Yoav Goldberg

Dealing with Sequences

- For an input sequence **x1**,...,**xn**, we can:
 - If *n* is **fixed**: *concatenate* and feed into an MLP.
 - sum the vectors (CBOW) and feed into an MLP.
 - Break the sequence into *windows*. Find n-gram embedding, sum into an MLP.
 - Find good ngrams using ConvNet, using pooling (either sum/avg or max) to combine to a single vector.

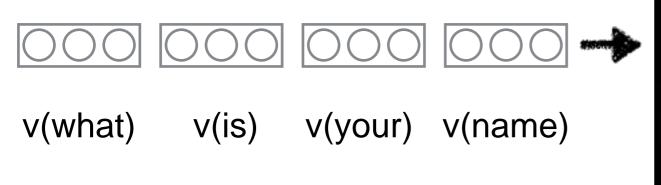
Dealing with Sequences

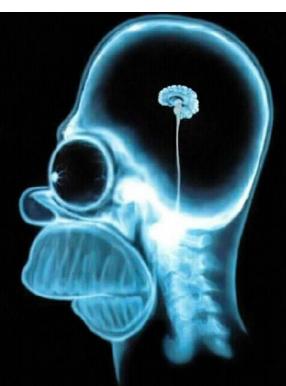
• For an input sequence **x1**,...,**xn**, we can:

Some of these approaches consider **local** word order (which ones?).

How can we consider **global** word order?

 Find good ngrams using ConvNet, using pooling (either sum/avg or max) to combine to a single vector.

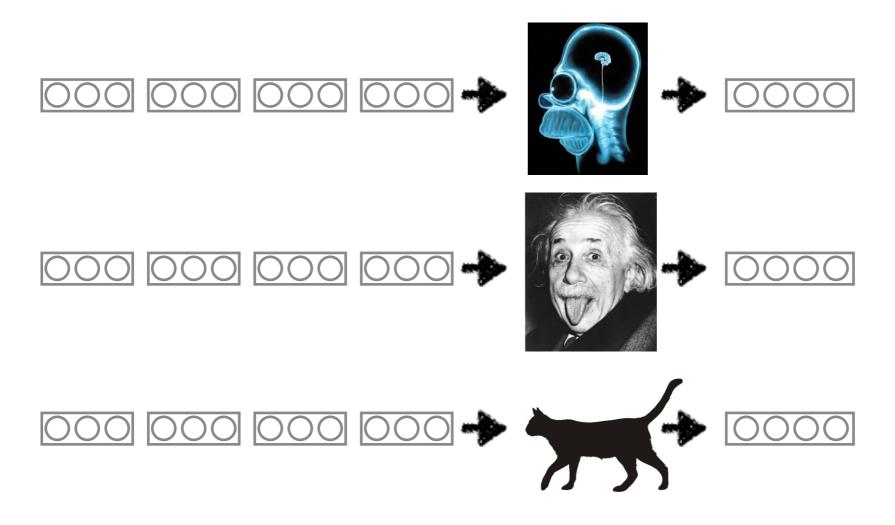






enc(what is your name)

- Very strong models of sequential data.
- Trainable function from *n* vectors to a single vector.



- There are different variants (implementations).
- So far, we focused on the interface level.

 $RNN(\mathbf{s_0}, \mathbf{x_{1:n}}) = \mathbf{s_n}, \mathbf{y_n}$

$$\mathbf{x_i} \in \mathbb{R}^{d_{in}}, \ \mathbf{y_i} \in \mathbb{R}^{d_{out}}, \ \mathbf{s_i} \in \mathbb{R}^{f(d_{out})}$$

- Very strong models of sequential data.
- Trainable function from *n* vectors to a single* vector.

$$RNN(\mathbf{s_0}, \mathbf{x_{1:n}}) = \mathbf{s_n}, \mathbf{y_n}$$

*this one is internal. we only care about the **y**

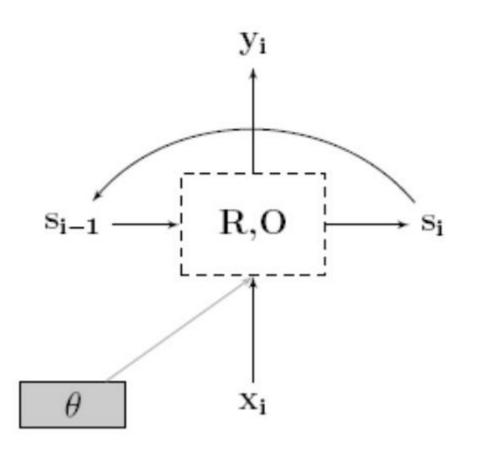
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$$RNN(\mathbf{s_0}, \mathbf{x_{1:n}}) = \mathbf{s_n}, \mathbf{y_n}$$
$$\mathbf{s_i} = R(\mathbf{s_{i-1}}, \mathbf{x_i})$$
$$\mathbf{y_i} = O(\mathbf{s_i})$$

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- Recursively defined.
- There's a vector \mathbf{y}_i for every prefix $\mathbf{x}_{1:i}$

