

Major, CSL630, 120 mins, 60 marks, Entry No. _____

Please write answers in the space provided. No extra sheet will be given.

Some coins are spread in the cells of a $n \times m$ board, one coin per cell. A robot, located in the upper left cell of the board, needs to collect as many of the coins as possible and bring them to the bottom right cell. On each step, the robot can move either one cell to the right or one cell down from its current location. When the robot visits a cell with a coin, it picks up that coin. Devise an efficient algorithm to find the maximum number of coins the robot can collect and a path it needs to follow to do this. (10)

Let A be an $n \times m$ matrix and define

2 marks [$A[i,j]$ = maximum number of coins robot can pick up in going from $(1,1)$ to (i,j)

Then

4 marks [$A[i,j] = \max(A[i-1,j], A[i,j-1]) + c[i,j]$
where $c[i,j] = \begin{cases} 1 & \text{if cell } (i,j) \text{ contains coin} \\ 0 & \text{otherwise} \end{cases}$

2 marks [The matrix A can be computed in $O(mn)$ time since it takes $O(1)$ time to fill each cell

2 marks. [$A[m,n]$ contains the solution to the problem.