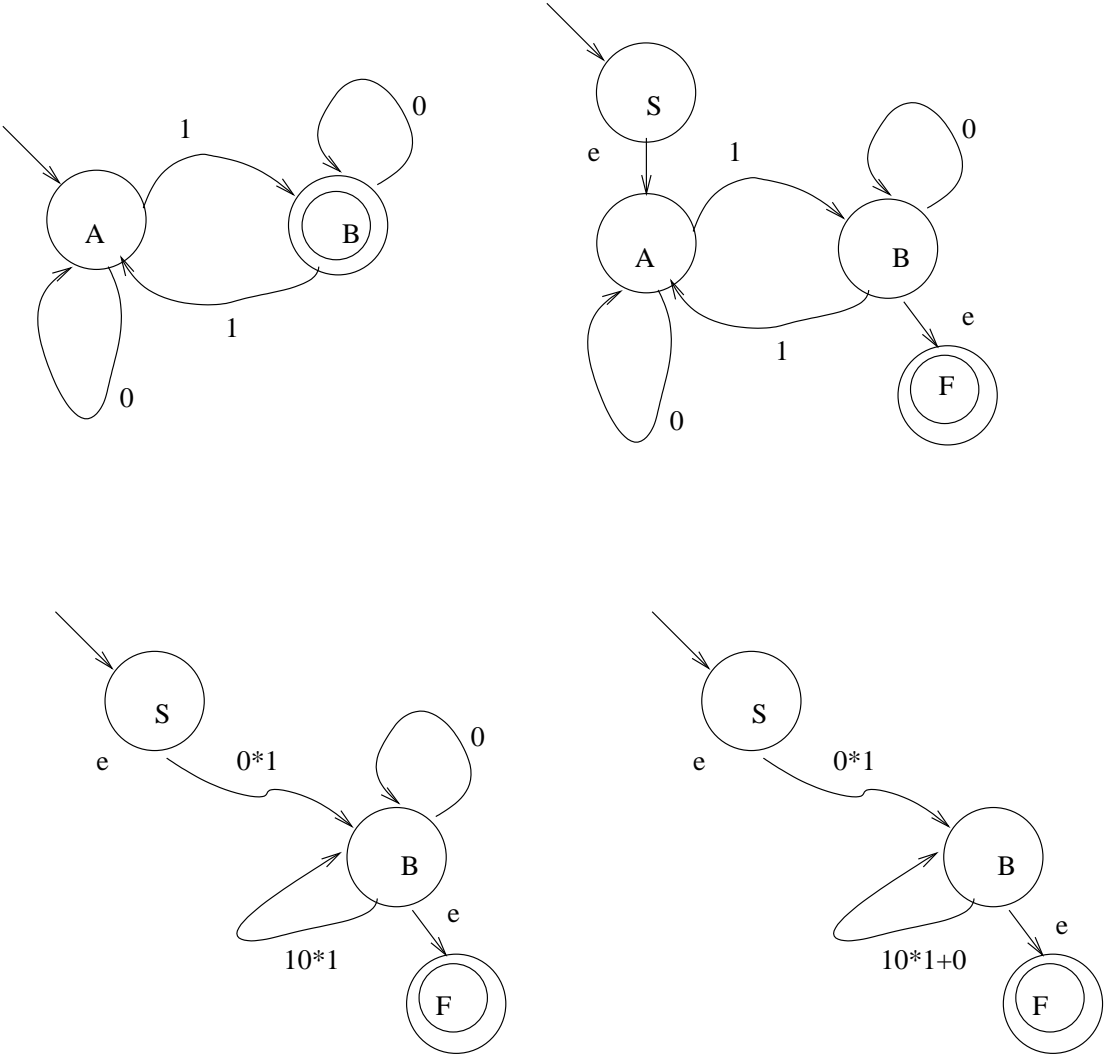
NFA to DFA using the subset construction: example/assignment 1



	a	b
$\{s_0\}$	$\{s_0, s_1\}$	$\{s_0\}$
$\{s_0, s_1\}$	$\{s_0, s_1\}$	$\{s_0, s_2\}$
$\{s_0, s_2\}$	$\{s_0, s_1\}$	$\{s_0, s_3\}$
$\{s_0, s_3\}$	$\{s_0, s_1\}$	$\{s_0\}$



DFA to RE



$0^*1 (10^*1+0)^*$

Limits of regular languages

Not all languages are regular

One cannot construct DFAs to recognize these languages:

- $L = \{p^k q^k\}$
- $L = \{wcw^r \mid w \in \Sigma^*\}$

Note: neither of these is a regular expression!

(DFAs cannot count!)

But, this is a little subtle. One can construct DFAs for:

- alternating 0's and 1's
 $(\epsilon \mid 1)(01)^*(\epsilon \mid 0)$
- sets of pairs of 0's and 1's
 $(01 \mid 10)^+$

So what is hard?

Language features that can cause problems:

reserved words

PL/I had no reserved words

```
if then then then = else; else else = then;
```

significant blanks

FORTRAN and Algol68 ignore blanks

```
do 10 i = 1,25
```

```
do 10 i = 1.25
```

string constants

special characters in strings

```
newline, tab, quote, comment delimiter
```

finite closures

some languages limit identifier lengths

adds states to count length

FORTRAN 66 → 6 characters

These can be swept under the rug in the language design

How bad can it get?

```
1      INTEGERFUNCTIONA
2      PARAMETER(A=6,B=2)
3      IMPLICIT CHARACTER*(A-B)(A-B)
4      INTEGER FORMAT(10),IF(10),D09E1
5      100  FORMAT(4H)=(3)
6      200  FORMAT(4 )=(3)
7          D09E1=1
8          D09E1=1,2
9          IF(X)=1
10         IF(X)H=1
11         IF(X)300,200
12      300  CONTINUE
13         END
        C   this is a comment
        $  FILE(1)
14         END
```

Example due to Dr. F.K. Zadeck of IBM Corporation