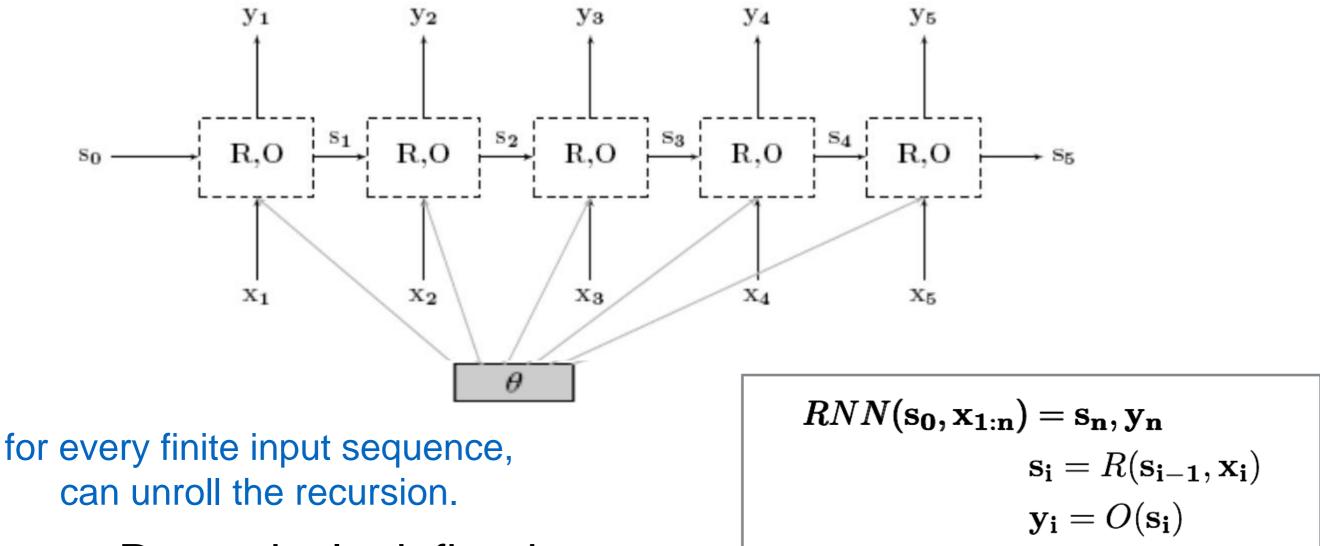


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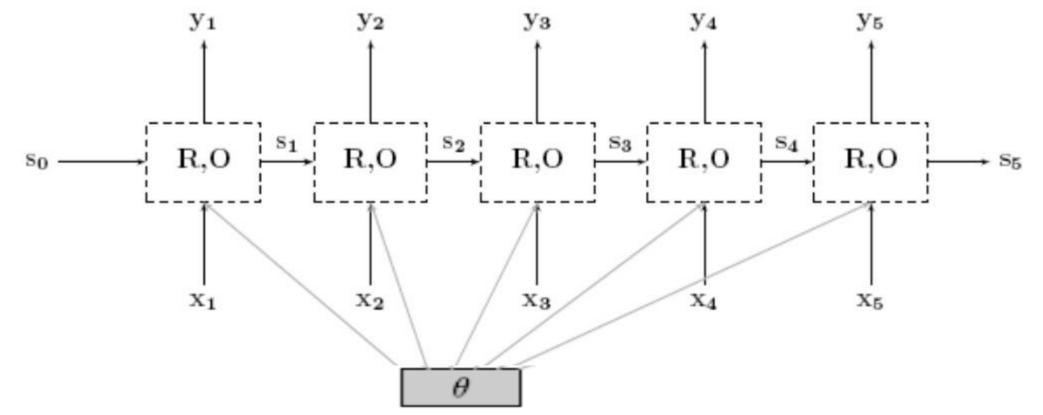
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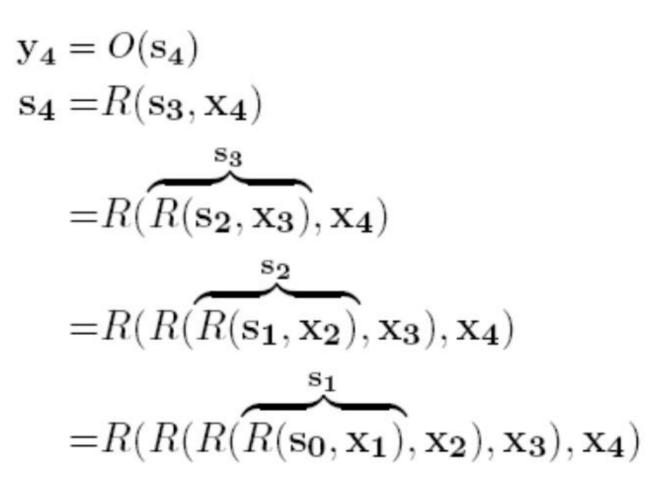
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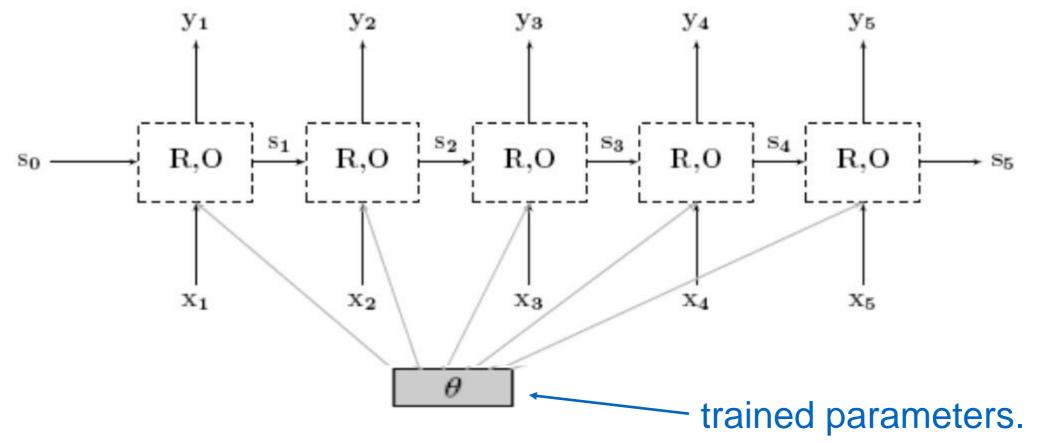


