COL 106 Data-Structures Minor Test 1 (B) 29th August, 2014

Name ______________________ Entry Number ____________ Gp No.____

Your Lab day________________

Answer all the questions in the space provided for each question.

Q1.[15 marks] Answer the following questions with proper justification (no marks will be awarded if the reasoning is incorrect.)

a. What is the running time complexity of the following code ? You can use order notation. Justify your answer.

```
sum = 0;
for (k=1; k<n; k++)
  for (i=1; i<k*k; i++)
    if (i % k == 0)
      for (j=0; j<i; j++)
        sum++;
```

$$\sum_{k=1}^{n} [0 + k + 2k + \dots + k \cdot k]$$

$$= \sum_{k=1}^{n} k [1 + 2 + \dots + k]$$

$$= \sum_{k=1}^{n} k^2 \frac{(k+1)}{2} = O(n^4).$$

b. Is it true that log 1 + log 2 + … + log n is θ(nlog n)? Explain your answer.

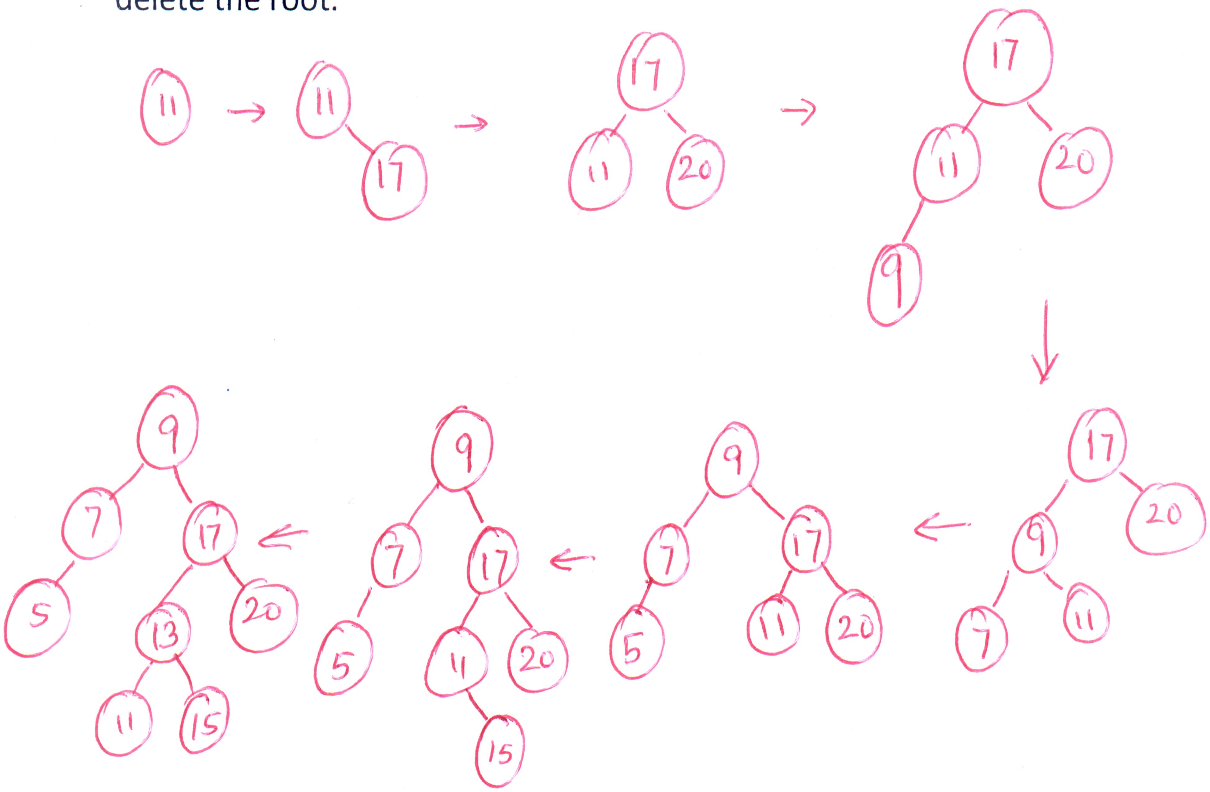
Yes. Two parts:

1. $\log 1 + \dots + \log n \le \log n + \dots + \log n = n \log n \Rightarrow$ LHS is $O(n \log n)$.

