INTRODUCTION

Markov Logic Networks (MLN) [Richardson & Domingos, 2006]

- A Markov Logic Network (MLN) is a set of weighted first order formulas.
- When a world violates a formula, it becomes less probable, but not impossible.
- Together with a set of constants, it defines a Markov network.

\[
\begin{align*}
\forall x \text{ Smokes}(x) & \Rightarrow \text{Cancer}(x) \\
\forall x, y \text{ Smokes}(x) \land \text{Friends}(x, y) & \Rightarrow \text{Smokes}(y)
\end{align*}
\]

\[\Delta x = \Delta y = \{A, B\}\]

Joint Probability:

\[P(y|x; w) = \frac{1}{Z_x} \exp \left( \sum w_i n_i(x, y) \right)\]

- Formula: Weight of formula \(i\)
- No. of true groundings of formula \(i\)

MOTIVATION

- Training: Small size domain
  - Unavailability of labelled datasets.
- Testing: Large size domain
- Observation: Extreme Marginal Probabilities
  
  \[1.0 : P(x) \Rightarrow Q(x, y)\]

\[|\Delta y| = 1 \quad |\Delta y| = 3 \quad |\Delta y| = 15\]

\[P(P(1)) \approx 0.41 \quad P(P(1)) \approx 0.24 \quad P(P(1)) \approx 0\]

- Use number of connections to adjust the weights.

RELATED WORK

- Identifying the problem:
  - [Poole et al 2014]: Characterized the cases of extreme marginals.
- Solving the Problem:
  - Adaptive MLNs [Jain et al 2010]
  - Relational Marginal Problems: Theory and Estimation [Kuzelka et al 2017]

DA-MLNs

Number of Connections

\[F = [P_1, P_2, ..., P_m]\]

Formula

\[c_j = \max \left( 1, \prod \left( \left| \Delta x \right| \right)^{-1} \right)\]

Predicates

\[
\begin{cases}
F(s) \land F(x, y) \Rightarrow S(y) \\
\Delta x = \Delta y = \{1, 2, ..., 10\}
\end{cases}
\]

\[c_1 = 10, c_2 = 1, c_3 = 10, s = 10\]

Scaling Factor

\[s = \max_{1 \leq j \leq m} c_j\]

Probability Distribution

\[
P(y|x; w) = \frac{1}{Z_x} \exp \left( \sum \frac{w_i}{s_i} n_i(x, y) \right)
\]

CHARACTERIZATION OF MLNS & DA-MLNS

<table>
<thead>
<tr>
<th>Formula Type</th>
<th>Dom</th>
<th>[P(1) = 1]</th>
<th>[P(1) \neq 1]</th>
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</thead>
<tbody>
<tr>
<td>MLN</td>
<td>Large (</td>
<td>\Delta y</td>
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<tr>
<td>DA-MLN</td>
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- In MLNs, Marginals either extreme or independent of weights.

EXPERIMENTS & RESULTS

- **Friends & Smokers**
  - Formulas:
    - \(S(x) \Rightarrow L(x)\)
    - \(S(x) \land F(x, y) \Rightarrow S(y)\)
  - Data Generation:
    - Communities of Smoker & Non-Smoker.
    - Smokers have high chances of having Cancer.

- **IMDB**
  - Formulas:
    - \(Act(x) \land Dir(y) \land Mov(m, x) \land WU(x, y) \Rightarrow Mov(m, y)\)
    - \(Act(x) \land Dir(y) \land Mov(m, y) \land WU(x, y) \Rightarrow Mov(m, x)\) (and 6 more)
  - Dataset:

![Graph showing connection weights and marginal probabilities.](image-url)