for every finite input sequence, can unroll the recursion.

An unrolled RNN is just a very deep Feed Forward Network with shared parameters across the layers, and a new input at each layer.

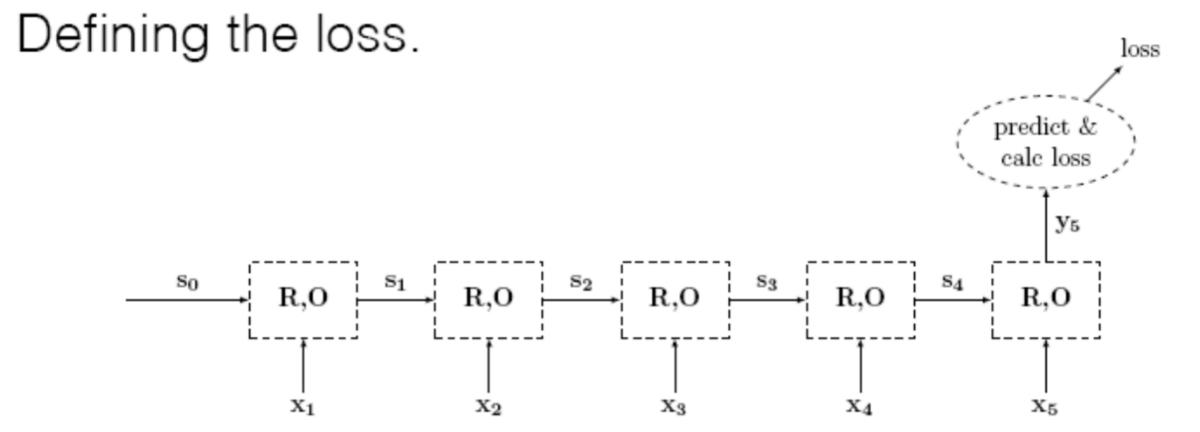


• The output vector y_i depends on **all** inputs $x_{1:i}$



But we can train them.
 define function form define loss

Recurrent Neural Networks for Text Classification



Acceptor: predict something from end state. Backprop the error all the way back. Train the network to capture meaningful information

$$R_{CBOW}(\mathbf{s_{i-1}}, \mathbf{x_i}) = \mathbf{s_{i-1}} + \mathbf{x_i}$$

(what are the parameters?)

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(what are the parameters?)

$$R_{CBOW}(\mathbf{s_{i-1}}, x_i) = \mathbf{s_{i-1}} + \mathbf{E}_{[x_i]}$$

Is this a good parameterization?

$$R_{CBOW}(\mathbf{s_{i-1}}, x_i) = \mathbf{s_{i-1}} + \mathbf{E}_{[x_i]}$$

how about this modification?

$$R_{CBOW}(\mathbf{s_{i-1}}, x_i) = \underline{\tanh(\mathbf{s_{i-1}} + \mathbf{E}_{[x_i]})}$$

Simple RNN (Elman RNN)

 $R_{SRNN}(\mathbf{s_{i-1}}, \mathbf{x_i}) = tanh(\mathbf{W^s} \cdot \mathbf{s_{i-1}} + \mathbf{W^x} \cdot \mathbf{x_i})$

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$$R_{SRNN}(\mathbf{s_{i-1}}, \mathbf{x_i}) = tanh(\mathbf{W^s} \cdot \mathbf{s_{i-1}} + \mathbf{W^x} \cdot \mathbf{x_i})$$

- Looks very simple.
- Theoretically very powerful.
- In practice not so much (hard to train).
- Why? Vanishing gradients.

Simple RNN (Elman RNN)

$$R_{SRNN}(\mathbf{s_{i-1}}, \mathbf{x_i}) = tanh(\mathbf{W^s} \cdot \mathbf{s_{i-1}} + \mathbf{W^x} \cdot \mathbf{x_i})$$

Another view on behavior:

- RNN as a "computer": input xi arrives, memory s is updated.
- In the Elman RNN, entire memory is written at each time-step.