2. $\log 1 + \dots + \log n \ge \log \frac{n}{2} + \dots + \log n \ge \log \frac{n}{2} + \dots + \log \frac{n}{2} = \frac{n}{2} \log \frac{n}{2}$ $= \Omega(n \log n)$.

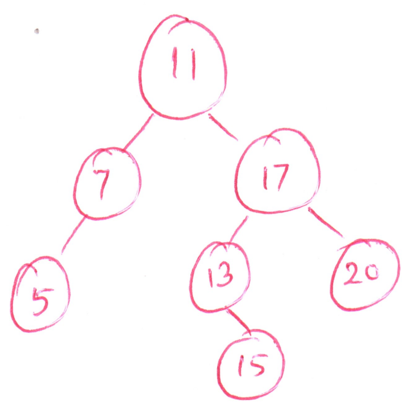
$\therefore$ LHS is both $O(n \log n)$ and $\Omega(n \log n) \Rightarrow \Theta(n \log n)$.

Q2.[10 marks] Starting from an empty AVL tree, the following elements are inserted (in this order) in the AVL tree: 11,17,20,9,7,5,15,13. You should use the rotation/balancing procedures described in the class. Show the AVL tree after each insert operation. After inserting the above numbers in the AVL tree, delete the root.



Delete 9

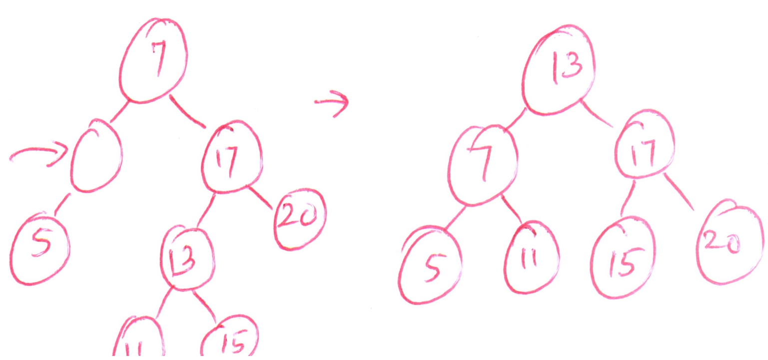
(i) Delete successor node of 9:



OR

(ii) Delete predecessor node:

Delete this



Q3.[15 marks] Given a node A in a binary search tree, write an efficient procedure to find its inorder predecessor. Fill in the details of the procedure below. Do not declare any other function.

```
Node* Predecessor ( Node* A)
{
  if (A->right is not a leaf) {
        p = A->right;
        while (p->left is not a leaf)
              p = p->left;
        return p; }
                                                  7 marks
       p = A;
 else { q = A->parent;
       while (p is the right child of q) {
             p = q;
             q = p->parent;     <- if q becomes NULL,
                                   A was the largest
       }                           node.
       return q;                                  8 marks
}
```

Q4.[20 marks] Given a stack, and a sequence of distinct numbers in an array A = [a_1, .., a_n] you can perform the following operations starting from the first number in A: either push the next number in the sequence on the stack, or pop from the stack.

Given a permutation B =[b_1, ..,b_n] of A=[a_1,..., a_n], we would like to know if it can be obtained from a_1...a_n by such operations.

For example, let A =[1 2 3 4 5] and B = [2 3 5 4 1]. Then B can be obtained from A by the following operations:

Push 1, Push 2. Pop 2. Push and Pop 3. Push 4, push 5. Pop 5. Pop 4. Pop 1.

(i) Let A = [1 2 3 4 5], B = [4 3 1 5 2]. Can B be obtained from A ? Why?

No. First we need to push 1,2,3,4 and then pop, pop, Now, 1 cannot be printed before 2. 3.

(ii) Write a **linear time** procedure which given A and B of length n each, outputs whether B can be obtained from A using the method above. If yes, the procedure should output the sequence of operations. Note that you can access the stack using **push, pop, is_empty** and **top** functions only. You should not modify the arrays A and B, and should use only a constant amount of extra space (besides using the stack and reading from the two arrays A and B).

```
i = 0;    (i scans A)
j = 0;    (j scans B)

while (j < n) {
    if (stack S is not empty   and   S.top() == B[j]) {
                S.pop();
                j++;
    }
    else {
        S.push( A[i])
            i++;   <- if  i exceeds n , "ERROR"
    }
}
```

7 marks

7 marks.

3 marks

If quadratic time algorithm, max. 5 marks.

Worse than quadratic time: NO MARKS.