LSTM RNN

better controlled memory access

continuous gates

Differentiable "Gates"

- The main idea behind the LSTM is that you want to somehow control the "memory access".
- In a SimpleRNN:

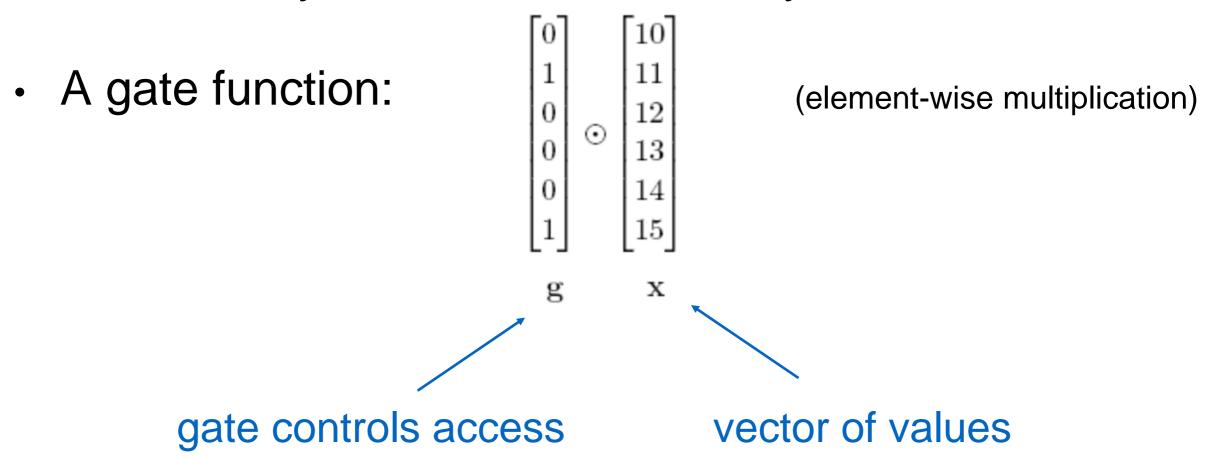
$$R_{SRNN}(\mathbf{s_{i-1}}, \mathbf{x_i}) = tanh(\mathbf{W^s} \cdot \mathbf{s_{i-1}} + \mathbf{W^x} \cdot \mathbf{x_i})$$

read previous state memory write new input

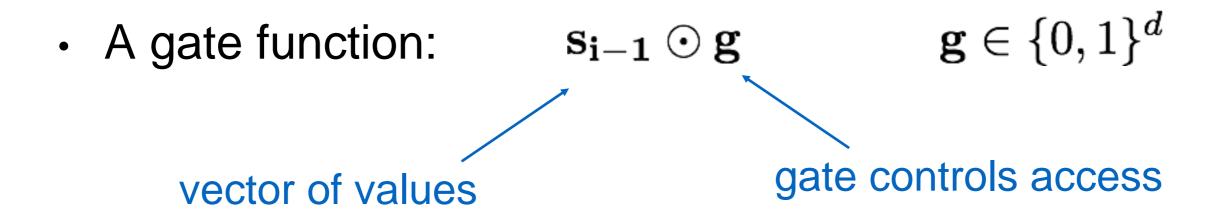
All the memory gets overwritten

- We'd like to:
 - * Selectively read from some memory "cells".
 - * Selectively write to some memory "cells".

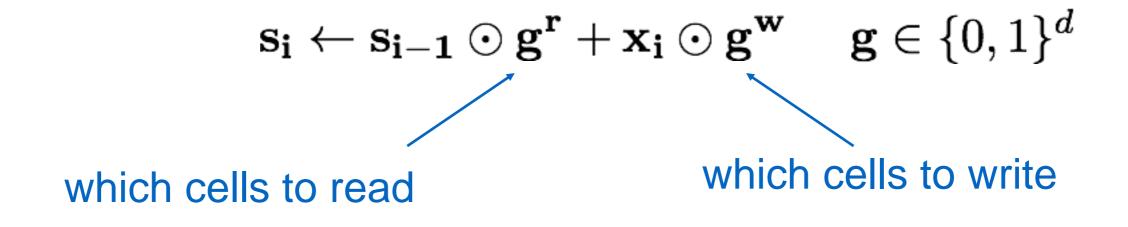
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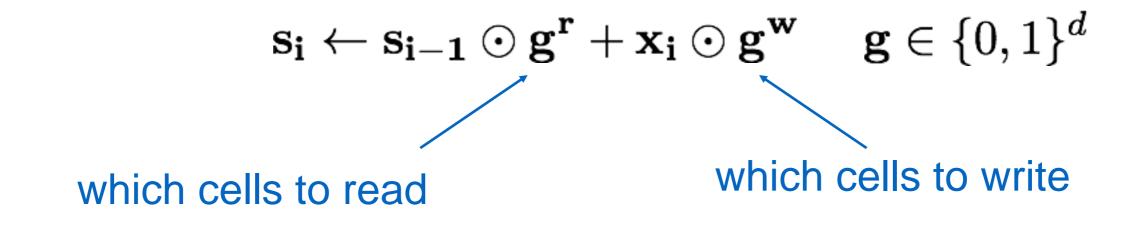
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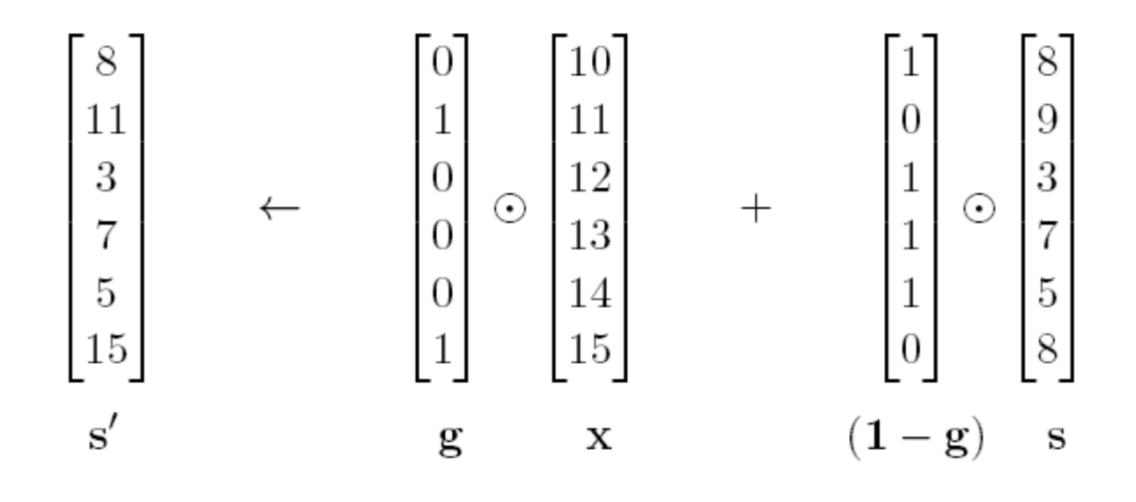
Using the gate function to control access:



Using the gate function to control access:



• (can also tie them: $\mathbf{g}^{\mathbf{r}} = 1 - \mathbf{g}^{\mathbf{w}}$)



Differentiable "Gates"

• Problem with the gates:

- * they are fixed.
- * they don't depend on the input or the output.

Differentiable "Gates"

• Problem with the gates:

- * they are fixed.
- * they don't depend on the input or the output.
- Solution: make them smooth, input dependent, and trainable. $\mathbf{g}^{\mathbf{r}} = \sigma(\mathbf{W} \cdot \mathbf{x}_{i} + \mathbf{U} \cdot \mathbf{s}_{i-1})$

"almost 0" or "almost 1"

The LSTM is a specific combination of gates.

 $R_{LSTM}(\mathbf{s_{j-1}}, \mathbf{x_j}) = [\mathbf{c_j}; \mathbf{h_j}]$ $\mathbf{c_j} = \mathbf{c_{j-1}} \odot \mathbf{f} + \mathbf{g} \odot \mathbf{i}$

$$i = \sigma(\mathbf{W^{xi}} \cdot \mathbf{x_j} + \mathbf{W^{hi}} \cdot \mathbf{h_{j-1}})$$
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The GRU is a different combination of gates.

$$\begin{split} \mathbf{s_j} &= R_{\mathrm{GRU}}(\mathbf{s_{j-1}}, \mathbf{x_j}) = (\mathbf{1} - \mathbf{z}) \odot \mathbf{s_{j-1}} + \mathbf{z} \odot \tilde{\mathbf{s_j}} \\ & \mathbf{z} = \sigma(\mathbf{x_j} \mathbf{W^{xz}} + \mathbf{s_{j-1}} \mathbf{W^{sz}}) \\ & \mathbf{r} = \sigma(\mathbf{x_j} \mathbf{W^{xr}} + \mathbf{s_{j-1}} \mathbf{W^{sr}}) \\ & \tilde{\mathbf{s_j}} = \tanh(\mathbf{x_j} \mathbf{W^{xs}} + (\mathbf{r} \odot \mathbf{s_{j-1}}) \mathbf{W^{sg}}) \end{split}$$

GRU vs LSTM

- The GRU and the LSTM are very similar ideas.
- Invented independently of the LSTM, almost two decades later.

• The GRU formulation:

$$\mathbf{s_j} = R_{\mathrm{GRU}}(\mathbf{s_{j-1}}, \mathbf{x_j}) =$$

Proposal state: $\tilde{\mathbf{s}_j} = \tanh(\mathbf{x_j}\mathbf{W^{xs}} + (\mathbf{r} \odot \mathbf{s_{j-1}})\mathbf{W^{sg}})$

The GRU formulation:

$$\mathbf{s_j} = R_{\mathrm{GRU}}(\mathbf{s_{j-1}}, \mathbf{x_j}) =$$

gate controlling effect of prev on proposal:

$$\begin{aligned} \mathbf{r} = &\sigma(\mathbf{x_j}\mathbf{W^{xr}} + \mathbf{s_{j-1}}\mathbf{W^{sr}}) \\ &\tilde{\mathbf{s_j}} = &\tanh(\mathbf{x_j}\mathbf{W^{xs}} + \mathbf{\mathbf{\hat{r}}} \odot \mathbf{s_{j-1}})\mathbf{W^{sg}}) \end{aligned}$$

blend of old state and proposal state $\mathbf{s_j} = R_{\text{GRU}}(\mathbf{s_{j-1}}, \mathbf{x_j}) = (1 - \mathbf{z}) \odot \mathbf{s_{j-1}} + \mathbf{z} \odot \tilde{\mathbf{s_j}}$

$$\mathbf{r} = \sigma(\mathbf{x_j}\mathbf{W^{xr}} + \mathbf{s_{j-1}}\mathbf{W^{sr}})$$
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Other Variants

- Many other variants exist.
- Mostly perform similarly to each other.
 - Different tasks may work better with different variants.
- The important idea is the differentiable gates.

LSTM (Long short-term Memory)

• The LSTM is formulation:

$$\begin{aligned} R_{LSTM}(\mathbf{s_{j-1}},\mathbf{x_j}) = & [\mathbf{c_j};\mathbf{h_j}] \\ \mathbf{c_j} = & \mathbf{c_{j-1}} \odot \mathbf{f} + \mathbf{g} \odot \mathbf{i} \\ \mathbf{h_j} = & \tanh(\mathbf{c_j}) \odot \mathbf{o} \\ & \mathbf{i} = & \sigma(\mathbf{W^{xi}} \cdot \mathbf{x_j} + \mathbf{W^{hi}} \cdot \mathbf{h_{j-1}}) \\ & \mathbf{f} = & \sigma(\mathbf{W^{xf}} \cdot \mathbf{x_j} + \mathbf{W^{hf}} \cdot \mathbf{h_{j-1}}) \\ & \mathbf{o} = & \sigma(\mathbf{W^{xo}} \cdot \mathbf{x_j} + \mathbf{W^{ho}} \cdot \mathbf{h_{j-1}}) \\ & \mathbf{g} = & \tanh(\mathbf{W^{xg}} \cdot \mathbf{x_j} + \mathbf{W^{hg}} \cdot \mathbf{h_{j-1}}) \end{aligned}$$

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Recurrent Additive Networks

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$$\mathbf{c_j} = \mathbf{c_{j-1}} \odot \mathbf{f} + \mathbf{g} \odot \mathbf{i}$$

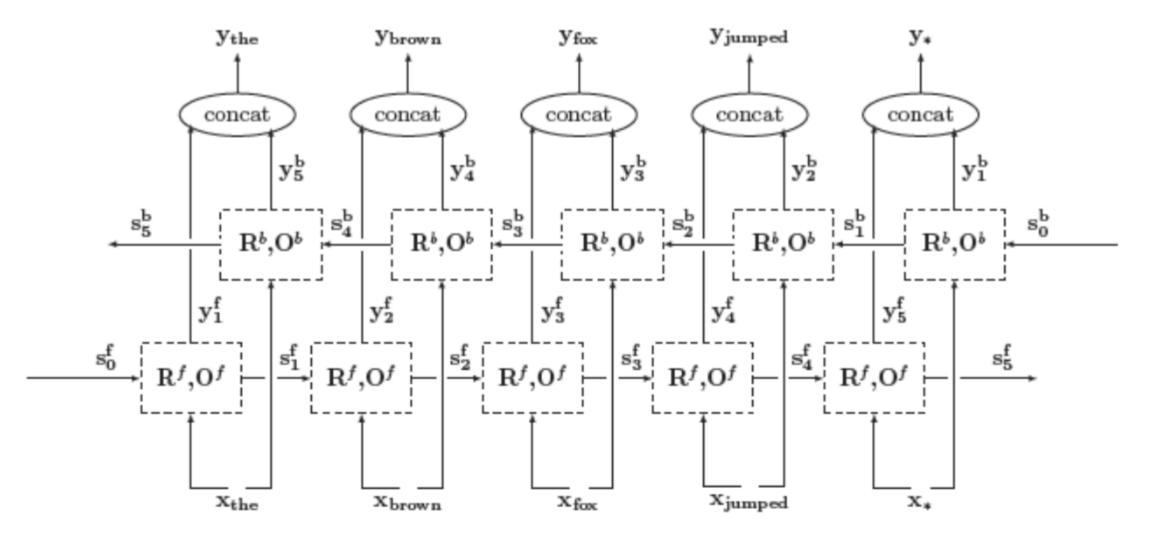
$$\mathbf{h_j} = \tanh(\mathbf{c_j})$$

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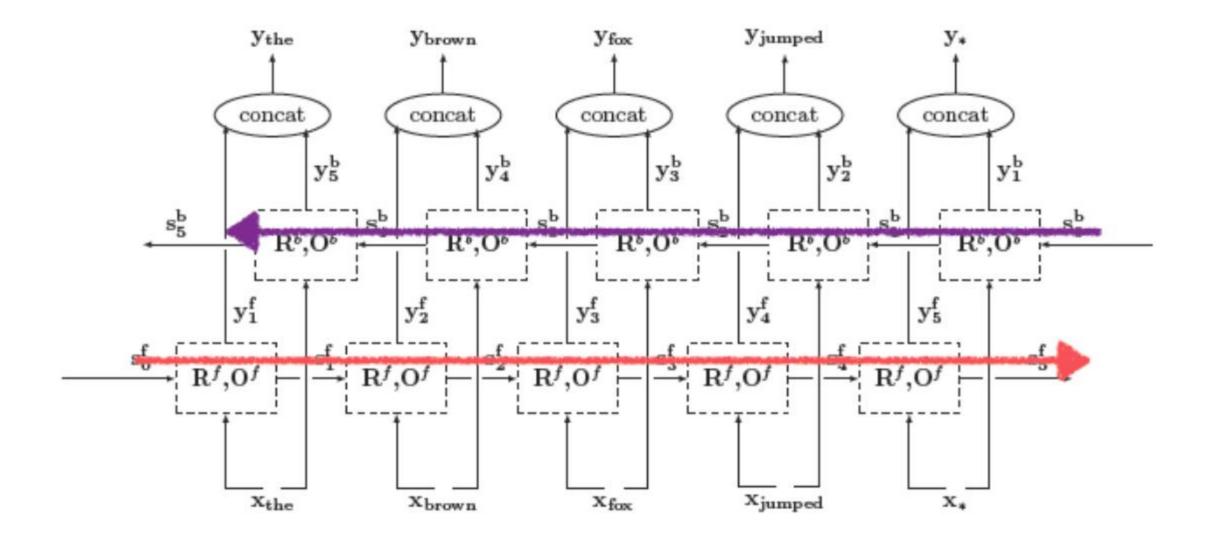
$$\mathbf{f} = \sigma(\mathbf{W^{xf}} \cdot \mathbf{x_j} + \mathbf{W^{hf}} \cdot \mathbf{h_{j-1}})$$

 $\mathbf{g} = \mathbf{W}^{\mathbf{xg}} \cdot \mathbf{x_j}$

Bidirectional LSTMs

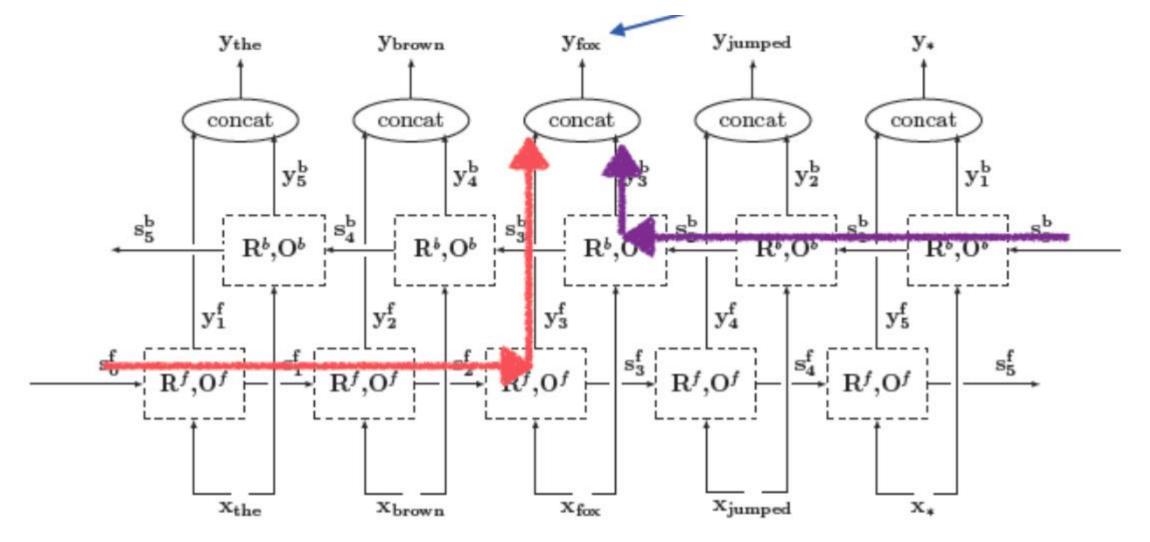


One RNN runs left to right. Another runs right to left. Encode **both future and history** of a word.



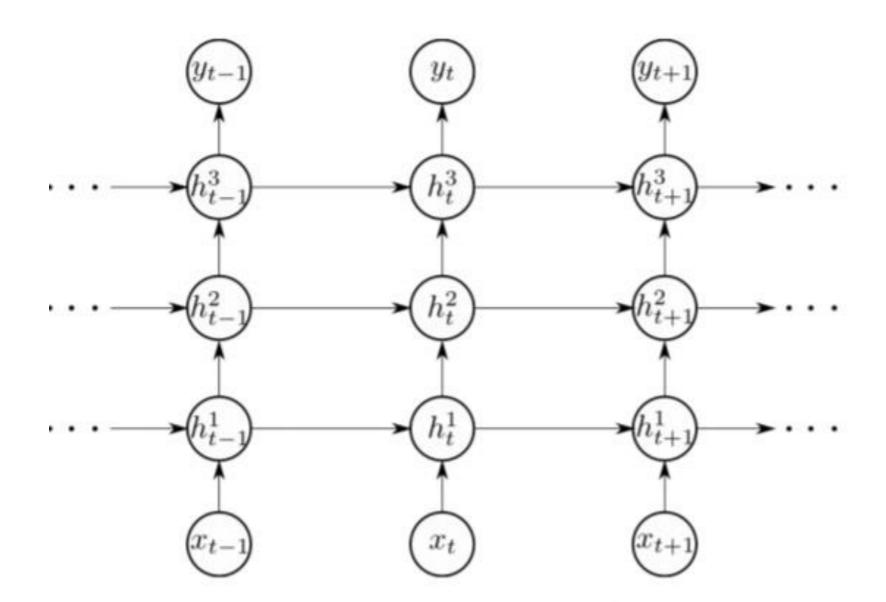
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Infinite window around the word



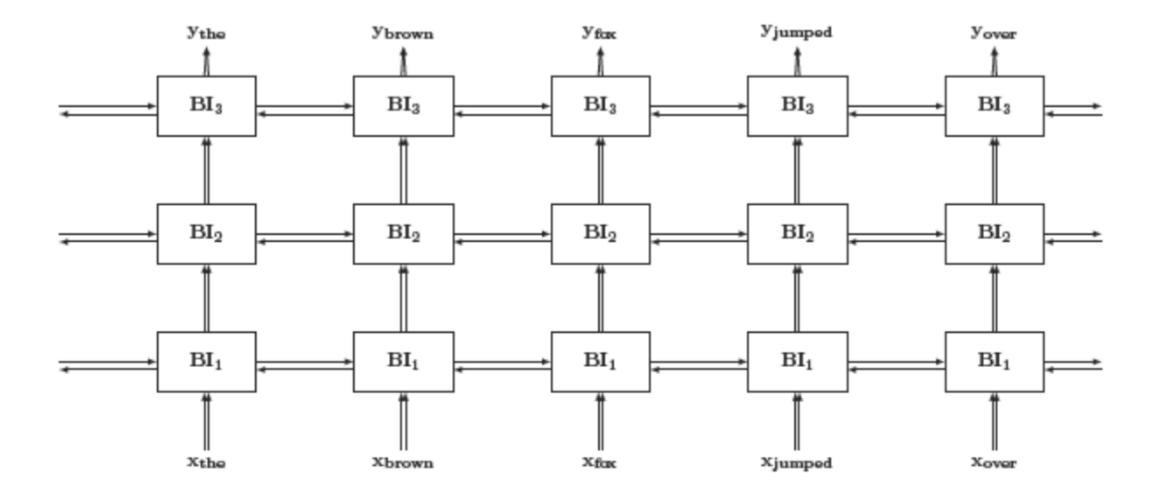
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Deep LSTMs



(a) Conventional stacked RNN

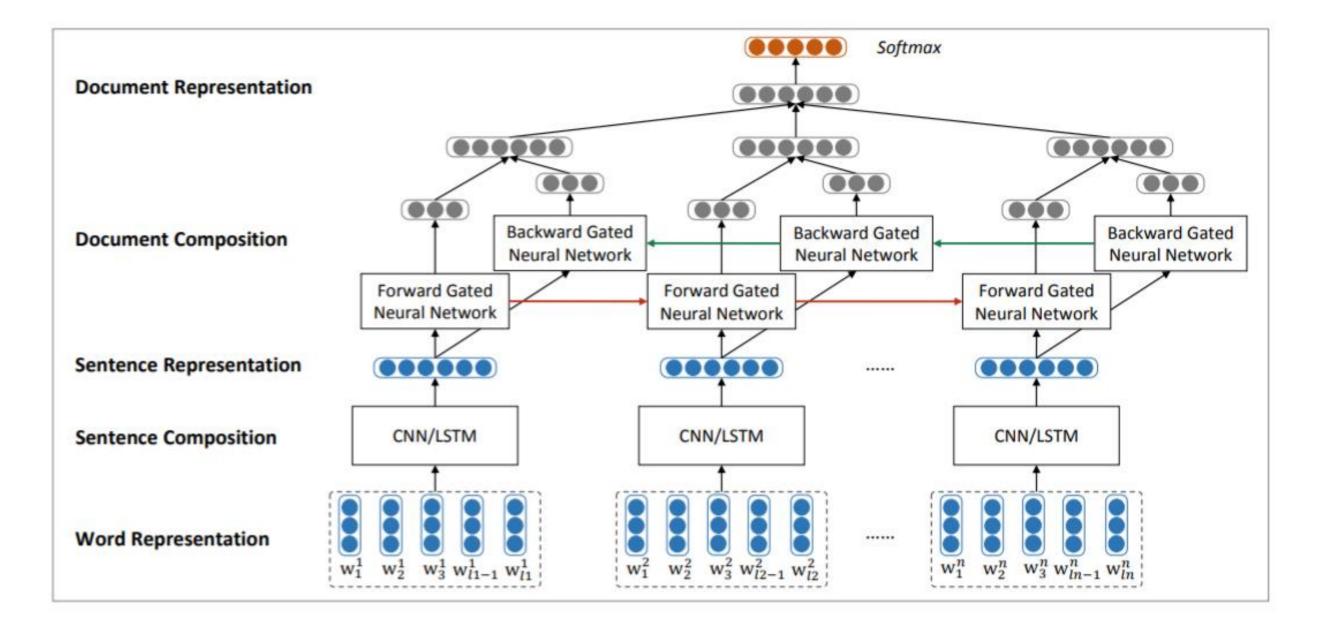
Deep Bi-LSTMs



Read More

- The gated architecture also helps the vanishing gradients problems.
- For a good explanation, see Kyunghyun Cho's notes: <u>http://arxiv.org/abs/1511.07916</u> sections 4.2, 4.3
- Chris Olah's blog post

Hierarchical RNN for Doc Classification



Tang et al 15